Statistical Physics
Assignment 9

Lecture: Prof. Dr. Otfried Gühne
Tutorial: Leonardo Novo, Tobias Moroder, Fri 8–10, Room: D115

Due to: Tue, 19.06

1. **Virial expansion** (4 Points)
   The classical virial expansion expresses the pressure of a many-particle system in equilibrium as a power series in the density $\varrho = N/V$, *i.e.*,

$$P = k_B T \varrho \left(1 + \sum_{n=1}^{\infty} B_n \varrho^n \right), \tag{1}$$

   where the virial coefficients $B_n(T)$ are characteristics of the interactions between the particles in the system.

   (a) Determine the coefficients for the van-der-Waals gas

$$\left(P + a \frac{n^2}{V^2} \right) (V - nb) = nRT \tag{2}$$

   with $n = N/N_A$, $R = N_A k_B$ and a, b appropriate empirical constants.

   (b) Repeat this computation for the Dieterici gas, given by

$$P = \frac{nRT}{V - nb} \exp\left(-\frac{na}{RTV} \right). \tag{3}$$

   (c) Compare the virial coefficients and investigate the limit of large temperature T.

2. **Spinless fermions** (3 Points)
   Fermions without interaction can for instance be described by a Hamiltonian of the form

$$H = \sum_{m,n=1}^{N} T_{mn} a_m^\dagger a_n, \tag{4}$$

   using a Hermitian $N \times N$ matrix T_{mn} and appropriate creation and annihilation operators.

   (a) Verify: For any unitary matrix U_{kl} the operators $c_k = \sum_l U_{kl} a_l$ form a new valid set of annihilation operators.

   (b) Employ this result in order to show that the Hamiltonian can always be written as

$$H = \sum_{n=1}^{N} \epsilon_n c_n^\dagger c_n, \tag{5}$$

   for appropriate chosen creation and annihilation operators.

3. **Fock space** *
   Discuss the following problems for the bosonic and fermionic case:

   (a) Consider the case of N indistinguishable particles, each of which having only two possible states $|0\rangle$, $|1\rangle$. Let $|n_0, N - n_0\rangle$ denote the state where exactly n_0 particles are in state $|0\rangle$, while $N - n_0$ particles are in state $|1\rangle$. Determine the explicit form of these states in the tensor product basis $|0 \dots 00\rangle$, $|0 \dots 01\rangle$, ..., $|1 \dots 11\rangle$.

(b) A single particle operator F can be expressed as

$$F = \sum_{kl} f_{kl} c_k^\dagger c_l, \tag{6}$$

while the number operator N is given by

$$N = \sum_{k} c_k^\dagger c_k. \tag{7}$$

Compare the action of $FN|\vec{n}\rangle$ vs. $NF|\vec{n}\rangle$.

(c) Compute the creation and annihilation operator in the Heisenberg picture for the Hamiltonian

$$H = \sum_{n=1}^{N} \epsilon_n c_n^\dagger c_n. \tag{8}$$

Verify that the time-dependent operators $c(t)^\dagger, c(t)$ are again valid creation and annihilation operators if they satisfy it at time instance $t = 0$.