Quantum effects and quantum paradoxes
Exercise sheet 10

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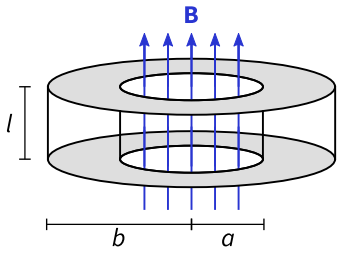
1. Aharonov–Bohm effect in bound states $(5+5+5+5+10)$

A particle with charge e and mass m is located in a hollow cylinder such that $a^2 \leq x^2+y^2 \leq b^2$ and $|z| \leq l/2$. Inside of the cylinder ($x^2+y^2 < a^2$) exists a magnetic field $\mathbf{B} = B_0\mathbf{e}_z$. Otherwise, the magnetic field is zero, i.e. $\mathbf{B} = \mathbf{0}$. We are interested in the wave function $\psi(\vec{x},t)$ in the cylinder, i.e. for $a^2 \leq x^2+y^2 \leq b^2$ and $|z| \leq l/2$.



(a) Determine the Hamiltonian of the particle in cylindrical coordinates.

(b) Assume that $l = a = b/2$ and make the following ansatz to solve the Schrödinger equation:

$$\psi(r,\phi,z) = \sin(\pi\alpha r/l)\mathrm{e}^{i\beta\phi}\cos(\pi\gamma z/l)r^u/N,$$

where α, β, γ are quantum numbers and u, N are constant.

Hint: Use the dimensionless quantities

$$\lambda = \frac{a^2 e}{\hbar}B_0 \quad \text{and} \quad \epsilon = \frac{l^2 m}{\pi^2\hbar^2}E_{\alpha,\beta,\gamma},$$

where $E_{\alpha,\beta,\gamma}$ is an eigenvalue of the Hamiltonian.

(c) The wave function has to be well-defined and has to fulfil the boundary conditions. Which conditions does this imply on α, β and γ?

(d) Determine u and ϵ as a function of α, β and γ.

(e) Which values of λ are allowed for this type of solution? In what sense does this confirms the effect of $\mathbf{B}_0$ on the solution?