Theory of Quantum Matter

Lecturer: Prof. Otfried Gühne (Mon 14:00, Fri 10:00, Room D120)
Exercises: Chau Nguyen (Fri 12:30, Room D201) **Sheet 10**

Hand in: not required
Discussion date: not required

Questions

1. What is a lattice? What is the reciprocal lattice?
2. Define the Brillouin zone. Can you define the Fourier transformation on a lattice?
3. What is the consequence of periodic boundary conditions?
4. Describe the Born-Oppenheimer approximation.
5. What is a phonon? Give the resulting Hamiltonian. How can one compute the dispersion relation?
6. What are optical and acoustic phonons? How are they related to the dimension of the lattice?
7. Define the phonon density. Why is this quantity useful?
8. What is the canonical ensemble? What is the partition function of it? Why is the partition function relevant?
9. Define the Debye model and the Einstein model. What are their central predictions? Are the models correct?
10. Formulate the Bloch theorem. What is the corresponding reduced Schrödinger equation in Fourier space?
11. How does a weak periodic potential influence the relation $\epsilon(k)$ for free electrons? Does this explain the occurrence of conductors and insulators?
12. Formulate the main idea of the tight-binding approximation.
13. What are Wannier functions?
14. How can one define an effective mass and the velocity of electrons in a periodic potential?
15. What are bosons and fermions? Why does one need to (anti-)symmetrize the wave function?
16. What is the Fock space?
17. Describe the action of creation and annihilation operators. What are their commutation or anticommutation relations?
18. How can one translate a one- or two-body operator to Fock space?
19. Describe the Jellium model. What is the Fermi surface?
20. What is the idea behind the Hartree-Fock approximation?
21. Describe the Wannier basis and the tight-binding approximation in second quantization. Can you give an example, where the tight-binding approximation is reasonable?

22. What is graphene? Why is it interesting? What is the Dirac cone?

23. Describe three typical interactions between electrons which occur, if the spin of the electron is considered.

24. Give the Hamiltonian of the Hubbard model. Describe the two typical phases of the model.

25. How can an antiferromagnetic interaction occur between spins? Describe the idea of the underlying superexchange mechanism.

26. Write down the Hamiltonian of the Heisenberg model and of the Ising model.

27. What is the mean field approximation for the Heisenberg model? What can be proven with it?

28. Describe the main idea behind the Jordan-Wigner transformation. Give the formula of it.

29. How can one solve the XY model with a magnetic field? What is meant by "solving" a model?