Genuine high-dimensional steering

S. DESIGNOLLE, V. SRIVASTAV, R. UOLA, N.H. VALENCIA,
W. MCCUTCHEON, M. MALIK, N. BRUNNER

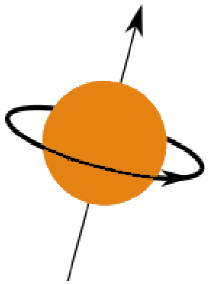
PHYS. REV. LETT. 126, 200404 (2021)

Outline

1. High-dimensional systems
2. Quantum steering
3. High-dimensional quantum steering
4. Criterion and experimental violation

High-dimensional quantum systems

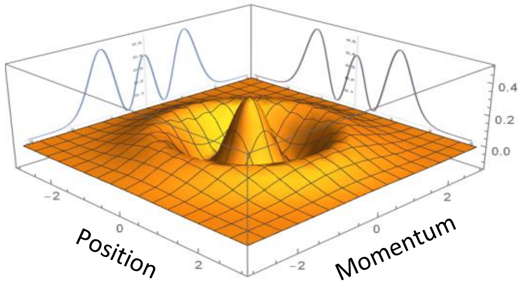
This project



Qubits

2

???



Continuous variables

d

∞

Theory

- Novel forms of quantum correlations
- Stronger uncertainty relations

Applications

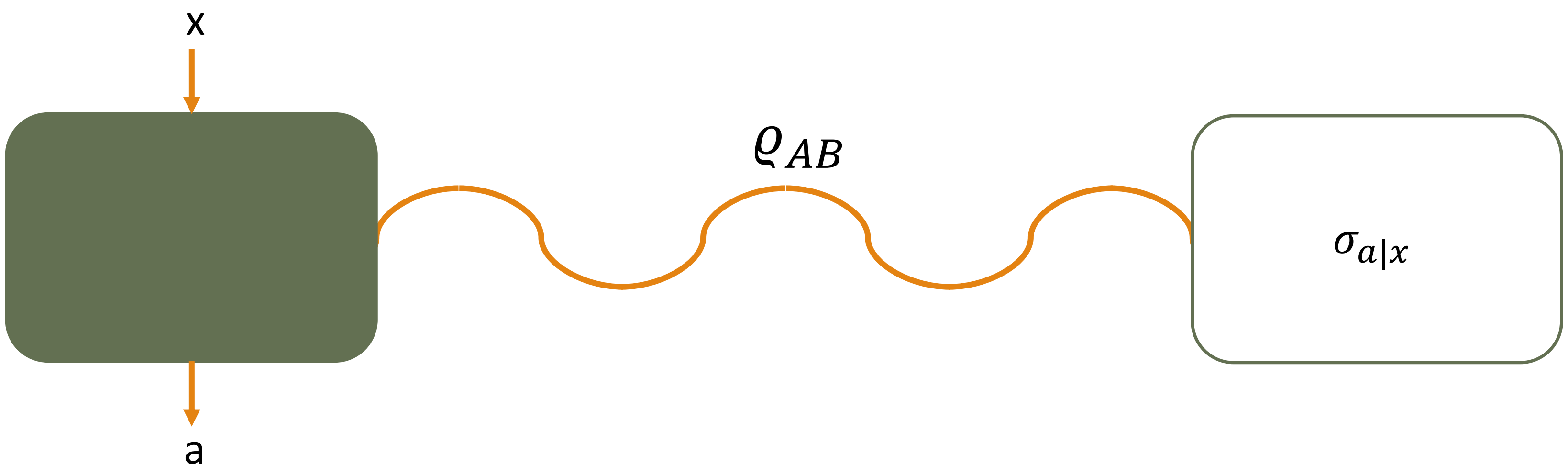
- Noise-tolerant protocols
- Experiments, e.g. OAM and time-bins

Main goal: genuine high-dimensional steering

Quantum steering

1. Entanglement verification method
2. Quantum correlation between entanglement and Bell non-locality Wiseman et al. PRL 98 (2007)
3. Applications: QKD, randomness generation, subchannel discrimination and measurement incompatibility see e.g. Cavalcanti and Skrzypczyk Rep. Prog. Phys. 80 (2017), Uola et al. Rev. Mod. Phys. 92 (2020)

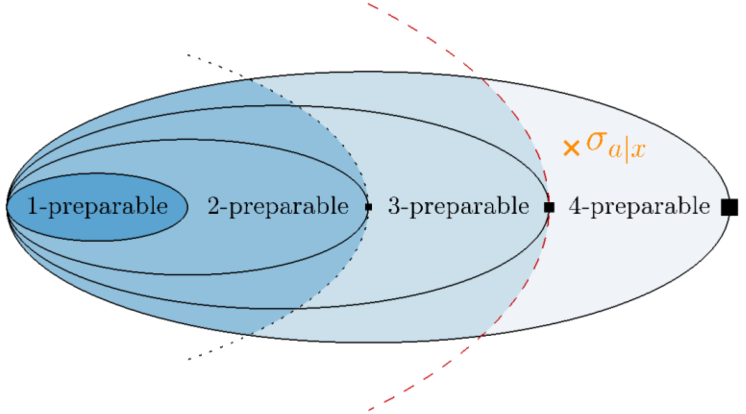
Quantum steering



1. Unsteerable: $\sigma_{a|x} = tr_A\left[\left(A_{a|x} \otimes I\right)\varrho_{AB}\right] = p(a|x)\sum_\lambda p(\lambda|a,x)\sigma_\lambda$
2. Separable ϱ_{AB} leads to unsteerable $\sigma_{a|x}$
3. Unsteerable $\sigma_{a|x}$ preparable with some separable state Kogias et al. PRL 115 (2015), Moroder et al. PRL 116 (2016)
4. High-dimensionality?

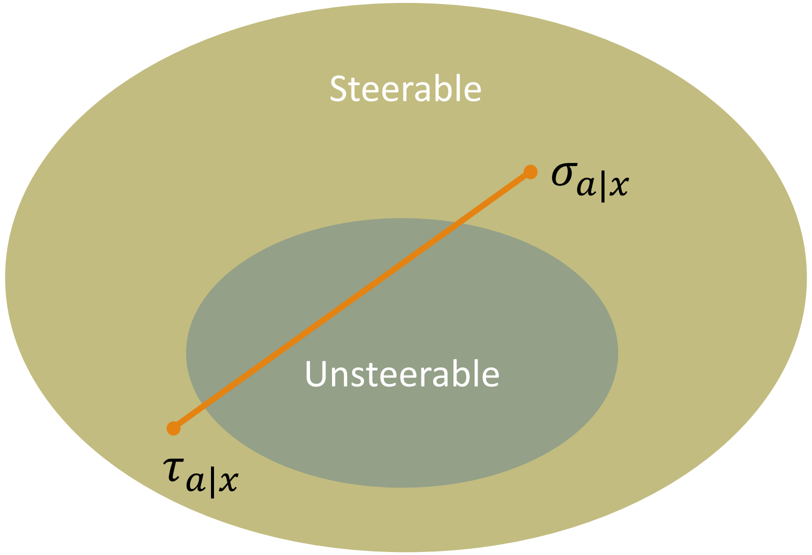
High-dimensional quantum steering

1. Unsteerable $\sigma_{a|x}$ preparable with some separable state
2. State separable iff Schmidt number is one
3. Reminder: Schmidt number $SN(\varrho_{AB}) = \min_{\varrho_{AB}=\sum_\lambda p(\lambda)|\psi_\lambda\rangle\langle\psi_\lambda|} \max SR(|\psi_\lambda\rangle)$
4. Genuine n-dimensional steering: $\sigma_{a|x}$ not preparable with a Schmidt number n-1 state



5. How to witness?

Steering robustness Piani and Watrous, PRL 114 (2015)



$$R(\sigma_{a|x}) = \min_{\tau_{a|x}} \{ t \geq 0 \mid \frac{\sigma_{a|x} + t\tau_{a|x}}{1+t} \text{ unsteerable} \}$$

Upper bound (non-technical):

1. Robustness is convex
2. Schmidt number n
   -> The convex components have a local dimension bounded by n
3. Dimension-dependent upper bounds on robustness

Upper bounding the robustness (technical)

1. Take $\sigma_{a|x}$ n-preparable:

$$\sigma_{a|x} = tr\left[(A_{a|x} \otimes I)\varrho_{AB}\right]$$

   where Schmidt number of ϱ_{AB} is n

2. Write $\varrho_{AB} = \sum_\lambda p(\lambda)|\psi_\lambda\rangle\langle\psi_\lambda|$ and $\sigma_{a|x} = \sum_\lambda p(\lambda)\tau^\lambda_{a|x}$
3. Convexity: $R(\sigma_{a|x}) \leq \sum_\lambda p(\lambda)R\left(\tau^\lambda_{a|x}\right)$
4. Define $\varrho_\lambda = \sum_a \tau^\lambda_{a|x}$ and $B^\lambda_{a|x} = \varrho_\lambda^{-1/2}\tau^\lambda_{a|x}\varrho_\lambda^{-1/2}$
5. Straight-forward: $R(\tau^\lambda_{a|x}) \leq IR(B^\lambda_{a|x})$
6. Here $IR(B_{a|x}) = \min_{C_{a|x}} \{\, t \geq 0 \mid \frac{B_{a|x} + t C_{a|x}}{1+t} \text{ jointly measurable} \,\}$
7. Bounds on IR from Designolle et al. New J. Phys. 21 (2019)

Explicit upper bound

Given that an assemblage $\sigma_{a|x}$ (with two inputs) is preparable with a Schmidt number n state, then

$$R(\sigma_{a|x}) \leq \frac{\sqrt{n} - 1}{\sqrt{n} + 1}$$

In other words

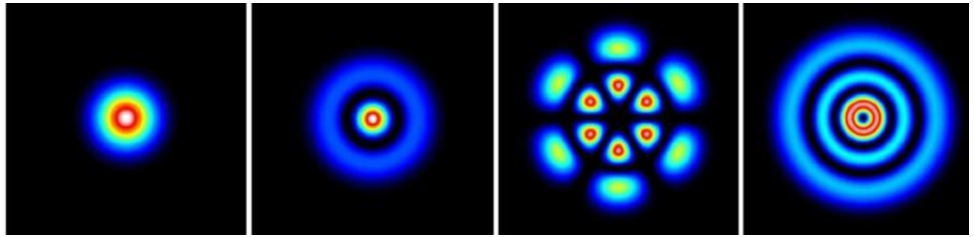
$$\delta(\sigma_{a|x}) \coloneqq \left(\frac{1 + R(\sigma_{a|x})}{1 - R(\sigma_{a|x})} \right)^2 \leq n$$

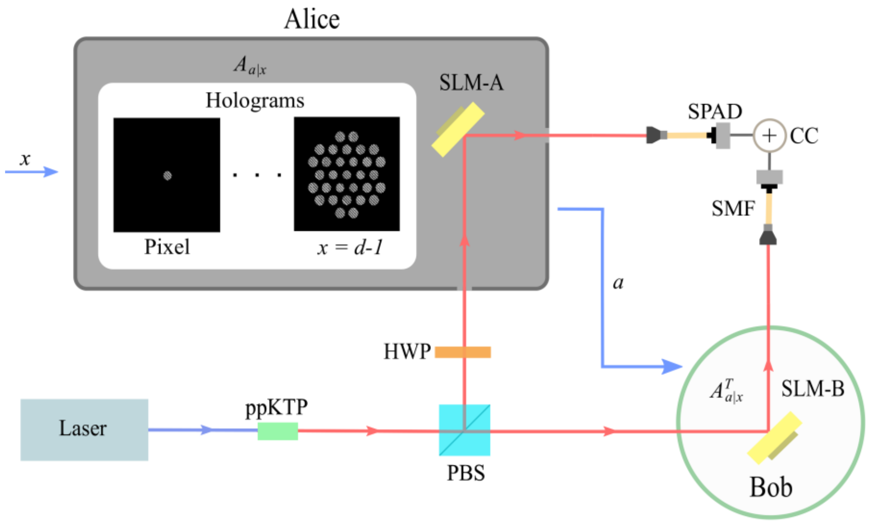
How to measure the robustness?

Steering robustness is an SDP -> Lagrange dual gives a witness

MUBs on Alice and a noisy singlet state -> witness given by MUBs on Bob

In the lab: use OAM of photons





Experimental results

Dimension d	Lower bound on $\delta(\sigma_{a|x})$ Minimum	Maximum	Max. certified dimension n
5	4.1 ± 0.1	4.7 ± 0.1	5
7	5.1 ± 0.2	6.4 ± 0.1	7
11	6.3 ± 0.3	9.1 ± 0.2	10
13	7.0 ± 0.3	10.1 ± 0.3	11
17	9.3 ± 0.3	12.4 ± 0.3	13
19	10.1 ± 0.5	13.6 ± 0.5	14
23	11.4 ± 0.5		12
29	12.1 ± 0.6		13
31	14.1 ± 0.6		15

Conclusions

1. Defined the concept of high-dimensional steering
2. Criterion based on most incompatible pairs of measurements
3. Experimental verification up to Schmidt number 15 (in dimension 31)
4. Future directions:
   i. dimensionality of POVMs
   ii. capacity of channels (in terms of dimension)
   iii. most incompatible sets of measurements