Quantum theory of light

Lecturer: Matthias Kleinmann (Tue 14:15, Room B030)
Exercises: Chau Nguyen (Mon 16:15, Room D120) **Sheet 3**

Hand in: Tue 05.11.2019 *(questions marked as $*$ are optional)*
Discussion date: Mon 11.11.2019

6. The thermal state

[5pts] Consider a single mode field of frequency ω in the thermal state at temperature T. Show that the photon statics (probably of detecting n photons when counting) is given by

$$P(n) = \frac{1}{1+\bar{n}} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n, \tag{1}$$

where $\bar{n} = 1/(e^{\hbar\omega/kT} - 1)$ is the mean photon number. This shows that at any temperature, the thermal state is very different from the single photon state of light. Compute the photon fluctuation $\langle n^2 \rangle - \langle n \rangle^2$ for the thermal state.

7. Properties of the displacement operator and coherent states

Recall that the displacement operator is given by $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ and the coherence state is give by $|\alpha\rangle = D(\alpha)|0\rangle$.

(a) [5pts] Compute $[a, D(\alpha)]$. From the result, deduce that $a|\alpha\rangle = \alpha|\alpha\rangle$.

(b) [5pts] Compute the mean photon number $\langle n \rangle$, the photon fluctuation $\langle n^2 \rangle - \langle n \rangle^2$, and the full photon statistics (probably of detecting n photons when counting) for the coherent state $|\alpha\rangle$.

(c) [$*$, 5pts] Show that all right-eigenvectors of a are coherent states.

(d) [5pts] Show that $D(\alpha)D(\beta) = \exp[i\mathrm{Im}(\alpha\beta^*)]D(\alpha+\beta)$.

(d) [5pts] Show that

$$(-1)^n |\alpha\rangle = |-\alpha\rangle \tag{2}$$

where $n = a^\dagger a$ is the number operator.

(e) [$*$, 5pts] Show that

$$a^\dagger |\alpha\rangle \langle\alpha| = \left(\alpha^* + \frac{\partial}{\partial\alpha} \right) |\alpha\rangle \langle\alpha| \tag{3}$$