Statistical Physics
Assignment 6

Lecture: Prof. Dr. Otfried Gühne
Tutorial: Leonardo Novo, Tobias Moroder, Fri 8–10, Room: D115

Due to: Tue, 22.05

1. **System specifications** (4 Points)
Consider a system having the following properties:

   i) The work done at the system if one expands V_0 to V at constant temperature T_0 is

$$\Delta W(T_0) = RT_0 \ln \frac{V}{V_0}. \tag{1}$$

   ii) The entropy of the system is given by

$$S = R\frac{V_0}{V}\left(\frac{T}{T_0}\right)^a \tag{2}$$

   Here V_0, T_0 and $a \neq -1$ are given real constants.

   (a) Compute the free energy $F = F(T, V)$ and the pressure $P = P(T, V)$.
   (b) Compute the expansion work $\Delta W(T_1)$ from V_0 to V at an arbitrary temperature T.

2. **Usefulness of thermodynamic relations** (5 Points)
In the following we want to see the usefulness of thermodynamic relations. For that consider a system with inner energy U, temperature T and volume V.

   (a) At first prove the relation

$$\left(\frac{\partial U}{\partial V}\right)_T = -p + T\left(\frac{\partial P}{\partial T}\right)_V, \tag{3}$$

   where the subscripts denote the variables that are kept constant.

   Now let us concentrate on a more specific example of a photon-gas. For such a system consider that

$$P = \alpha\epsilon(T), \tag{4}$$

   with $\alpha = const.$, while $\epsilon(T)$ is the inner energy per unit volume.

   (b) Determine the explicit temperature dependence of $U = U(T, V)$.
   (c) Compute the entropy $S = S(T, V)$.

3. **Spring** *
A spring follows Hooke's law, *i.e.*, as long as it is not stretched or compressed beyond the elastic limit, the force with which the spring pushes back is linearly proportional to the distance from its equilibrium length. More precisely it holds

$$F_r = -kx. \tag{5}$$

   with an expansion coefficient k depending on the temperature. Assume that this dependence is given by

$$k = \frac{a}{T} \tag{6}$$

   with $a = const. > 0$.

   (a) What are the natural variables of the free energy F and state its differential dF?
   (b) How does the inner energy U changes if we stretch the spring up to x while keeping the temperature constant.