Foundations of Quantum Mechanics

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Sheet 7 — SS 2019 — Due: Mon., 03.06.2019

Exercise 20: Tensor product II (10+5)

a) Calculate the reduced density matrices for systems A and B of the following states:

i) $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)$

ii) $|\psi\rangle_{AB} = \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right)$

iii) $\varrho_{AB} = \frac{1}{4}\begin{pmatrix} \frac{13}{8} & 0 & \frac{33}{32} - \frac{1}{16}i & 0 \\ 0 & \frac{5}{8} & 0 & \frac{1}{32} - \frac{1}{16}i \\ \frac{33}{32} + \frac{1}{16}i & 0 & \frac{11}{8} & 0 \\ 0 & \frac{33}{32} + \frac{1}{16}i & 0 & \frac{3}{8} \end{pmatrix}$

iv) $\varrho_{AB} = \frac{1}{4}|+0\rangle\langle +0| + \frac{3}{4}|1-\rangle\langle 1-|$

v) $|\psi\rangle_{ABC} = \frac{1}{2\sqrt{2}}\left[|0\rangle\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) + |1\rangle\left(|00\rangle - |01\rangle - |10\rangle + |11\rangle\right)\right]$
For this case also calculate the reduced state $\varrho_{AB} = \mathrm{tr}_C\left[|\psi\rangle\langle\psi|_{ABC}\right]$.

b) Show that $(X \otimes X)|\Psi^-\rangle = \det(X)|\Psi^-\rangle$, for $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ and X an arbitrary 2×2 matrix. What does this result imply for the case when X is a unitary and what is the physical interpretation of this property in that case? Does there exist another two qubit pure state with that property?
Hint: Show that $(X \otimes \mathbb{1})|\Phi^+\rangle = (\mathbb{1} \otimes X^T)|\Phi^+\rangle$ for $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and that $X\sigma_y X^T = \det(X)\sigma_y$.

Exercise 21: Schmidt decomposition (3+2)

a) Show that for a bipartite pure state $|\psi\rangle_{AB} = \sum_k \sqrt{\lambda_k}|\alpha_k\rangle \otimes |\beta_k\rangle$ the squares of the Schmidt coefficients λ_k are the eigenvalues of the reduced states $\varrho_{A/B}$ and furthermore the Schmidt basis on A (B) is given by the eigenbasis of the reduced state ϱ_A (ϱ_B).

b) What are the Schmidt coefficients and bases for the states in Ex. 20a (i) and (ii) ?