Mathematical Methods in Quantum Theory WS 2020/2021

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Sheet 5 — Hand-Out: Wed, 13.01.2021 — Due: Wed, 20.01.2021

Exercise 15: Purification

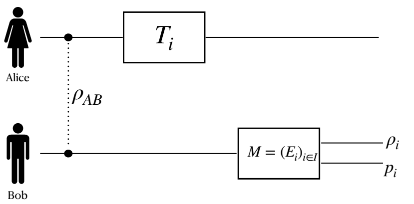
Let $\mathcal{H}$ be a finite dimensional Hilbert space and $\rho \in \mathcal{L}(\mathcal{H})$ a density matrix. Show that there exists another Hilbert space $\mathcal{H}_B$ such that one can find a pure state $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ with $\mathrm{tr}_B(|\psi\rangle\langle\psi|) = \rho$.

Exercise 16: No-cloning theorem

Let $\mathcal{H}$ be a finite dimensional Hilbert space and $\rho \in \mathcal{L}(\mathcal{H})$ a quantum state. Prove that there cannot exist a channel $\mathcal{C} : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H} \otimes \mathcal{H})$ such that $\mathcal{C}(\rho) = \rho \otimes \rho$ for all $\rho \in \mathcal{L}(\mathcal{B})$.

Exercise 17: No-signalling in QM

Consider the following situation. Two parties, say Alice and Bob, share a quantum state $\rho_{AB} \in \mathcal{L}(\mathcal{H}_A \otimes \mathcal{B})$. Assume that Bob can make a measurement $M = (E_i)_{i \in I}$ with $|I| = n < \infty$. Further, Alice has access to a set of channels $(T_j)_{j \in J}$ with $|J| = m$. Show that the probability distribution of Bob p_B does not depend on the choice of channel which Alice applies to her system.



Exercise 18: Quantum instruments

Let $\mathcal{I}$ be a quantum instrument. Prove that if $\tilde{\rho} = \rho$ for all $\rho \in \mathcal{L}(\mathcal{H})$, then the outcome distribution p_x is independent of ρ. Why is this statement also called the *no information without disturbance* theorem ?