Statistical Physics
Assignment 12

Lecture: Prof. Dr. Otfried Gühne
Tutorial: Leonardo Novo, Tobias Moroder, Fri 8–10, Room: D115

Due to: Tue, 10.07

1. **Bose-Einstein condensate in 2D** (4 Points)
   Consider an ideal Bose gas in $2D$ with single particle energies $\epsilon(\vec{k}) = \hbar^2\vec{k}^2/(2m)$. Show that a Bose-Einstein condensate does not occur in this case.

   Hint: Compute the critical temperature T_c. Here the Bose-Einstein functions might be helpful,

   $$g_\alpha(z) = \frac{1}{\Gamma(\alpha)}\int_0^\infty dx \frac{x^{\alpha-1}}{z^{-1}e^x - 1} = \sum_{k=1}^{\infty} \frac{z^k}{k^\alpha}. \tag{1}$$

2. **Photon number** (4 Points)
   The expectation value $\langle \hat{n}_\lambda(\vec{k})\rangle$ gives the mean photon number of the mode with wave-vector $\vec{k}$ and polarization λ.

   (a) Compute the total mean photon number $\langle N\rangle = \sum_{\lambda,\vec{k}}\langle \hat{n}_\lambda(\vec{k})\rangle$ of a photon gas at temperature T and volume V.

   (b) Via this result one can formally obtain $PV = x\langle N\rangle k_B T$. Determine x and discuss its difference with respect to the equation of state of the ideal gas.

   (c) Compute the variance of $\langle \hat{n}_\lambda(\vec{k})\rangle$.