Statistical Physics
Assignment 11

Lecture: Prof. Dr. Otfried Gühne
Tutorial: Leonardo Novo, Tobias Moroder, Fri 8–10, Room: D115

Due to: Tue, 03.07

1. **Semiconductor** (5 Points)
   A semiconductor consists of two energy bands, the valence and the conductor band. They are characterized by their energy difference $\Delta\epsilon = \epsilon_c - \epsilon_v > 0$. At $T = 0$ the valence band is fully occupied while the conductor band is empty.

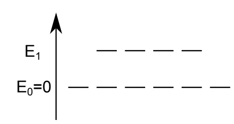
   (a) Determine the conductor electron density N_c^-/V and the valence hole density N_v^+/V as a function of T and μ. At this step the densities should still be considered general.

   (b) Using that the number of electrons is conserved provides a conditional equation for the chemical potential μ. Moreover, the chemical potential typically satisfies, $k_B T \ll \epsilon_c - \mu$ and $k_B T \ll \mu - \epsilon_v$. Determine μ.

   (c) Consider that the energy of electrons in the conductor band is given by $\epsilon_c(\vec{k}) = \epsilon_c + \hbar^2\vec{k}^2/(2m_c)$, while electrons in the valence band attain $\epsilon_v(\vec{k}) = \epsilon_v - \hbar^2\vec{k}^2/(2m_v)$. Determine the particle density and the chemical potential μ? Is the previous limit satisfied?

2. **Chemical potential** (3 Points)

   (a) A semiconductor with the following energy levels contains (on average) $N = 4$ electrons. The degeneracy of the levels is $g(E_0) = 6$ and $g(E_1) = 4$, including already the spin degeneracy. At which temperature $T_0 > 0$ passes the chemical potential $\mu = 0$.

   (b) What is the behaviour of the ratio μ/T in the limit of $T \to \infty$.

3. **Pauli-Paramagnetism** *
   Consider a gas of free electrons in three dimensions in a homogeneous magnetic field. Of course we neglect any spin-orbit interaction.

   (a) What is the density for an electron with spin-↑, spin-↓?

   (b) Compute the magnetization at $T = 0$. Here you may expand the expression with respect to the field strength and drop quadratic terms.

   (c) Evaluate the magnetization for the non-vanishing temperatures. Here you need to Sommerfeld expansion

$$\int_{-\infty}^{\infty} \mathrm{d}\varepsilon\, g(\varepsilon) \frac{1}{\mathrm{e}^{(\varepsilon-\mu)/(k_B T)} + 1} = \int_{-\infty}^{\mu} \mathrm{d}\varepsilon\, g(\varepsilon) + \frac{\pi^2}{6}(k_B T)^2 g'(\mu) + \mathcal{O}(k_B T)^4. \qquad (1)$$