Exercise 28: Partial transposition

For a bipartite state

$$\rho_{AB} = \sum_{ijkl} p^{ij}_{kl} |i\rangle\langle j| \otimes |k\rangle\langle l| \tag{1}$$

the partial transposition with respect to Bobs system is defined by

$$\rho_{AB}^{T_B} := I \otimes T\,(\rho_{AB}) = \sum_{ijkl} p^{ij}_{kl}\,|i\rangle\langle j| \otimes (|k\rangle\langle l|)^T = \sum_{ijkl} p^{ij}_{kl}\,|i\rangle\langle j| \otimes |l\rangle\langle k| = \sum_{ijkl} p^{ij}_{lk}\,|i\rangle\langle j| \otimes |k\rangle\langle l|\,, \tag{2}$$

and similarly for Alice, where $\rho_{AB}^{T_A} := T \otimes I\,(\rho_{AB})$.

a) For a two qubit density matrix in the computational basis, which of the entries get exchanged by the partial transposition on Alices and Bobs side?

b) Consider the state

$$\begin{aligned} \rho_{AB}(t) &= p_+|00\rangle\langle 00| + p_-|01\rangle\langle 01| + p_-|10\rangle\langle 10| + p_+|11\rangle\langle 11| & (3)\\ &+ e^{-t/T_2}/2|00\rangle\langle 11| + e^{-t/T_2}/2|11\rangle\langle 00|, & (4) \end{aligned}$$

where $p_\pm = \frac{1}{4}\left(1 \pm e^{-t/T_1}\right)$. Write this state as a matrix, compute the partial transpose with respect to Bobs system.

c) Calculate the time after which the state becomes separable.

Exercise 29: Schur complement

Consider a matrix M in the 2×2 block form

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{5}$$

If D is invertible the matrix $S = A - BD^{-1}C$ is called the Schur complement of D in M.

a) Prove for M being symmetric, $M > 0$ is positive definite, iff $D > 0$ and $S > 0$.

b) Prove if $D > 0$, then $M > 0$ iff $S > 0$.