Quantum Information Theory
Exercise sheet 8

Lecture: Prof. Dr. Otfried Gühne Exercise: Costantino Budroni
Lecture: Tuesday, 10-12, Room D 308
Exercise: Monday, 15-17, Room B 107

19. Universal single-qubit gates

Show that any single-qubit unitary operation can be implemented with Hadamard gates, phase shift gates and multiplication with a global phase. (Recall that any single-qubit unitary can be written as $U = \mathrm{e}^{\mathrm{i}\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$, where R_y and R_z are rotations about the y and z axis.)

20. Simple quantum circuits

(a) Construct a CNOT gate from CPHASE and single-qubit gates.

(b) Construct a quantum half-adder, that is, a circuit which for input $|x\rangle$ and $|y\rangle$ (where $x, y \in \{0, 1\}$) gives $|x \oplus y\rangle$ and $|xy\rangle$ as output. Why can no two-qubit circuit realize this operation?

(c) For many applications, e. g. for teleportation, one has to measure a pair of qubits in the Bell basis. Design a quantum circuit that implements this measurement, using the usual one- and two-qubit gates, but only single-qubit measurements.