Quantum theory of light

Lecturer: Matthias Kleinmann (Tue 14:15, Room B030)
Exercises: Chau Nguyen (Mon 16:15, Room D120) **Sheet 4**

Hand in: Tue 12.11.2019 *(questions marked as ∗ are optional)*
Discussion date: Mon 18.11.2019

8. Some physical properties of the coherent states

[5pts] Compute the mean electromagnetic field for a coherent state $|\alpha\rangle$ to convince yourselves that it is closed to the notion of a classical electromagnetic wave.

9. Representation of operators by the coherent state basis

[∗, 5pts] As coherent states form a basis, it can be used to represent state vectors and operators. One has to be careful, though: since the basis is non-orthogonal and over-compelete, the representation does not always behave in the same way as for orthonormal bases.

Let $F(a, a^\dagger)$ is a operator polynomial in a and $a^\dagger$. Show that

$$F(a, a^\dagger) = \frac{1}{\pi^2} \int d\alpha \int d\beta \exp\left[-\frac{1}{2}(|\alpha|^2 + |\beta|^2)\right] F(\beta^*, \alpha) |\beta\rangle \langle\alpha| . \tag{1}$$

10. Optical cat states

Coherent states and their poissonian distribution are regarded as being 'classical'. 'Cat states', as Schrödinger introduced, are superpositions of (macroscopically) classical states. Consider the superpositions

$$|\psi_e\rangle = \frac{1}{N_e} (|\alpha\rangle + |-\alpha\rangle) , \tag{2}$$

$$|\psi_o\rangle = \frac{1}{N_o} (|\alpha\rangle - |-\alpha\rangle) , \tag{3}$$

which are known as even and odd optical cat state, respectively. These are to be distinguished with the statistical mixture

$$\rho_c = \frac{1}{2}(|\alpha\rangle \langle\alpha| + |-\alpha\rangle \langle-\alpha|). \tag{4}$$

(a) [5pts] Compute the normalisation factors N_e and N_o.

(b) [5pts] Compute the electromagnetic field for the three states.

(c) [10pts] Compute the photon statistics for the three states, in particular, the Fano factors $F = (\langle n^2\rangle - \langle n\rangle^2)/\langle n\rangle$. Conclude if the the statistics are poissonian ($F = 1$), subpoissonian ($F < 1$) or superpoissonian ($F > 1$).