Quantum theory of light

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Exercises: Chau Nguyen (Mon 16:15, Room D120) **Sheet 8**

Hand in: Tue 17.12.2019 *(questions marked as ∗ are optional)*
Discussion date: Mon 06.01.2020

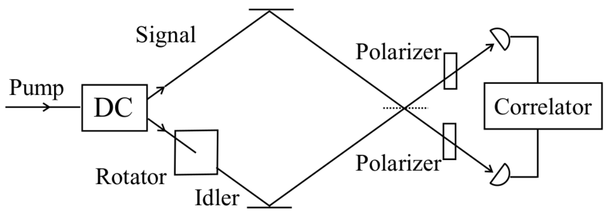
In this sheet, we discuss some remarkable experiments in quantum optics that stir our classical intuition and that touch the foundation of quantum mechanics, namely, the quantum eraser and the induced coherence experiment. Basis to these experiments is the down conversion process that have been discussed in the parametric approximation in Problem 15. Here, we consider the case where the pump consists of a single photon. Sweeping all the technical details under the carpet, the down conversion (type I) process with the Hamiltonian in Problem 15 is simply given by

$$|1\rangle_p |0\rangle_s |0\rangle_i \to \gamma |0\rangle_p |1\rangle_s |1\rangle_i \,, \tag{1}$$

where a photon in the pump is converted by a nonlinear crystal to two photons in two beams of identical polarisation called the *signal* and the *idler* (naming is purely due to historical reason.) Here γ denotes the conversion rate. More details of the process are irrelevant to our discussion.

19. Quantum eraser

The following setup describes the Hong-Oh-Mandel interference as discussed in Problem 18. It was crucial in this experiment that the two incoming beams are of identical polarisation (say horizontal) so that the two photons are actually *indistinguishable*. In comparison to Problem 18, here we also include the source that generates the two photons for the completeness (the down converter *DC*.) Moreover, a rotator is inserted into one of the incoming beam to rotate the polarisation of the field by an angle θ. Also two polarisers are used to filter light with horizontal polarisation before coming into the detectors.



(a) [10pts] Suppose the polarizers do not present, compute the coincident probability at the two detectors.

Hint: The rotator implements the transformation of the state $|H\rangle \to |\theta\rangle = \cos\theta\,|H\rangle + \sin\theta\,|V\rangle$, where $|H\rangle$ and $|V\rangle$ are the single photon in the horizonal and vertical polarisation.

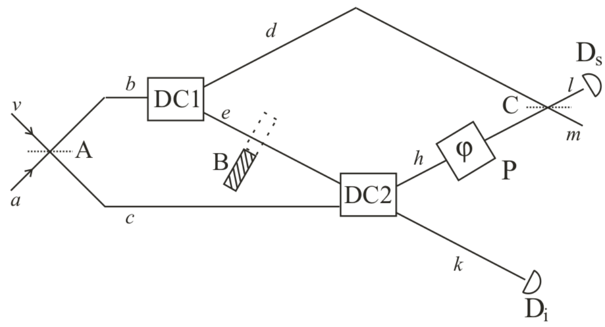
(b) [∗, 10pts] Consider now the full setup with the polarisers in their places. Compute the coincident probability.

Hint: Quantum mechanically, the polarisers implement a measurement projecting the states of the field back into the horizontal polarisation.

20. Induced coherence

[10pts] The following setup implements an experiment which is regared as 'mind boggling'.[1]

[1] Greenberge, Horn and Zeilinger, Phys. Today 1993, p22.



A photon is injected in mode a, which is splited into two modes b and c by a beam spliter A. Each of the mode b and c excites a down converter, $DC1$ and $DC2$, respectively. The outcome signal mode d and h meet at a beam spliter C and the detector D_s is to detect the interference. The interference is observed by inserting a refractive meadia P giving a phaseshift on the photon, $|0\rangle_h \to |0\rangle_h$, and $|1\rangle_h \to e^{i\varphi}|1\rangle_h$, and observe the dependence of the intensity measured at D_s on the variable phase φ. Here B is to block the transmission of the idler beam e. The notable feature is that without B, the idler mode e is *exactly aligned* with the idler mode k.

Compute the probability of detecting the photon at detector D_s and D_i if the mode e is blocked and not blocked. What would be 'mind boggling' to you?