Unified picture for spatial, temporal and channel steering

Roope Uola, Fabiano Lever, Otfried Gühne, Juha-Pekka Pellonpää

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Outline

- Three forms of steering

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- Temporal steering and macrorealism
- Conclusions and outlook

Spatial and temporal steering

- In spatial scenario one is interested in assemblages consisting of unnormalised states

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- The steerability of these assemblages is decided by checking the existence of a local hidden state model (LHS):

$$\rho_{a|x} = \sum_{\lambda} p(\lambda)p(a|x,\lambda)\rho_{\lambda}, \tag{2}$$

where $p(\cdot)$ and $p(\cdot|x,\lambda)$ are probability distributions and $\{\rho_\lambda\}_\lambda$ are quantum states.

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- In temporal scenario the assemblage originates from instruments on a single system:

$$\rho_{a|x} = \mathcal{I}_{a|x}(\rho). \quad (3)$$

Channel steering

- The unsteerability of an instrument assemblage is defined as

$$\mathcal{I}_{a|x}(\rho) = \sum_{\lambda} p(a|x,\lambda)\Lambda_{\lambda}(\rho) \ \forall \rho, \tag{4}$$

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  where $p(\cdot|x,\lambda)$ are probability distributions and Λ_λ is an instrument with $\sum_\lambda \Lambda_\lambda = \sum_a \mathcal{I}_{a|x}$.
- The question of instrument steerability makes sense only for non-signalling instrument assemblages, i.e. $\sum_a \mathcal{I}_{a|x} = \sum_a \mathcal{I}_{a|x'}$.

Channel steering

- Non-signalling instrument assemblages are given through a minimal Stinespring dilation of their total channel Λ as

$$\mathcal{I}_{a|x}(\rho) = \mathrm{tr}_A[(A_{a|x} \otimes \mathbb{I}) V \rho V^*] \ \forall \rho, \tag{5}$$

where $V|\psi\rangle = \sum_k \varphi_k \otimes K_k|\psi\rangle$, $\{K_k\}_k$ is a linearly independent set of Kraus operators of Λ and $\{\varphi_k\}_k$ is an orthonormal basis.

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- In the case of minimal dilation, the connection between instruments and the POVMs on the dilation is one-to-one.

The joint measurability connection

- **Theorem**

  An instrument assemblage $\{\mathcal{I}_{a|x}\}_{a,x}$ given through their total channel's minimal dilation as

  $$\mathcal{I}_{a|x}(\rho) = tr_A[(A_{a|x} \otimes \mathbb{I})V\rho V^*] \;\forall\rho \tag{6}$$

  is unsteerable if and only if the POVMs $A_{a|x}$ are jointly measurable.

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  - Proof. For minimal dilation mother instruments and joint observables are one-to-one connected.

Exploiting the connection

Corollary

Joint measurement uncertainty relations are universal steering inequalities.

Exploiting the connection

- As an example, for a given state assemblage $\{\rho_{a|x}\}_{a,x}$ the interesting measurements are $\rho_B^{-1/2}\rho_{a|x}\rho_B^{-1/2}$, where $\rho_B = \sum_a \rho_{a|x}$.

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- The Busch criterion[1]:

$$\|\vec{a}+\vec{b}\| + \|\vec{a}-\vec{b}\| \leq 2. \tag{7}$$

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- To see this, define the instruments $\mathcal{I}_{a|x}$ through a total channel having the Kraus operators $K_i = |i\rangle\langle i|$, $i = 1, ..., d$.
- With these Kraus operators one has the purification of $\sum_a \rho_{a|x}$ in the range of the Stinespring isometry V.

Theorem

Non-signalling temporal steering on a d-level system and spatial steering on a $d*d$ system are fully equivalent problems. Namely, temporal steering can be embedded into the spatial scenario (through the Stinepring dilation) and the two can produce exactly the same assemblages.

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  - Proof. Tailoring Kraus operators such that the range of the Stinespring isometry includes a steerable but local state, and noticing that macrorealistic hidden variable models have the same structure as local hidden variable models does the job.

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- Joint measurement uncertainty relations are steering inequalities for all three steering scenarios.
- Showing an equivalency between spatial steering and non-signalling temporal steering.
- The set of temporally unsteerable correlations is a proper subset of macrorealistic correlations.

References for the steering vs joint measurement connection

- Measurement uncertainty relations from steering inequalities.[2]

[2] RU et al. PRA 94, 022109 (2016)
[3] S.L. Chen et al. PRL 116, 240401 (2016), RU et al. PRL 115, 230402 (2015)
[4] RU et al. PRL 113, 160403 (2014), see also M.T. Quintino et al. PRL 113, 160402 (2014)
[5] J. Kiukas et al. arXiv:1707.09237

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- Measurement uncertainty relations from steering inequalities.[2]
- (Semi-device independent) quantificatiers for incompatibility.[3]
- Joint measurements from hidden state models.[4]
- Jukka's talk.[5]

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