Foundations of Quantum Mechanics

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Sheet 8 — SS 2019 — Due: Tue., 11.06.2019

Exercise 22: Simple Hardy-like proof of quantum contextuality (4+5)

Consider five boxes. The probability, that box i is empty and box j is filled is given by $p(01|ij)$. The system is prepared in such a way that

$$p(01|12) + p(01|23) = 1, \tag{1}$$

and

$$p(01|34) + p(01|45) = 1. \tag{2}$$

These conditions can be rewritten as

$$p(11|12) = p(00|23) = 0 \tag{3}$$

and

$$p(11|34) = p(00|45) = 0. \tag{4}$$

a) Assuming non-contextuality, argue that

$$p(01|51) = 0 \tag{5}$$

must hold.

b) Show that in the following scenario Eq. (1) and (2) are satisfied, whereas Eq. (5) is violated.
Consider the state $|\eta\rangle = \frac{1}{\sqrt{3}}(1,1,1)^T$. Opening the box i is represented by measuring $\{P_0 = \mathbb{1} - |v_i\rangle\langle v_i| \ , \ P_1 = |v_i\rangle\langle v_i|\}$, where P_0 corresponds to 0 and P_1 corresponds to 1 and the vectors $|v_i\rangle$ are given by:

$$\begin{aligned}
|v_1\rangle &= \frac{1}{\sqrt{3}}(1,-1,1)^T \\
|v_2\rangle &= \frac{1}{\sqrt{2}}(1,1,0)^T \\
|v_3\rangle &= (0,0,1)^T \\
|v_4\rangle &= (1,0,0)^T \\
|v_5\rangle &= \frac{1}{\sqrt{2}}(0,1,1)^T
\end{aligned}$$

Please turn!

Exercise 23: Joint measurability and steering (4+4)

A set of measurements $\{A_{a|x}\}$, where a labels the outcomes and x labels the measurement setting, is jointly measurable if $A_{a|x} = \sum_\lambda p(a|x,\lambda)G_\lambda$.

a) Show that, if Alice performs jointly measurable POVMs $\{A_{a|x}\}$ on any state ϱ_{AB} she cannot steer the state of Bob. *Hint:* Calculate the conditional states on Bobs side and compare that to the definition of unsteerability, i.e. the existence of a local hidden state model $\varrho_{a|x} = \sum_\lambda p(a|x,\lambda)p(\lambda)\hat{\sigma}_\lambda$, where $\hat{\sigma}_\lambda$ is normalized.

b) Show that, if Alice performs incompatible POVMs $\{A_{a|x}\}$ there exists a state which demonstrates steering. *Hint:* You might want to use one of the results from the last exercise sheet.