Quantum effects and quantum paradoxes
Exercise sheet 2

Lecture: PD. Dr. M. Kleinmann — Due: Thursday, 04.11.2021
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1. Projectors (10)
Suppose that the three projectors P_1, P_2 and P_3 form a partition of unity, i.e. $P_1 + P_2 + P_3 = \mathbf{1}$. Show that $P_1 P_2 = 0$.

2. Entanglement (10)
Are the following states entangled or separable? For this purpose, determine the coefficient matrix (ψ_{ij}) and calculate its rank. In case the state is separable, write down the state in the form $|\psi\rangle = |a\rangle\,|b\rangle$.

$$\begin{aligned}
|\psi_1\rangle &= |00\rangle + |11\rangle \\
|\psi_2\rangle &= |00\rangle + |01\rangle + |10\rangle + |11\rangle \\
|\psi_3\rangle &= |00\rangle + |01\rangle + |10\rangle - |11\rangle \\
|\psi_4\rangle &= |00\rangle + |01\rangle + |10\rangle
\end{aligned}$$

3. Local hidden variable model ($10 = 5 + 5$)
We consider the 3-qubit GHZ state $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$.

(a) Show that the GHZ state $|\mathrm{GHZ}\rangle$ is an eigenstate of the operators $M_1 = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)} := \sigma_x \otimes \sigma_y \otimes \sigma_y$, $M_2 = \sigma_y^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}$, $M_3 = \sigma_y^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}$ and $M_4 = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}$. Determine the corresponding eigenvalues.

(b) Assume that the measurement results of the observables $\sigma_x^{(1)}$, $\sigma_x^{(2)}$, $\sigma_x^{(3)}$ and also of $\sigma_y^{(1)}$, $\sigma_y^{(2)}$, $\sigma_y^{(3)}$ are described by some local hidden variable model. Within the model the observables σ_α^i have preset values $s_\alpha^i \in \{-1, +1\}$. Is there a choice of the values s_α^i which is consistent with the measurement outcomes of the GHZ state for the observables in (a)?