Quantum Information Theory
Exercise sheet 3

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Lecture: Tuesday, 10-12, Room D 120
Exercise: Monday, 15-17, Room B 107

7. The GHZ argument

(a) Verify that the GHZ state $|\text{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ is an eigenstate of the operators $\sigma_x \otimes \sigma_y \otimes \sigma_y$, $\sigma_y \otimes \sigma_x \otimes \sigma_y$, $\sigma_y \otimes \sigma_y \otimes \sigma_x$ and $\sigma_x \otimes \sigma_x \otimes \sigma_x$. What are the respective eigenvalues?

(b) Suppose that Alice, Bob and Charlie each have one qubit of the GHZ state. If one of them measures σ_x and the other two measure σ_y simultaneously, what does (a) imply for the measurement results? What if they all measure σ_x ?

(c) From a local realistic viewpoint, the values of the local observables σ_x and σ_y are independent of any measurement carried out on the other qubits. Show that this assumption leads to a contradiction. (Hint: Calculate the product of the first three operators in (a)).

(d) What are the local hidden variable bound and the maximal quantum mechanical violation of the Mermin inequality

$$\langle \sigma_x \otimes \sigma_x \otimes \sigma_x \rangle - \langle \sigma_x \otimes \sigma_y \otimes \sigma_y \rangle - \langle \sigma_y \otimes \sigma_x \otimes \sigma_y \rangle - \langle \sigma_y \otimes \sigma_y \otimes \sigma_x \rangle \text{ ?}$$

8. Teleportation with a non-maximally entangled state

Alice wishes to teleport a one-qubit state $|\varphi_A\rangle$ to Bob, but instead of a Bell state they share only the non-maximally entangled state $\cos\theta|01\rangle - \sin\theta|10\rangle$. For a given, but unknown input state $|\varphi_A\rangle$, calculate the average fidelity of the teleportation (that is, the squared overlap $|\langle\varphi_A|\varphi_B\rangle|^2$ of the input state $|\varphi_A\rangle$ with the result of the teleportation $|\varphi_B\rangle$).

9. Classical teleportation

Alice tries to teleport a one-qubit quantum state $|\varphi_A\rangle$ to Bob classically in the following way: She measures σ_z on the state $|\varphi_A\rangle$ and sends the result to Bob, who then prepares the state $|\varphi_B\rangle = |0\rangle$ or $|\varphi_B\rangle = |1\rangle$, according to Alice's result.

(a) What is the average fidelity $|\langle\varphi_A|\varphi_B\rangle|^2$ of this protocol for a given state $|\varphi_A\rangle$?

(b) What is this fidelity, averaged over all pure one-qubit states $|\varphi_A\rangle$?

(The half-angle formulas $\sin(\theta/2) = \pm\sqrt{\frac{1}{2}(1-\cos\theta)}$ and $\cos(\theta/2) = \pm\sqrt{\frac{1}{2}(1+\cos\theta)}$ may be helpful to solve the integral.)