Quantum theory of light

Lecturer: Matthias Kleinmann (Tue 14:15, Room B030)
Exercises: Chau Nguyen (Mon 16:15, Room D120) **Sheet 11**

Hand in: Tue 21.01.2020 *(questions marked as $*$ are optional)*
Discussion date: Mon 27.01.2020

24. Coupled qubits

Consider two two-level atoms A and B coupled via the Hamiltonian

$$H = \hbar\Omega(a \otimes a^\dagger + a^\dagger \otimes a), \tag{1}$$

where $a = |g\rangle\langle e|$

(a) [5pts] Compute the super operator $\Lambda(t)$ for atom A, $\rho_A(t) = \Lambda(t)\rho_A(0)$ with the initial condition $\rho(0) = \rho_A(0) \otimes |g\rangle\langle g|$. On which basis one can conclude that $\Lambda(t)$ is completely positive?

(c) [5pts] Show that $\Lambda(t + t') = \Lambda(t)\Lambda(t')$ does not hold.

25. Dephasing process

[10pts] Consider the dephasing channel, where, for $t > 0$,

$$\rho(t) = \Lambda(t)[\rho(0)] = \begin{pmatrix} a & e^{-t/\tau}b. \\ e^{-t/\tau}b^* & 1-a \end{pmatrix}. \tag{2}$$

Follow the steps as in the lectures to arrive at a master equation of the form

$$\dot{\rho}(t) = \frac{-i}{\hbar}[H,\rho] + \sum_k \gamma_k \left(A_k\rho A_k^\dagger - \frac{1}{2}\left\{A_k^\dagger A_k, \rho\right\}\right), \tag{3}$$

with H hermitian and $\gamma_k \geq 0$.

26. The decaying process

[5pts] Consider a two-level atom with the Hamiltonian

$$H = \frac{\hbar\omega}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right) \tag{4}$$

and two single Lindblad operators

$$A_1 = |g\rangle\langle e|, A_2 = |e\rangle\langle g|. \tag{5}$$

Write down and solve the Lindblad equation.