Mário Ziman
RCQI, Institute of Physics, Bratislava, Slovakia

Incompatibility and nonlocality for quantum process theories

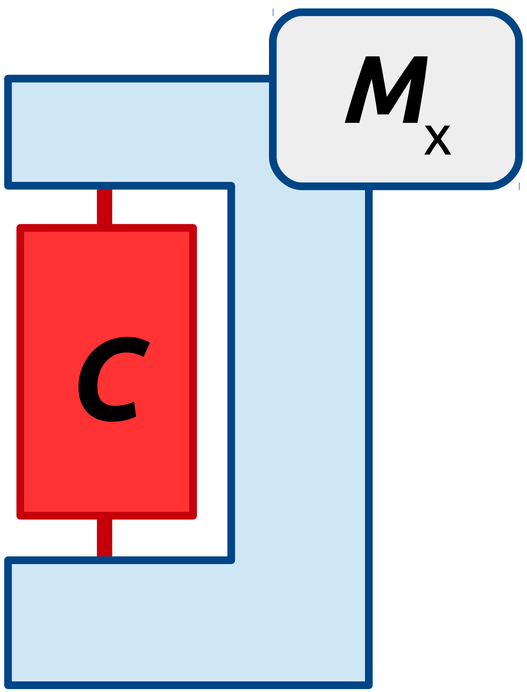
joint work with
Giulio Chiribella, Daniel Reitzner, Michal Sedlák, Martin Plávala,

PLAN

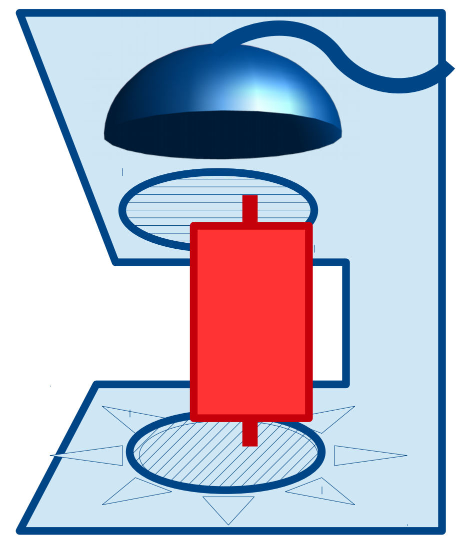
Quantum process GPT

A ⚭ B

CHSH with quantum processes

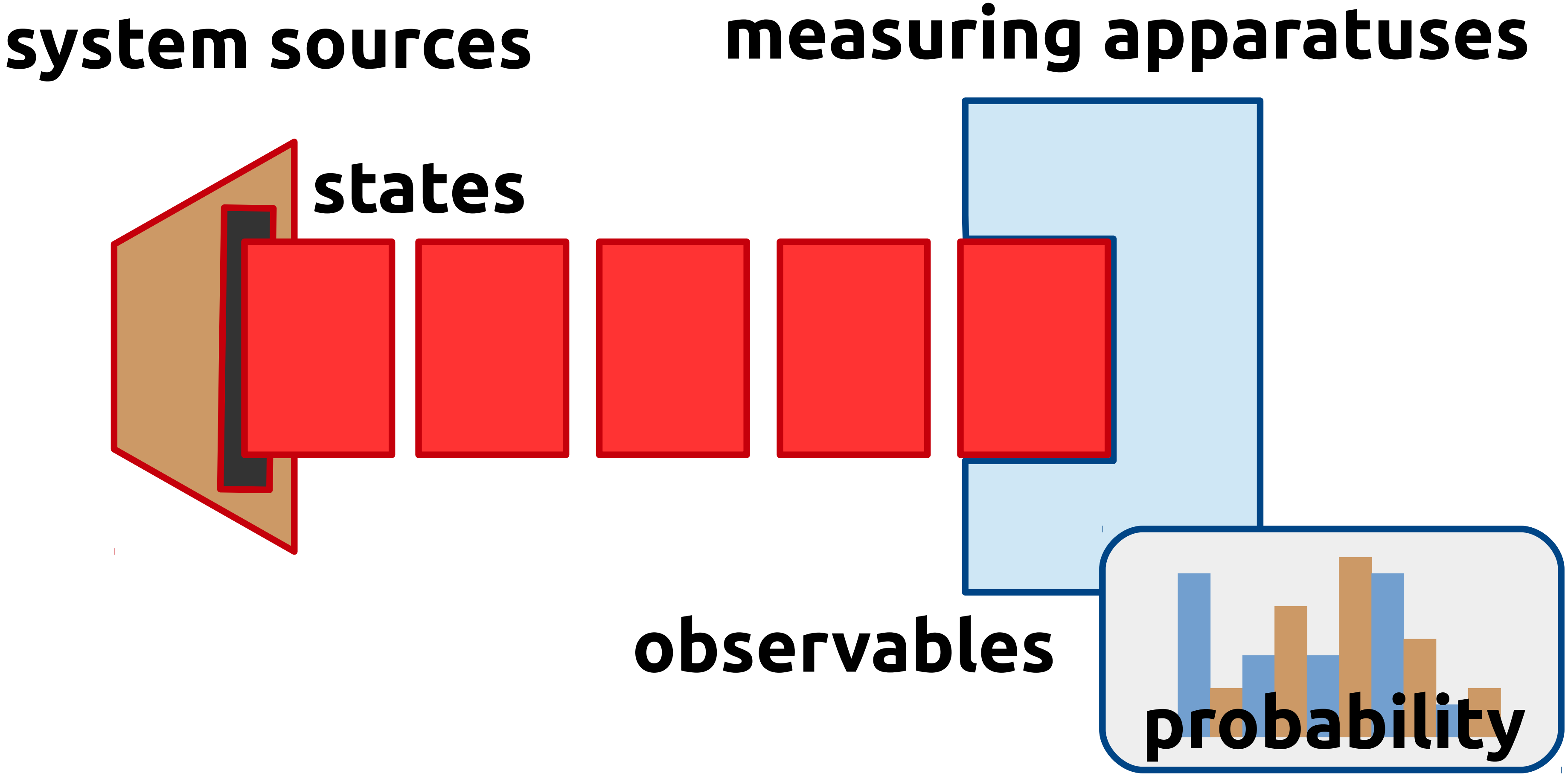


Incompatibility of process observables



Quantum process GPT

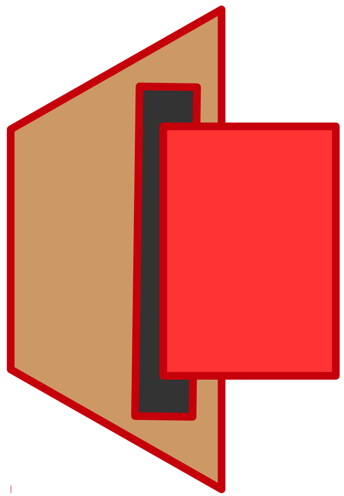
basic model of measurement



probability = f(state, observable)

Quantum process GPT

examples

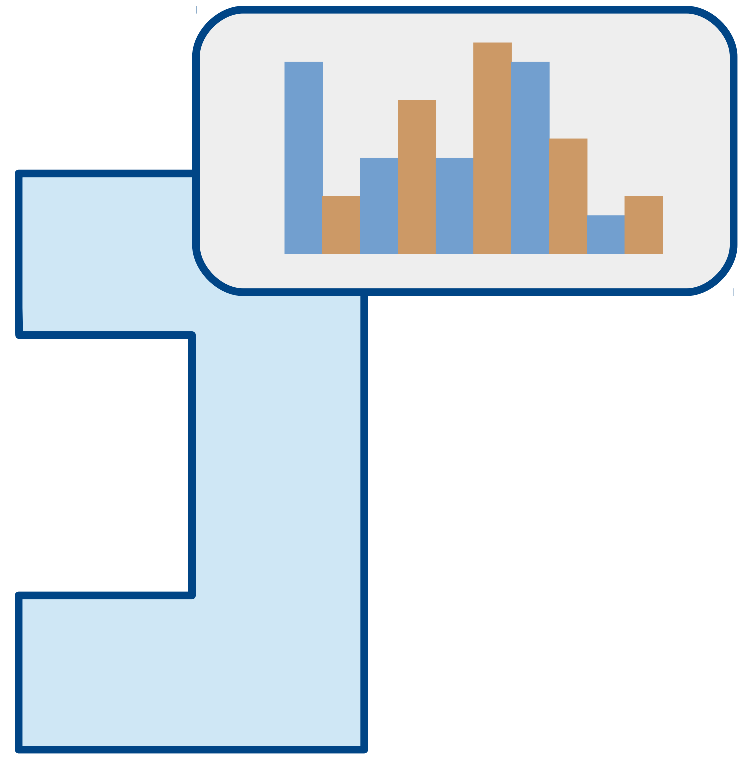


states

density operators

probabilities

channels



observables

positive opvalued measures

positive functionals

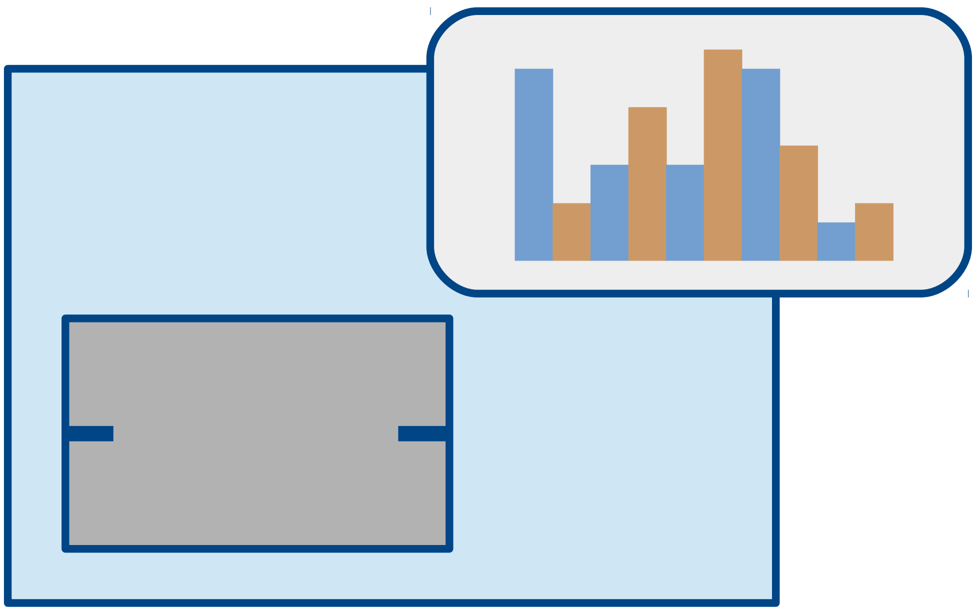
process povms

Quantum process GPT

process observables

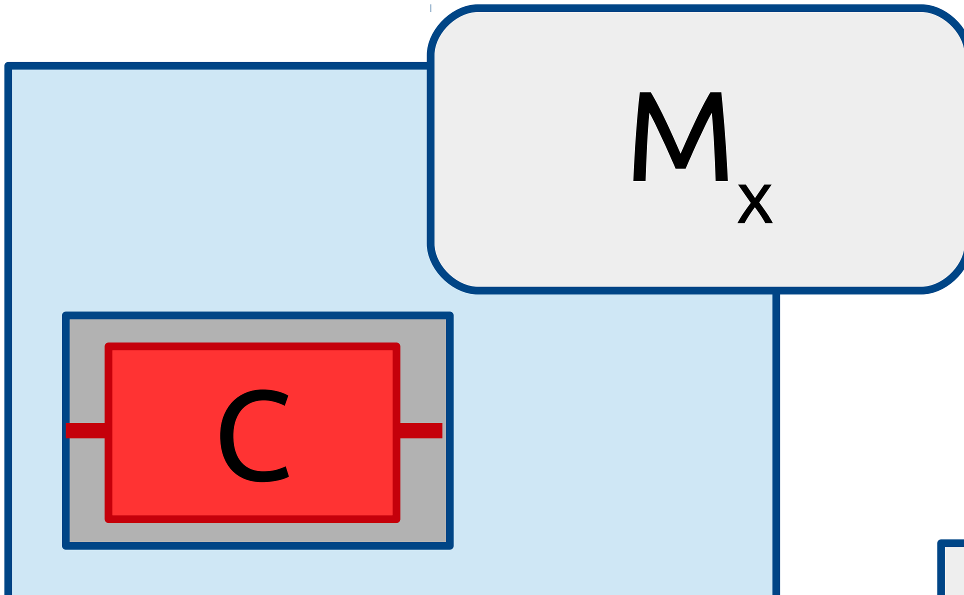
quantum process = channel
- CP+TP linear map on L(H)
- qq input/output box

process observable
- channel → probability
- affine assignement



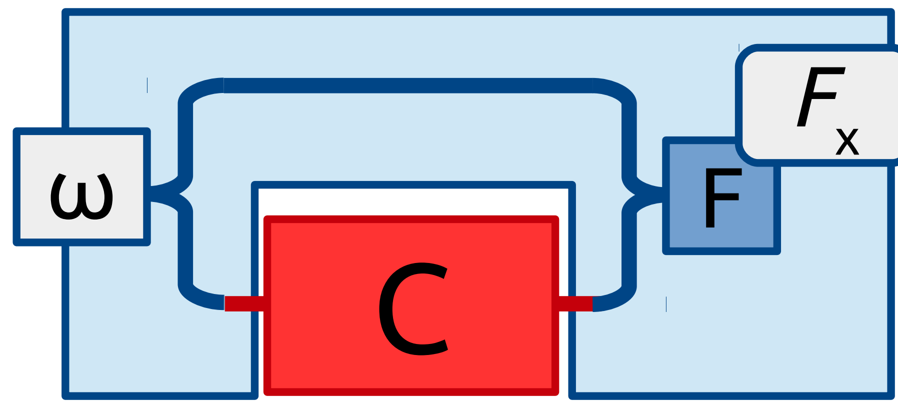
Quantum process GPT

process observables



process observable

outcome M_x describes choice of in-state ω and out-POVM F (resulting in effect F_x)



$$\text{prob}(x)=\text{tr}[\boldsymbol{C}\boldsymbol{M}_x]$$

$$\boldsymbol{C} = (\text{Id}\otimes \text{C})[\Phi_+]$$

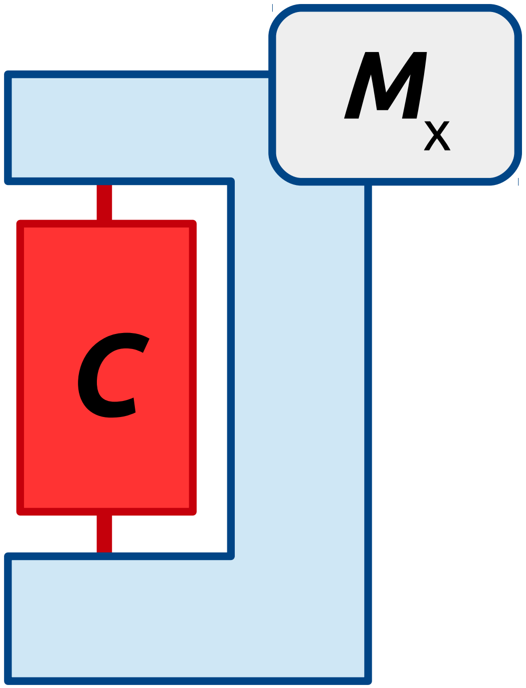
$$\boldsymbol{M}_x = (\text{R}^*_\omega\otimes \text{Id})[F_x]$$

where $(\text{R}_\omega\otimes\text{Id})[\Phi_+] = \omega$

- are operators in $L(H_d\otimes H_d)$

Quantum process GPT

process observables = process POVM = 1-testers



$$M_x : M_x \geq O\,,\ \sum_x M_x = \rho \otimes \mathrm{id}_d$$

density operator

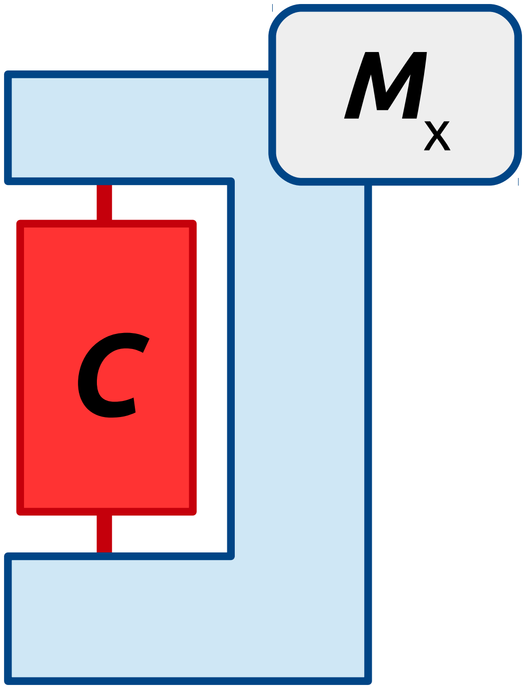
$$\mathrm{prob}(x) = \mathrm{tr}[C M_x]$$

Choi-Jamiolkowski channel representation

$$C : d\,\mathrm{tr}_1[C] = \mathrm{id}_d$$

Quantum process GPT

process observables = process POVM = 1-testers



never POVM

$$M_x : M_x \geq O\,,\ \sum_x M_x = \rho \otimes \mathrm{id}_d$$

density operator

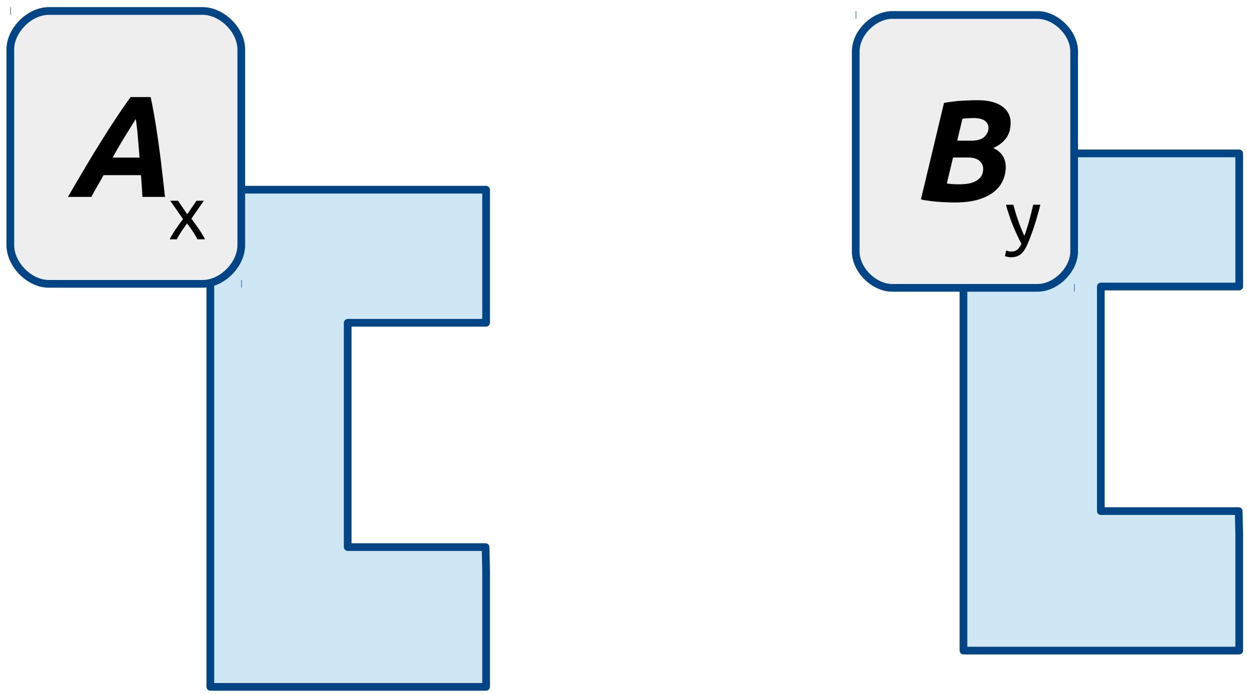
$$\mathrm{prob}(x) = \mathrm{tr}[C M_x]$$

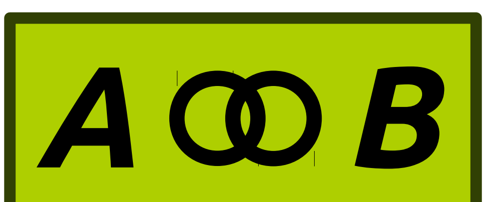
Choi-Jamiolkowski $C : d\,\mathrm{tr}_1[C] = \mathrm{id}_d$

always density operator

Incompatibility

definition of compatibility

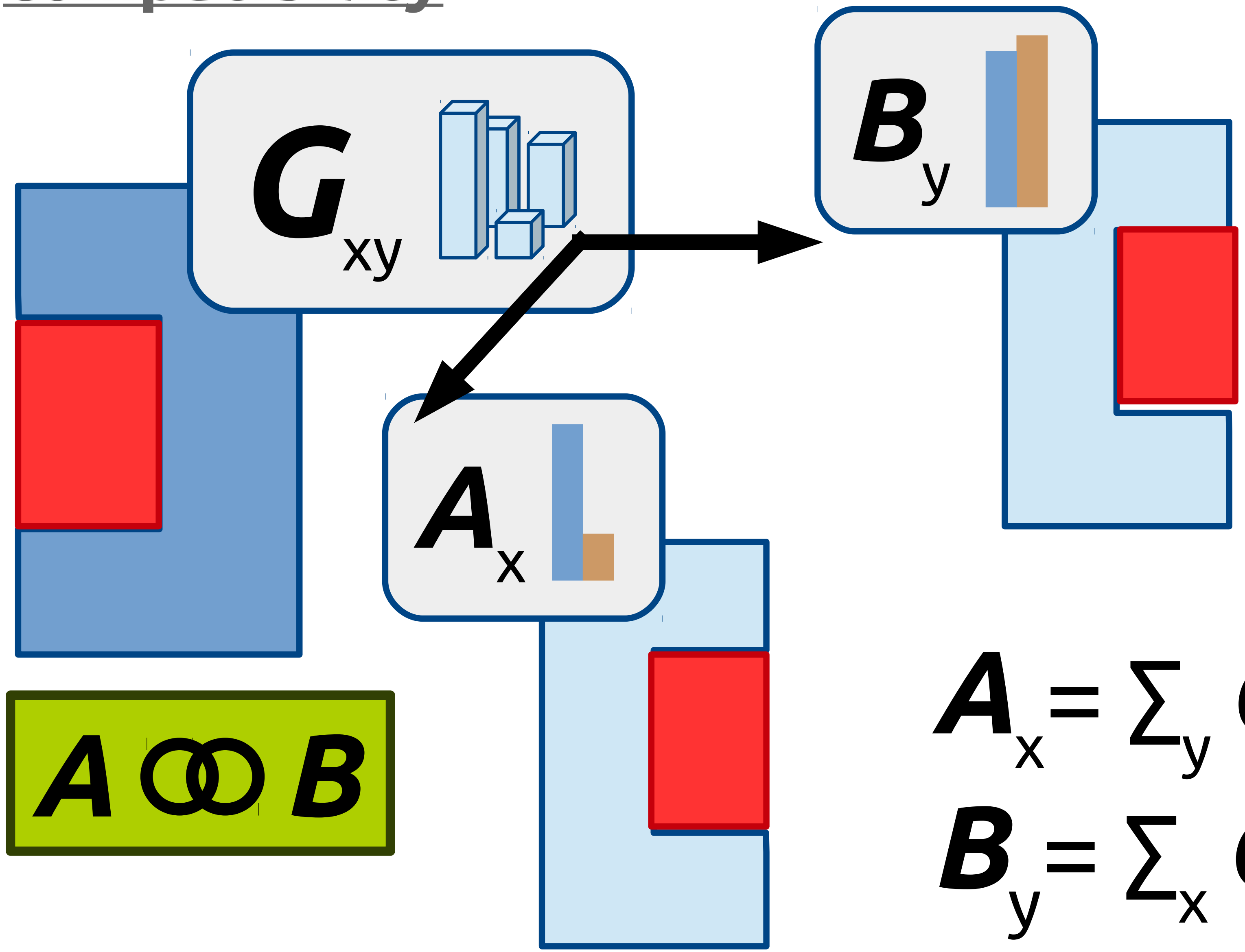




if ***A*, *B*** are marginals of some ***G***

Incompatibility

compatibility



$$A_x = \sum_y G_{xy}$$

$$B_y = \sum_x G_{xy}$$

Incompatibility

compatibility of testers

$\boldsymbol{A} : \boldsymbol{A}_x$ such that $\sum_x \boldsymbol{A}_x = \rho \otimes \mathrm{id}_d$

$\boldsymbol{B} : \boldsymbol{B}_y$ such that $\sum_y \mathbf{B}_y = \sigma \otimes \mathrm{id}_d$

A ⚭ B iff $\rho = \sigma$ and P ⚭ Q

P,Q are POVMs for cannonical realization

$$P_x = (\rho^{-1/2} \otimes \mathrm{id})\, \boldsymbol{A}_x\, (\rho^{-1/2} \otimes \mathrm{id})$$

$$Q_y = (\sigma^{-1/2} \otimes \mathrm{id})\, \boldsymbol{B}_y\, (\sigma^{-1/2} \otimes \mathrm{id})$$

Incompatibility

commutativity does not imply compatibility

Incompatibility

commutativity does not imply compatibility

A* ⚭ *B iff $\rho = \sigma$ and P ⚭ Q

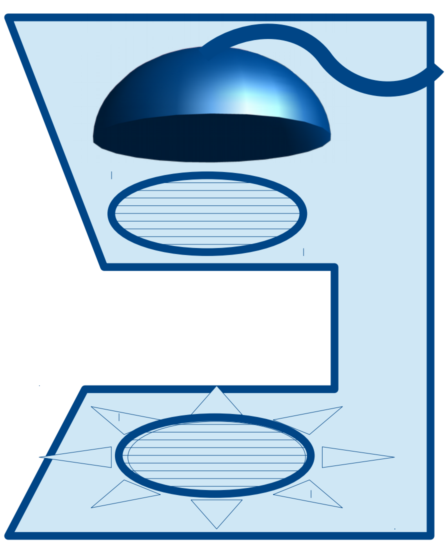
There exist commuting ***A*** and ***B*** with different normalization states.

Incompatibility

commutativity does not imply compatibility

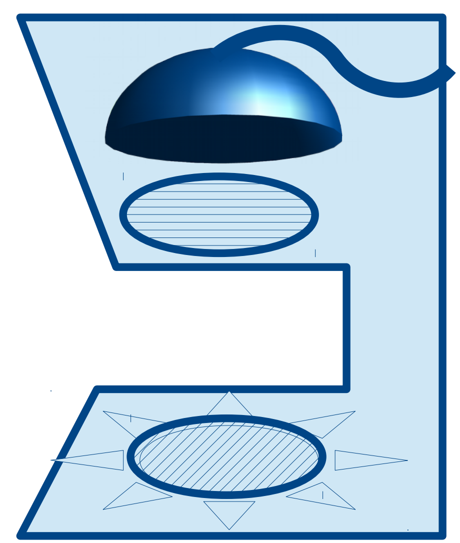
A* ⚭ *B iff $\rho = \sigma$ and P ⚭ Q

There exist commuting ***H*** and ***V*** with different normalization states.



$$\boldsymbol{V}_0 = \Pi_v \otimes \Pi_h$$
$$\boldsymbol{V}_1 = \Pi_v \otimes \Pi_v$$

$$\boldsymbol{H}_0 = \Pi_h \otimes \Pi_h$$
$$\boldsymbol{H}_1 = \Pi_h \otimes \Pi_v$$

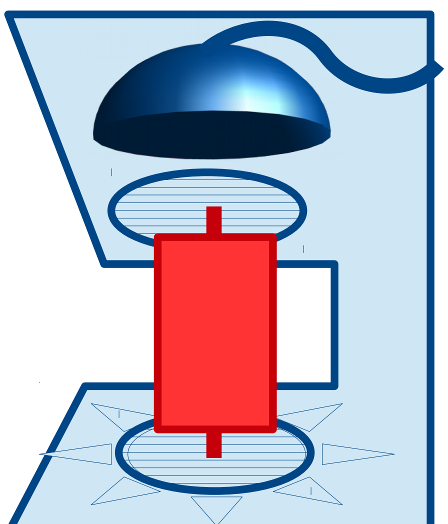


Incompatibility

commutativity does not imply compatibility

$A \bigcirc\!\!\!\bigcirc B$ iff $\rho = \sigma$ and $P \bigcirc\!\!\!\bigcirc Q$

There exist commuting $\boldsymbol{H}$ and $\boldsymbol{V}$ with different normalization states.

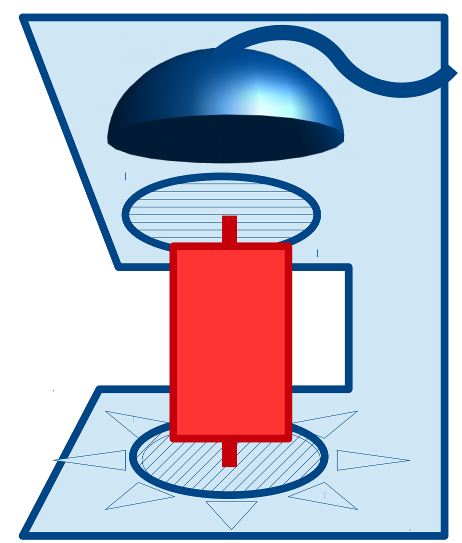


$\boldsymbol{V}_0 = \Pi_v \otimes \Pi_h$

$\boldsymbol{V}_1 = \Pi_v \otimes \Pi_v$

$\boldsymbol{H}_0 = \Pi_h \otimes \Pi_h$

$\boldsymbol{H}_1 = \Pi_h \otimes \Pi_v$



Incompatibility

quantification of incompatibility

A, B are λ **compatible** if there exist $\boldsymbol{N^{(A)}}, \boldsymbol{N^{(B)}}$:

$$(1-\lambda)\, A + \lambda N^{(A)} \;\text{CO}\; (1-\lambda)\, B + \lambda N^{(B)}$$

robustness of incompatibility R_t

- minimal λ such that ***A, B*** are λ compatible

Incompatibility

quantification of incompatibility

A, B are λ **compatible** if there exist $\boldsymbol{N^{(A)}}, \boldsymbol{N^{(B)}}$:

$$(1-\lambda)\, \boldsymbol{A} + \lambda \boldsymbol{N^{(A)}} \;\text{ꝏ}\; (1-\lambda)\, \boldsymbol{B} + \lambda \boldsymbol{N^{(B)}}$$

robustness of incompatibility R_t

- minimal λ such that ***A, B*** are λ compatible

$$\mathbf{0 \leq R_t \leq 1/2}$$

Incompatibility

robustness bounds

A* ⚭ *B iff ρ = σ and P ⚭ Q

- robustness of *A* and *B* … R_t
- robustness of ρ and σ … R_s
- robustness of P and Q … R_m

Incompatibility

robustness bounds

A* ꝏ *B iff $\rho = \sigma$ and P ꝏ Q

- **robustness of *A* and *B* … R_t**
- **robustness of ρ and σ … R_s**
- **robustness of P and Q … R_m**

$$0 \leq R_s \leq R_t \leq R_m \leq 1/2$$

only if $R_s = 0$

Incompatibility

maximal incompatibility, i.e. $R = 1/2$

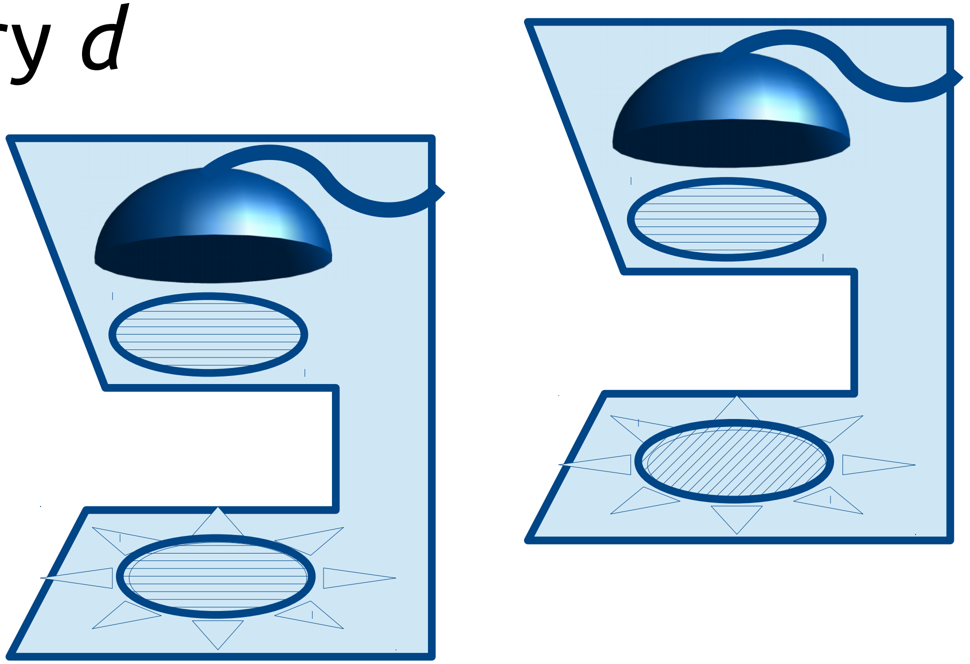
- for POVMs achieved **only for** $d = \infty$

- for testers **for arbitrary** d

Incompatibility

maximal incompatibility, i.e. $R = 1/2$

- for POVMs achieved only for $d = \infty$
- for testers for arbitrary d
- ***H*,*V*** example before



$R_s(\Pi_v,\Pi_h) = ½$ implies $R_t = ½$ (but $R_m = 0$)

CHSH with testers

<u>**the setting**</u> **same as usual**

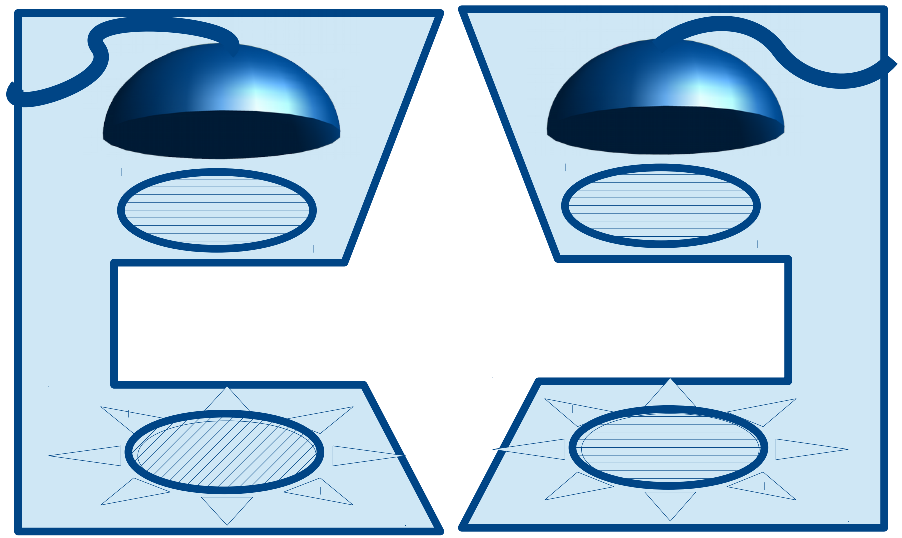
- Alice and Bob choosing testers $\boldsymbol{A}, \boldsymbol{A'}, \boldsymbol{B}, \boldsymbol{B'}$,
- each with outcomes labeled ±1
- CHSH-Bell inequality (for mean values)

$$-2 \leq A \otimes (B + B') + A' \otimes (B - B') \leq 2$$

- our task: **maximize over testers and channels**
- **motication:** maximal incompatibility

CHSH with testers

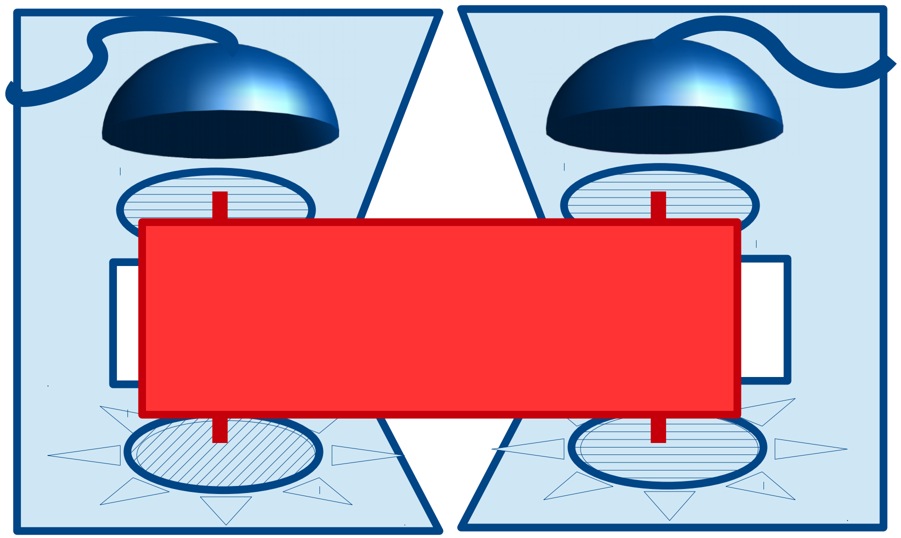
- Alice and Bob choosing testers being either ***H*** or ***V***, i.e. $\mathbf{-2 \leq V \otimes (V+H) + H \otimes (V-H) \leq 2}$



- measurement of the channel output: vertical polarization measurement
- test states: vertical or horizontal polarization

CHSH with testers

- Alice and Bob choosing measurements being either ***H*** or ***V***, i.e. $\mathbf{-2 \leq V\otimes(V+H)+H\otimes(V-H) \leq 2}$



- consider channel $\mathcal{C}[\omega]=(1-\kappa)\xi_{cor}+\kappa\xi_{acor}$

$\kappa = tr[\omega\Pi_h\otimes\Pi_h]$

$\xi_{cor}=(\Pi_h\otimes\Pi_h+\Pi_v\otimes\Pi_v)/2$

$\xi_{acor}=(\Pi_v\otimes\Pi_h+\Pi_v\otimes\Pi_h)/2$

CHSH with testers

- Alice and Bob choosing measurements being either ***H*** or ***V***, i.e.

- consider channel $\boldsymbol{C}[\omega]=(1-\kappa)\xi_{cor}+\kappa\xi_{acor}$

$$\kappa = tr[\omega\Pi_h\otimes\Pi_h]$$
$$\xi_{cor}=(\Pi_h\otimes\Pi_h+\Pi_v\otimes\Pi_v)/2$$
$$\xi_{acor}=(\Pi_v\otimes\Pi_h+\Pi_v\otimes\Pi_h)/2$$

- then $\boldsymbol{V}\otimes\boldsymbol{V} = \boldsymbol{V}\otimes\boldsymbol{H} = \boldsymbol{H}\otimes\boldsymbol{V} = \mathbf{1}$ and $\boldsymbol{H}\otimes\boldsymbol{H} = \mathbf{-1}$

$$\boldsymbol{V}\otimes(\boldsymbol{V}+\boldsymbol{H})+\boldsymbol{H}\otimes(\boldsymbol{V}-\boldsymbol{H}) = \mathbf{4}$$

CHSH with testers

- Alice and Bob choosing measurements being either ***H*** or ***V***, i.e.

- consider channel $\boldsymbol{C}[\omega]=(1-\kappa)\xi_{cor}+\kappa\xi_{acor}$

$$\kappa = tr[\omega\Pi_h\otimes\Pi_h]$$
$$\xi_{cor}=(\Pi_h\otimes\Pi_h+\Pi_v\otimes\Pi_v)/2$$
$$\xi_{acor}=(\Pi_v\otimes\Pi_h+\Pi_v\otimes\Pi_h)/2$$

- then $\boldsymbol{V\otimes V = V\otimes H = H\otimes V = 1}$ and $\boldsymbol{H\otimes H = -1}$

$$\boldsymbol{V\otimes(V+H)+H\otimes(V-H) = 4}$$

- Popescu-Rohrlich box correlations

Nonlocality with testers

- Popescu-Rohrlich box correlations

$$V \otimes (V + H) + H \otimes (V - H) = 4$$

- **Nonlocal?**
- yes in systems, but not spatially nonlocal

Nonlocality with testers

- Popescu-Rohrlich box correlations

$$V \otimes (V + H) + H \otimes (V - H) = 4$$

- **Nonlocal?**
- yes in systems, but not spatially nonlocal
- distributing bipartite channel over space(-time) is pure fantasy

Classical PR box analogue

there is nothing quantum in our example

Classical PR box analogue

there is nothing quantum in our example

- channel is measure-and-prepare (QC) channel
- measurement and initial states are diagonal
- conclusion: maximal violation observed also for classical processes

$$V \otimes (V + H) + H \otimes (V - H) = 4$$

- question: is there purely quantum example?

Conclusion

GPT of processes accommodate both qualitatively and quantitatively different incompatibility.

Conclusion

GPT of processes accommodate both qualitatively and quantitatively different incompatibility.

Maximal incompatibility $R_t(H,V) = 1/2$

PR box $V\otimes(V+H)+H\otimes(V-H) = 4$

- mathematical curiosity
- conceptual differences in incompatibility between classical and quantum processes?

Thank you

REFERENCES:

[arXiv:1511.00976] Michal Sedlák, Daniel Reitzner, Giulio Chiribella, Mário Ziman: **Incompatible measurements on quantum causal networks**, Phys. Rev. A 93, 052323 (2016)

[arXiv:1708.07425] Martin Plávala, Mário Ziman: **Popescu-Rohrlich box implementation in general probabilistic theory of processes**

joint work with
Giulio Chiribella, Daniel Reitzner, Michal Sedlák, Martin Plávala,