Quantum Information Theory
Exercise sheet 9

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Lecture: Tuesday, 10-12, Room D 308
Exercise: Monday, 15-17, Room B 107

21. Period finding on a quantum computer

Find the period of the function

$$f(x) = \frac{1}{2}\big(\cos(\pi x) + 1\big),$$

assuming x is stored in a 3-qubit quantum register, i.e. $0 \leq x < L = 2^3$ and $x \in \mathbb{N}$, using the following recipe:

(a) Calculate the state

$$|\psi\rangle = \frac{1}{\sqrt{L}} \sum_{x=0}^{L-1} |x\rangle|f(x)\rangle.$$

(b) Compute the state of the first register (left ket) for the case that a measurement on the second register (right ket) resulted in a "0".

(c) Perform the quantum Fourier transform

$$|x\rangle \mapsto \frac{1}{\sqrt{L}} \sum_{k=0}^{L-1} e^{2\pi i x k/L}|k\rangle.$$

(d) The period r can be determined from the relation

$$\frac{\ell}{r} = \frac{c}{L},$$

where c is the result of the final measurement on the first register, and ℓ is an integer such that ℓ/r is an irreducible fraction. What happens if $c = 0$ is measured?

(e) Can you explain why the relation in (d) holds?