Quantum Information Theory
Exercise sheet 7

Lecture: Prof. Dr. Otfried Gühne Exercise: Costantino Budroni
Lecture: Tuesday, 10-12, Room D 308
Exercise: Monday, 15-17, Room B 107

17. Turing machines

(a) Consider a Turing machine with internal states q_s, q_h, q_1 and the program

$$\begin{aligned}
&(q_s, \triangleright, q_1, \triangleright, +1) \\
&(q_1, 0, q_1, 0, +1) \\
&(q_1, 1, q_1, 1, +1) \\
&(q_1, \#, q_h, 0, 0).
\end{aligned}$$

Assume that the initial state of the tape is of the form

$$[\triangleright, x, \#, \#, \ldots],$$

where $x \in \{0,1\}^n$ is the binary representation of an integer number.

What does this algorithm compute?

(b) Construct a Turing machine that adds 1 to an integer number.

18. Complexity classes

(a) The factoring decision problem is the question: Given an integer m and $l < m$, does m have a nontrivial factor less than l ?

Show that that a polynomial-time algorithm for finding the factors of an integer exists if and only if the factoring decision problem is in **P**.

(b) The travelling salesman decision problem is as follows: Given n cities and a nonnegative integer distance d_{ij} between each pair of cities, is there a tour of all cities with a total length less than a given d ?

The Hamiltonian cycle problem is the question: Given an undirected graph, does it contain a Hamiltonian cycle, i. e., a cycle visiting each vertex exactly once?

Show that the Hamiltonian cycle problem can be reduced to the travelling salesman decision problem.