Foundations of Quantum Mechanics

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Sheet 6 — SS 2019 — Due: Mon., 27.05.2019

Exercise 18: Tensor product I (6+6)

Decide if the following states are separable or entangled and write them as vectors in the standard basis:

(i) $|\Psi\rangle = \frac{1}{\sqrt{2}}(\alpha\,|00\rangle + i\beta\,|11\rangle + i\alpha\,|01\rangle + \beta\,|10\rangle)$ with $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$;

(ii) $|\Psi\rangle = \frac{1}{\sqrt{2}}(|{++}\rangle + |{--}\rangle)$;

(iii) $|\Psi\rangle = \frac{1}{2}(|000\rangle + |010\rangle - |001\rangle - |011\rangle)$.

Write $A \otimes B$ and $B \otimes A$ as a matrix in the standard basis:

(i) $A = \alpha\sigma_x + \beta\sigma_y$ und $B = \gamma\sigma_z + \mathbb{1}$;

(ii)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

and

$$B = \begin{pmatrix} a & \alpha & \beta \\ \alpha^* & b & \gamma \\ \beta^* & \gamma^* & c \end{pmatrix} \tag{2}$$

(iii) $A = a\,|+\rangle\langle+| + b\,|-\rangle\langle-|$ with $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ and $B = \sigma_y \otimes \sigma_y$.

Please turn!

Exercise 19: The mean king's problem (5)

A ship wrecked physicist gets stranded on a far-away island that is ruled by a mean king who loves cats and hates physicists since the day when he first heard what happened to Schrödinger's cat. A similar fate is awaiting the stranded physicists. Yet, mean as he is, the king enjoys defeating physicists on their own turf, and therefore he maliciously offers an apparently virtual chance of rescue.
He takes the physicist to the royal laboratory, a splendid place where experiments of any kind can be performed perfectly. There the king invites the physicist to prepare a certain silver atom in any state she likes. The king's men will then measure one of the three cartesian spin components of this atom - they'll either measure σ_x, σ_y, or σ_z without, however, telling the physicist which one of these measurements is actually done. Then it is again the physicist's turn, and she can perform any experiment of her choosing. Only after she's finished with it, the king will tell her which spin component had been measured by his men. To save her neck, the physicist must then state correctly the measurement result that the king's men had obtained.
Much to the king's frustration, the physicist rises to the challenge - and not just by sheer luck: She gets the right answer any time the whole procedure is repeated. How does she do it?

a) Describe a simple strategy, that gives the physicist a success probability of $\frac{5}{6}$.

b) Show that the following strategy always succeeds: The physicist prepares two particles in the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. One of the particles she gives to the king. After the king has performed his measurement, the phsicist measures the two particles in the basis

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{2}\left(e^{i\frac{\pi}{4}}|01\rangle + e^{-i\frac{\pi}{4}}|10\rangle\right), \qquad |\phi_2\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{2}\left(e^{i\frac{\pi}{4}}|01\rangle + e^{-i\frac{\pi}{4}}|10\rangle\right),$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}|11\rangle + \frac{1}{2}\left(e^{-i\frac{\pi}{4}}|01\rangle + e^{i\frac{\pi}{4}}|10\rangle\right), \qquad |\phi_4\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{2}\left(e^{-i\frac{\pi}{4}}|01\rangle + e^{i\frac{\pi}{4}}|10\rangle\right).$$