Quantum theory of light

Lecturer: Matthias Kleinmann (Tue 14:15, Room B030)
Exercises: Chau Nguyen (Mon 16:15, Room D120) **Sheet 7**

Hand in: Tue 10.12.2019 *(questions marked as ∗ are optional)*
Discussion date: Mon 16.12.2019

16. Coherence functions

For simplicity, we consider a one-dimensional cavity of length L and concentrate on one polarisation of the field. Recall that the two-point correlation function is defined by

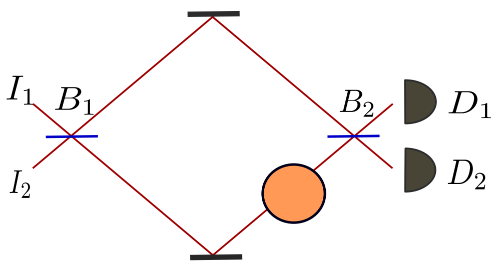
$$G^{(1)}(x_1, x_2) = \mathrm{Tr}[\rho E^{(-)}(x_1) E^{(+)}(x_2)]. \quad (1)$$

where $E^{(+)}(x)$ and $E^{(-)}(x)$ are the positive and negative frequency parts of the field operator as defined in the lecture. The first order coherence function is defined as $g^{(1)}(x_1, x_2) = G^{(1)}(x_1, x_2)/\sqrt{G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2)}$.

(a) [5pts] Compute the first order coherence function for a two-mode coherent state $|\alpha_{k_1}, \alpha_{k_2}\rangle$ at two wave vectors k_1 and k_2.

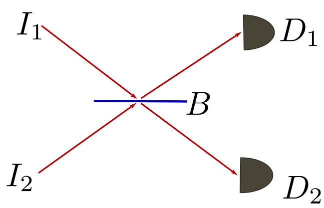
(b) [∗, 10pts] Compute the first order coherence function for the thermal state at temperature T in the high temperature limit. Discuss the physical implication in the interference experiment.

17. Elitzur–Vaidman bomb tester



[5pts] A manufacturer produces a bomb which is so sensitive that a single photon can ignite it to explode. Unfortunately, for 50% of the cases, the engineers forget to install the trigger, making it a dud. The manufacturer wants to test the bombs to make sure that any bomb that goes out of the test is surely not a dud. Here is how they can do it: Consider the Mach–Zehnder interferometer with balanced beam-splitters B_1 and B_2 as sketched at the right. The bomb (round, orange) is inserted into the lower arm of the interferometer. Show that if a single photon is fed into the first channel I_1, whenever counter D_1 clicks, the bomb does not explode even though one can be sure that the trigger presents. Compute the fraction of bombs that past the test. What if a coherent state is fed into the first channel I_1?

18. Hong–Oh–Mandel interference



Two light beams with identical polarisation and frequency I_1 and I_2 are sending to a balanced beam-splitter (50/50) B as illustrated below. Two detectors D_1 and D_2 are placed to detect the outgoing light.

(a) [5pts] Show that if the incoming beams are in the single photon states, then the coincidence rate of the detectors vanishes.

(a) [5pts] Compute the counting values of the detectors for the case the incoming beams are states of higher numbers of photons.

(a) [5pts] Compute the counting values of the detectors for the case the incoming beams contain two coherent states.