Quantum effects and quantum paradoxes
Exercise sheet 9

Lecture: PD. Dr. M. Kleinmann
Exercises: Jan Bönsel

Due: Thursday, 13.01.2022

---

1. Change of the wave function under canonical transformations $(10 + 10)$

(a) We consider a solution $\psi(x,t)$ of the free Hamiltonian $H = \frac{\hat{p}^2}{2m}$. Carry out the canonical transformation which maps $\hat{p} \mapsto -\hat{x}$ and $\hat{x} \mapsto \hat{p}$. How does the wave function $\psi(x,t)$ change?

(b) Consider a particle in an electromagnetic field, which is described by the Schrödinger equation

$$\left[\frac{1}{2m}(-i\hbar\boldsymbol{\nabla} - e\mathbf{A}(\vec{x},t))^2 + e\Phi(\vec{x},t)\right]\psi(\vec{x},t) = i\hbar\partial_t\psi(\vec{x},t). \tag{1}$$

A gauge transformation changes the potentials such that

$$\mathbf{A} \mapsto \mathbf{A}' = \mathbf{A} + \boldsymbol{\nabla}\chi \quad \text{and} \quad \Phi \mapsto \Phi' = \Phi - \partial_t\chi.$$

Determine the effect of the gauge transformation on the wave function.
Hint: Start by multiplying the Schrödinger equation in Eq. (1) with $e^{\frac{ie}{\hbar}\chi(\vec{x},t)}$ from the left.

Merry Christmas and a happy New Year!