Quantum Theory of Light (WS17/18)

Exercises 4

(For exercise class on Thu, February 1st 2018.)

Lectures: Matthias Kleinmann, Otfried Gühne, Mondays 8:30 a.m., room B205
Exercise classes: Ana Costa, Thursdays 12:30 p.m., room B019

1. **Swapping**
   Consider two two-level atoms coupled via the Hamiltonian

   $$H_I = \hbar\Omega(a \otimes a^\dagger + a^\dagger \otimes a),$$

   where $a = |g\rangle\langle e|$.

   (a) Compute the "super operators" $\Lambda(t)$ for $\rho_A(t) = \Lambda(t)\rho_A(0)$, where $\rho_A = \mathrm{tr}_B(\rho)$ is the reduced state of the first atom and $\rho(0) = \rho_A(0) \otimes |g\rangle\langle g|$.

   (b) Show, that $\Lambda(t)$ is completely positive, for each t.

   (c) Find an instance, where $\Lambda(t + t') = \Lambda(t)\Lambda(t')$ does not hold.

2. **Dephasing**
   Consider the dephasing channel, where, for $t/\tau \geq 0$,

   $$\rho(t) = \Lambda(t)\rho(0) = \begin{pmatrix} a & \mathrm{e}^{-t/\tau}\, b \\ \mathrm{e}^{-t/\tau}\, b^* & 1-a \end{pmatrix}.$$

   Follow the steps of from the lecture to arrive at a master equation of the form

   $$\dot\rho(t) = -\frac{i}{\hbar}[H, \rho] + \sum_k \gamma_k \left(A_k \rho A_k^\dagger - \frac{1}{2}\{A_k^\dagger A_k, \rho\} \right), \qquad (*)$$

   with H Hermitean and all $\gamma_k > 0$.

3. **Decay**
   The master equation for a two-level atom in the form $(*)$ can be approximated by letting

   $$H = \frac{\hbar}{2}\omega(|e\rangle\langle e| - |g\rangle\langle g|),$$

   and having only two terms in the sum with

   $$A_1 = |g\rangle\langle e| \text{ and } A_2 = |e\rangle\langle g|.$$

   Compute $\rho(t)$ and interpret the result. (Hint: Use the Bloch representation for the density matrix, $\vec v(t) = \mathrm{tr}(\vec\sigma \rho(t))$.)