

Aufgabe 1: Tensorprodukt II

a) Stellen Sie die folgenden Zustände in der Standardbasis dar.

i) $|\psi\rangle = |+++\rangle$

ii) $|\psi\rangle = |\Psi^-\rangle |\Phi^+\rangle |y^+\rangle$, mit $|y^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

b) Zeigen Sie, dass für alle 3-Qubit Zustände $|\psi\rangle$ gilt, $\langle\psi^*|\sigma_y \otimes \sigma_y \otimes \sigma_y|\psi\rangle = 0$

Aufgabe 2: The mean king's problem

“A ship wrecked physicist gets stranded on a far-away island that is ruled by a mean king who loves cats and hates physicists since the day when he first heard what happened to Schrödinger's cat. A similar fate is awaiting the stranded physicists. Yet, mean as he is, the king enjoys defeating physicists on their own turf, and therefore he maliciously offers an apparently virtual chance of rescue.

He takes the physicist to the royal laboratory, a splendid place where experiments of any kind can be performed perfectly. There the king invites the physicist to prepare a certain silver atom in any state she likes. The king's men will then measure one of the three cartesian spin components of this atom - they'll either measure σ_x , σ_y , or σ_z without, however, telling the physicist which one of these measurements is actually done. Then it is again the physicist's turn, and she can perform any experiment of her choosing. Only after she's finished with it, the king will tell her which spin component had been measured by his men. To save her neck, the physicist must then state correctly the measurement result that the king's men had obtained.

Much to the king's frustration, the physicist rises to the challenge - and not just by sheer luck: She gets the right answer any time the whole procedure is repeated. How does she do it?” [1]

- a) Beschreiben Sie eine einfache Strategie, die unter Verwendung eines einzelnen Qubits eine Erfolgswahrscheinlichkeit von $\frac{5}{6}$ erreicht.
- b) Zeigen Sie, dass die folgende Strategie [2] immer zum Erfolg führt. Die Physikerin präpariert zwei Teilchen im Bellzustand $|\Phi^+\rangle$. Eines der Teilchen übergibt sie dem König, Nachdem die Diener des Königs ihre Messung durchgeführt haben,

misst die Physikerin beide Teilchen in der Basis

$$\begin{aligned} |\phi_1\rangle &= \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{2} (e^{i\frac{\pi}{4}} |01\rangle + e^{-i\frac{\pi}{4}} |10\rangle), & |\phi_2\rangle &= \frac{1}{\sqrt{2}} |00\rangle - \frac{1}{2} (e^{i\frac{\pi}{4}} |01\rangle + e^{-i\frac{\pi}{4}} |10\rangle), \\ |\phi_3\rangle &= \frac{1}{\sqrt{2}} |11\rangle + \frac{1}{2} (e^{-i\frac{\pi}{4}} |01\rangle + e^{i\frac{\pi}{4}} |10\rangle), & |\phi_4\rangle &= \frac{1}{\sqrt{2}} |00\rangle - \frac{1}{2} (e^{-i\frac{\pi}{4}} |01\rangle + e^{i\frac{\pi}{4}} |10\rangle). \end{aligned}$$

Wie kann sie von ihren Messergebnissen auf die Messergebnisse der Diener des Königs schließen?

Literatur

- [1] Y. Aharonov and B.-G. Englert, Z. Naturforsch., **56a**, 16-19 (2001)
- [2] L. Vaidman, Y. Aharonov, and D. Z. Albert, Phys. Rev. Lett., **58**, 1385-1387 (1987)