Quantum theory of light

Lecturer: Matthias Kleinmann (Tue 14:15, Room B030)
Exercises: Chau Nguyen (Mon 16:15, Room D120) **Sheet 10**

Hand in: Tue 14.01.2020 *(questions marked as $*$ are optional)*
Discussion date: Mon 20.01.2020

22. Rivial of the Rabi oscillations

Consider the Jaynes–Cummings model at exact resonance

$$H = \frac{1}{2}\hbar\omega\sigma_z + \hbar\omega a^\dagger a + \hbar\lambda(\sigma_+ a + \sigma_- a^\dagger). \tag{1}$$

The solution for the Schrödinger equation with the atom initially in the excited state $|e\rangle$ and the field in a general state $\sum_{n=0}^{\infty} C_n |n\rangle$ is given by

$$|\psi(t)\rangle = |\psi_g(t)\rangle |g\rangle + |\psi_e(t)\rangle |e\rangle , \tag{2}$$

where $|\psi_g(t)\rangle = -i\sum_{n=0}^{\infty} C_n \sin(\beta t\sqrt{n+1}) |n+1\rangle$ and $|\psi_e(t)\rangle = \sum_{n=0}^{\infty} C_n \cos(\beta t\sqrt{n+1}) |n\rangle$.

(a) [$*$,5pts] Verify that (2) is indeed the solution to the Schrödinger equation of the Hamiltonian (1).

(b) [5pts] Find the atomic inversion $W(t)$ for case where the field is initially in a number state $|m\rangle$. Sketch $W(t)$ over time.

(c) [5pts] Find the atomic inversion $W(t)$ for case where the field is initially in a coherent state $|\alpha\rangle$. Sketch $W(t)$ over time with a suitable software of your choice.

(d) [5pts] Derive the density operator for the atom at general time t from the solution (2).

23. Dressed states

Consider again the Jaynes–Cummings Hamiltonian in the Schrödinger picture

$$H = \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega a^\dagger a + \hbar\beta(\sigma_+ a + \sigma_- a^\dagger), \tag{3}$$

where we do not assume exact resonance, $\omega \neq \omega_0$.

(a) [5pts] Show that if the field is initially in a number state $|m\rangle$, the dynamic of the system is restricted to a two-dimensional subspace. Derive the reduced Hamiltonian in this two-dimensional subspace.

(b) [5pts] Find the energy levels and the eigenstates of the derived Hamiltonian.

(c) [5pts] Assuming exact resonace, $\omega = \omega_0$, compute the density operator for the atom when the system is the energy eigenstates.