Quantum Information Theory
Exercise sheet 10

Lecture: Prof. Dr. Otfried Gühne Exercise: Costantino Budroni
Lecture: Tuesday, 10-12, Room D 308
Exercise: Monday, 15-17, Room B 107

22. A circuit for three-qubit error correction

(a) Design a quantum circuit that implements the collective two-qubit measurement $ZZ = \sigma_z \otimes \sigma_z$.

(b) Use this result to design a circuit for the three-qubit error correction scheme as introduced in the lecture.

Recall that this scheme uses the codewords $|0\rangle_{\mathrm{L}} = |000\rangle$ and $|1\rangle_{\mathrm{L}} = |111\rangle$ and the operators $ZZ1$ and $1ZZ$.

23. The Steane code

Suppose we have represented a logical qubit as a seven-qubit state according to the Steane code, and suppose further that an error occurs such that all information concerning the sixth qubit is lost.

(a) Show that this error is described by the mapping

$$\rho \mapsto \frac{1}{4}\big(\rho + X_6\rho X_6 + Y_6\rho Y_6 + Z_6\rho Z_6\big).$$

(b) Show that the Steane code can correct this error.

Recall that the Steane code uses the codewords

$$\begin{aligned}
|0\rangle_{\mathrm{L}} = \frac{1}{2^{3/2}}\big(&|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle \\
&+ |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle\big), \\
|1\rangle_{\mathrm{L}} = \frac{1}{2^{3/2}}\big(&|1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle \\
&+ |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle\big)
\end{aligned}$$

and the operators

$$\begin{aligned}
J_1 &= \mathbf{111}XXXX, & J_2 &= \mathbf{1}XX\mathbf{11}XX, & J_3 &= X\mathbf{1}X\mathbf{1}X\mathbf{1}X, \\
K_1 &= \mathbf{111}ZZZZ, & K_2 &= \mathbf{1}ZZ\mathbf{11}ZZ, & K_3 &= Z\mathbf{1}Z\mathbf{1}Z\mathbf{1}Z.
\end{aligned}$$