Bell’s Theorem(s) and Causation

based in part on

“*Causarum Investigatio* and the Two Bell's Theorems of John Bell”

in *Quantum [Un]speakables II* (Springer, 2017)

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Why do people care?

- "Bell's theorem is the most profound discovery in science" – H. P. Stapp (1977).
- "Bell's theorem is the most profound ramification of quantum theory that has been experimentally confirmed" – H. M. Wiseman (2014).
- "[Bell's theorem] had a huge impact on our modern view of quantum theory [and is] at the heart of many protocols and applications in quantum information processing" – N. Brunner *et al.* (2014).

Outline

- A cartoon history of quantum metaphysics
- An example of Bell correlations
- Bell's two Bell's theorems
- Teaching the controversy
- Seeking a resolution via causality
- Summary.

A cartoon history of quantum metaphysics

The Copenhagen Interpretation (1927)

The matrix representing, for example, the spin of an electron does not tell us its **value**. It tells us only the possible values we may find **if we measure it**.

Before the measurement the spin **has no value**. It is the act of measurement itself which **creates** the measured value.

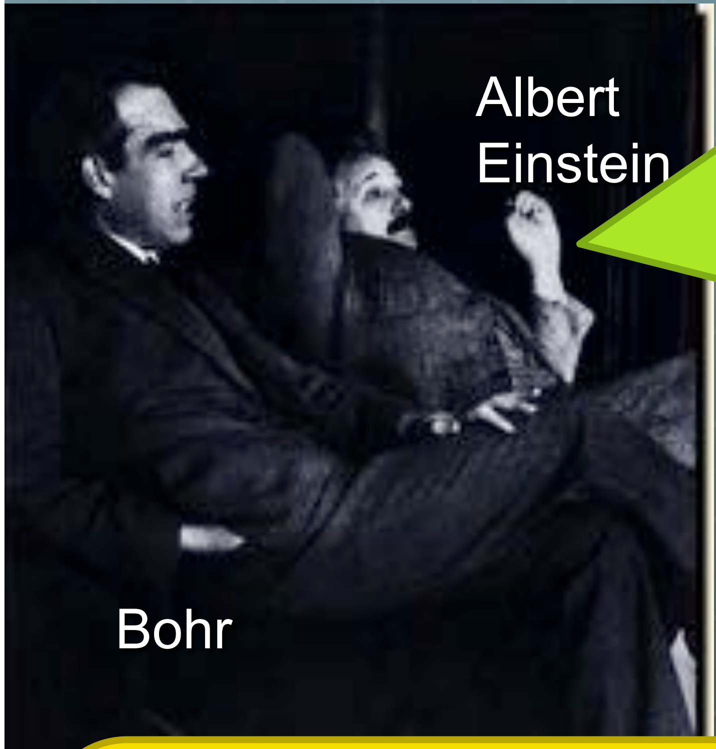
$$S_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{measured } S_x = \frac{1}{2} \text{ or } -\frac{1}{2}$$

In other words, our understanding of reality in the normal (macroscopic or *classical*) world does not apply to the microscopic or quantum world.

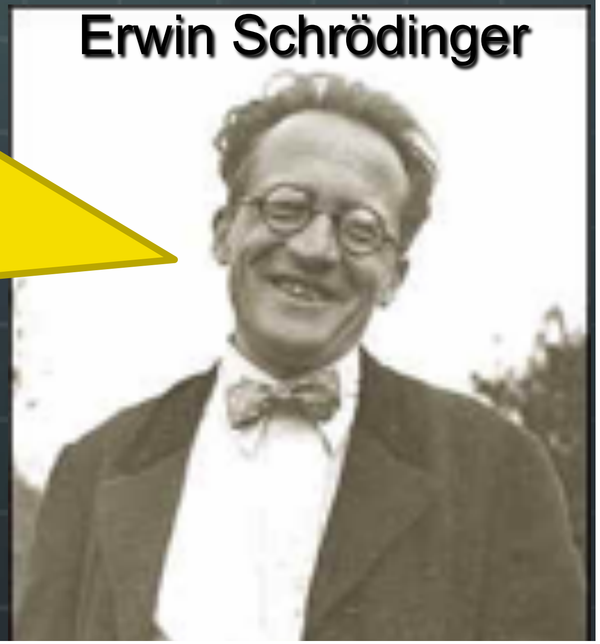
The nature of the quantum world is, and will forever be, unknowable.

Dissenting Opinions (1935)



I can't accept the way measurements create reality in your interpretation, Bohr. The whole idea of "measurement" is vague – will a sidelong glance by a mouse suffice?

Physical quantities must have values prior to measurement. A proper theory, with *hidden variables*, will ensure this, and will also exorcise the **spooky action at a distance** that haunts your interpretation.



The **action at a distance** Einstein (rightly) worries about comes about when two particles have interacted and become **Entangled**.

It is because when one particle is measured, the result causes an *instantaneous change* in the other particle's state, no matter how distant.

Bell's Theorem (1964)

- For a generation, physicists forgot the debate. Then …

I have proven a **theorem** showing Einstein was wrong! His idea of hidden variables *cannot* remove the **action at a distance** in quantum mechanics.

Because of quantum entanglement, the theory predicts correlations between distant measurement results that *cannot be explained without* ***nonlocality*** (**instantaneous transfer of information across a distance**).

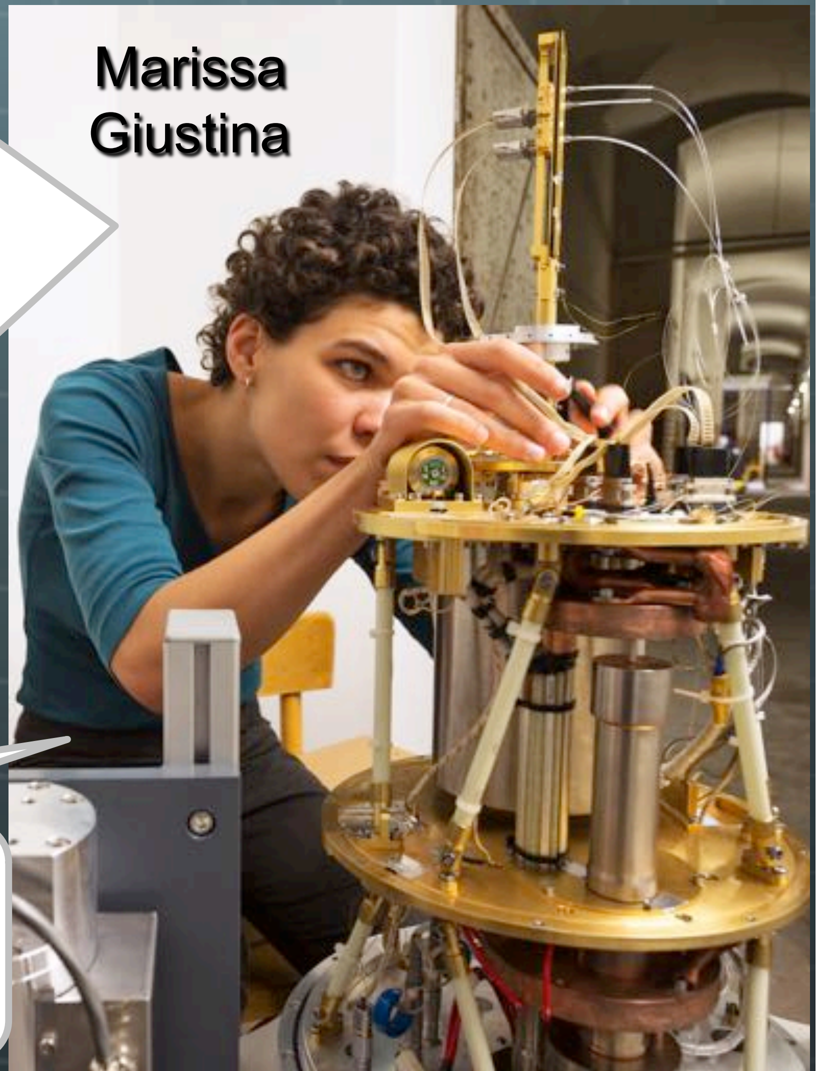
But, like Einstein and Schrödinger, I find this very unsettling. Can nature really be so strange?

Experimental Tests (1973-2015)

- So is there Bell-nonlocality, or is QM wrong (as Bell suspected)?

We have created pairs of entangled photons, and sent them to two labs, more than 50m apart. There we performed, at precisely the same time, measurements of their polarization (which has only two possible values, like the spin of an electron). The results of these measurements have correlations just as QM predicts, which violate the "locality inequality" John Bell introduced to prove his famous theorem.

But before you ask … **No, we cannot use these correlations to send signals faster than light.** It doesn't work like that.



An example of Bell correlations

Aravind's magic matrix

Bell's theorem can be proven in many different ways. This is my proof, which I published in 2002.

- Consider a 3x3 matrix for which each of the nine entries is a zero or a one, like this example:

0	1	0
0	1	1
0	0	1

- A *magic matrix* has the following property: each column has an even number of ones, and each row has an odd number of ones.

- Why is it *magic*? Because it **can't exist**!
  At least one row or column must fail to satisfy these requirements (e.g. the 2nd row above).

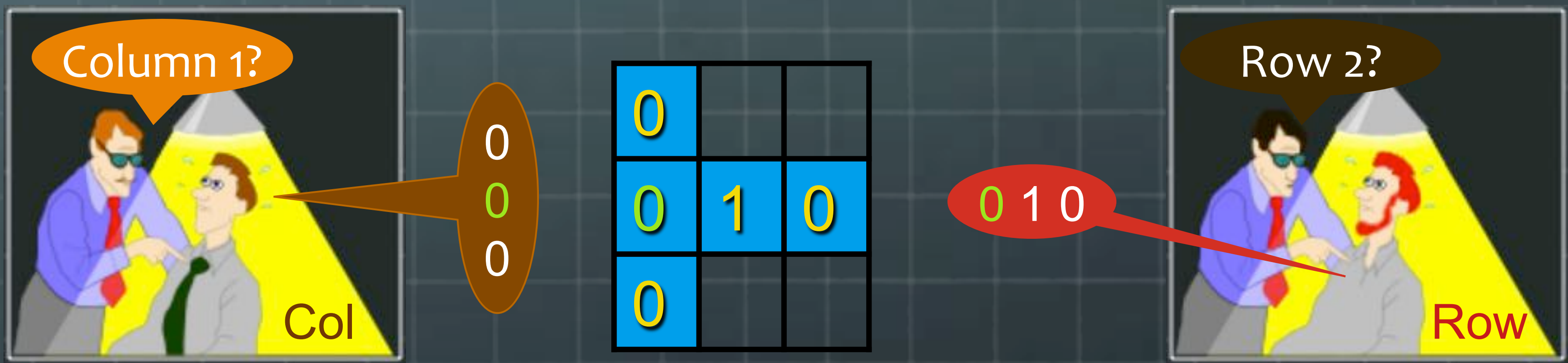
Aravind's Game (1)

- In this game the two players (Rowan and Colin) have to convince us (the referees) that they own one of these impossible *magic matrices.*
- To make life easier for them , we will not ask them to tell us all the entries of the magic matrix. Instead, we will ask Colin for one column, and Rowan for one row.
- Col's column should have an even number of ones. Row's row should have an odd number of ones. And the common entry should agree.
- Here is an example of a success where we asked Col for column 1 and Row for row 2.

0		
0	1	0
0		

Aravind's Game (2)

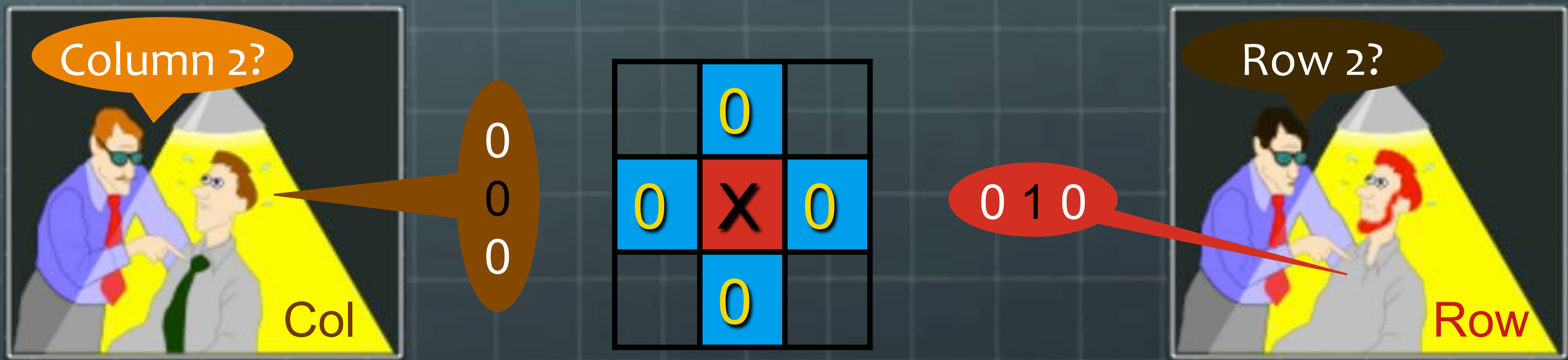
- If Col or Row can hear the question asked to the other player, then this game is too easy. E.g. Col can always answer 0,0,0, and Row can answer with a 1 and two 0s, making sure that he doesn't put the 1 in Col's column.
- So to make the game harder, we put Col and Row in distant concrete bunkers to prevent any communication.



In this case Col and Row succeed, but can they always win?

Aravind's Game (3)

- Now Col and Row have to agree on how to answer every question before they are separated.
- But because the magic matrix cannot exist, they will inevitably give incompatible answers some of the time.
- In fact, they must fail (as below) at least 1/9 of the time.

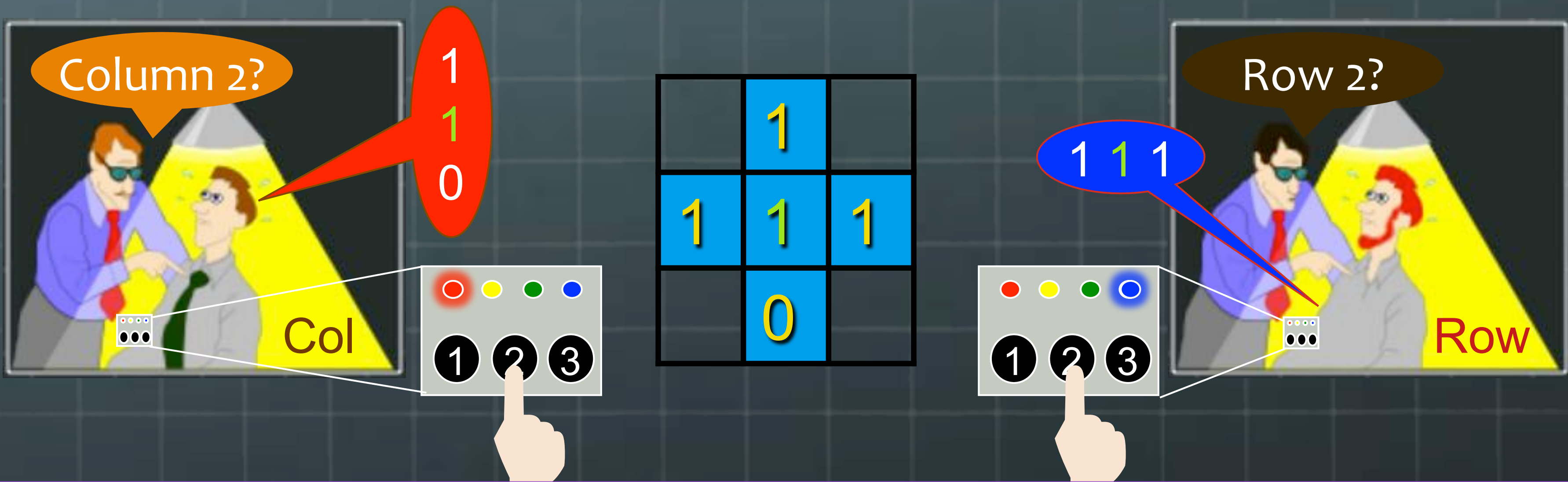


But what if Col and Row share entangled pairs of particles?

Aravind's Game (4)

If Col and Row share pairs of entangled particles, they can win 100% of the time! In theory, this would work no matter how far apart they are.

Col and Row each needs a device which can make one of three different measurements on one quantum particle (hidden inside the device), each of which can yield four possible different answers (flashing lights).



Since Col and Row can succeed with P>8/9, that proves Bell's theorem.

Bell’s two Bell’s theorems

Bell's 1st Bell's theorem (1964)

In a theory in which parameters are added to quantum mechanics to **determine the results of individual measurements**, without changing the statistical predictions, there **must be** a mechanism whereby **the setting of one measuring device can influence the reading of another instrument, however remote.** (1964)

i.e. quantum correlations violate the joint assumptions of:

- "Predetermination" (of measurement results).
- "Locality, [meaning] that the result of a measurement on one system be unaffected by operations on a distant system".

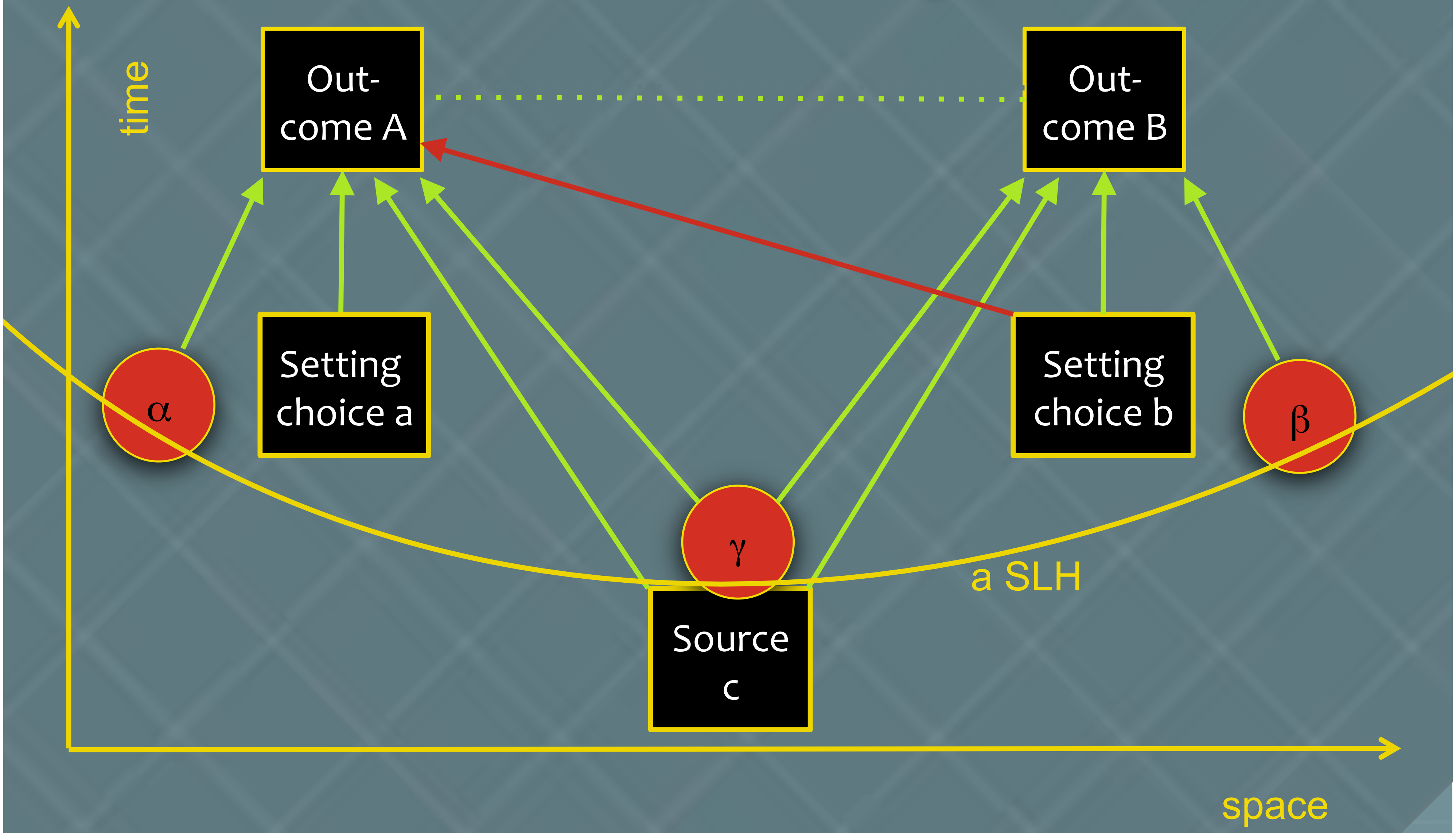
Bell's 2nd Bell's theorem (1976)

My … notion of local causality is that events in [one lab] should not be causes of events in [a *space-like separated* lab], and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the *overlap in their backward light cones*. (1976)

A consequence … of 'local causality' [is] the outcomes [in the two labs] having no [statistical] dependence **on one another** nor on the settings of the remote [measurement], but only on the local [settings] and on the [common] past causes. (1990)

Quantum mechanics … gives certain **correlations** which … **cannot be [reproduced by] a locally causal theory.** (1976)

Minkowski Diagram



Or, if you prefer probabilities …

- A quantum phenomena is described by a theory if

$$P_{obs}(A,B|a,b,c) = \Sigma_\lambda P(A,B|a,b,c,\lambda)P(\lambda|c), \text{ where } \lambda=(\alpha,\beta,\gamma)$$

- **Predetermination**: $P(A,B|a,b,c,\lambda) = 0$ or 1.
  ≠ **Predictability:** $P_{obs}(A,B|a,b,c) = 0$ or 1.
- **Locality**: $P(A|a,b,c,\lambda) = P(A|a,c,\lambda)$.
  ≠ **Signal locality**: $P_{obs}(A|a,b,c) = P_{obs}(A|a,c)$.
- **Bell 1964**: there is no theory reproducing quantum phenomena satisfying **locality** and **predetermination**.
- **Local causality**: $P(A|a,B,b,c,\lambda) = P(A|a,\alpha,c,\gamma)$.
- **Bell 1976**: L&OD can be replaced by **local causality** (LC).
- Both assume **no-superdeterminism**: $P(\lambda|a,c) = P(\lambda|c)$

Teaching the controversy

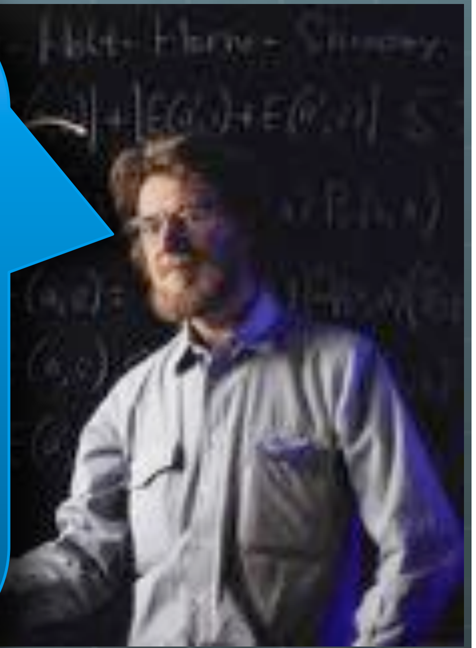
Two theorems, two camps

Unfortunately the confusion started even in 1976:

- Having defined “local causality” Bell immediately started using “locality” as a synonym for it, even though it was different from “locality” as he had used in 1964.
- Soon afterwards, Bell started claiming that by “locality” he had *always* meant local causality, and that this was the “sacred principle” which Einstein had believed in.
- Followers of Bell therefore often state that Bell’s theorem is that quantum phenomena are nonlocal.
- Most quantum physicists only know Bell’s 1964 theorem, and say that we can keep locality if we give up predetermination.

Realism vs Operationalism

My theorem uses only one assumption: local causality (or 'locality' as we might call it for short). This is the only non-piddling way to define the principle of relativity for statistical theories. It is essentially what EPR assumed in 1935. They showed that operational quantum mechanics is nonlocal, and I showed in 1964 that adding hidden variables cannot solve the problem. Experiments have thus proven the principle of relativity false. The world is nonlocal.



Bell's theorem uses two assumptions. The first assumption is locality. This means no signaling faster than light (which is all the principle of relativity implies) even for hidden variable theories. Operational quantum mechanics respects locality. The second assumption is realism/classicality/predetermination/…. Clearly it is this second assumption that we should abandon, whatever we call it. Locality is here to stay.

Terminology, but not only

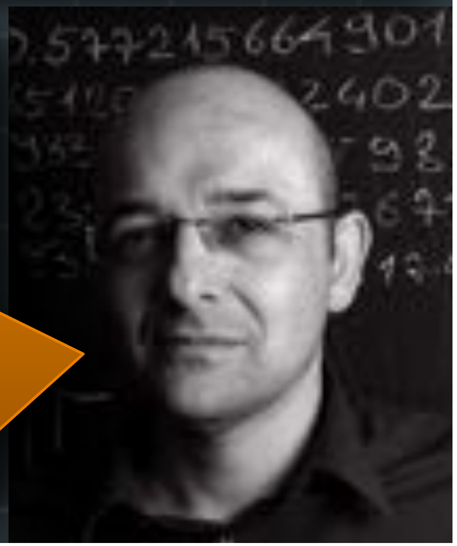
- For realists, locality = local causality, and Bell = Bell76, while for operationalists, locality ≈ no FTL signalling, and Bell = Bell64.

[To use] no signalling faster than light … as *the* expression of the fundamental causal structure of theoretical physics is hard for me to accept. For one thing we have lost the idea that **correlations can be explained**. More importantly, [it is] desperately vague, and immediately provokes the question: who de we think *we* are? – We who can make "measurements" etc.

Also, **predetermination** *is a sacrificial lamb - it is unnecessarily strong, and so too easily given up.*

As an operationalist, I don't see why we should expect all correlations in nature to be explicable in the way that they are in classical physics.

Also, how can you realists criticize us for using a vague, agent-centric notion like "measurement" in defining locality, when you still need to assume **no-superdeterminism**, which is agent-centric too?



Seeking a resolution via causality

AXIOMS

- **AXIOM 1: MACROREALITY.**
  An event observed by any observer is a real single event, and not 'relative' to anything or anyone.

- **AXIOM 2: MINKOWSKI** [TIME-ORIENTABLE LORENTZIAN] **SPACE-TIME.**
  Concepts like light-cones, space-time variables (STVs), space-like separated (SLS), space-like hypersurfaces (SLHs) *etc.* can be applied unambiguously in ordinary laboratory situations.

- **AXIOM 3:** (PARTIAL) **TEMPORAL ORDER.**
  For any STV *A*, there is at least one SLH containing *A* that separates events in *A*'s past from events with *A* in their past.

- **AXIOM 4: CAUSAL ARROW.**
  Any cause of a STV is another STV in its past.

(Axioms 2-4 guarantee that there are no causal loops.)

POSTULATES

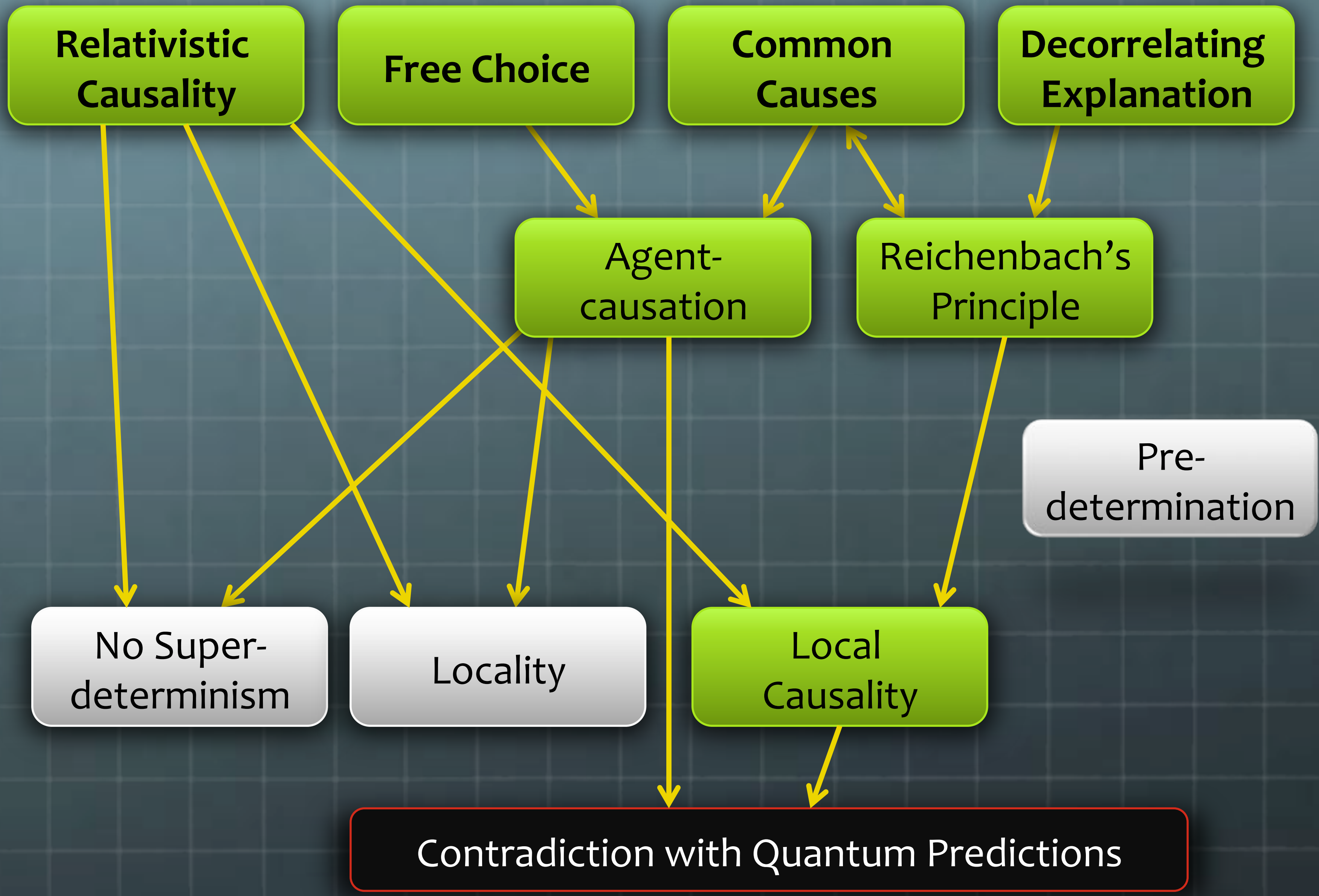
- **POSTULATE 1: FREE CHOICE.**
  A freely chosen action has no causes that we need to worry about.

- **POSTULATE 2: RELATIVISTIC CAUSALITY.**
  The past is the past light-cone.

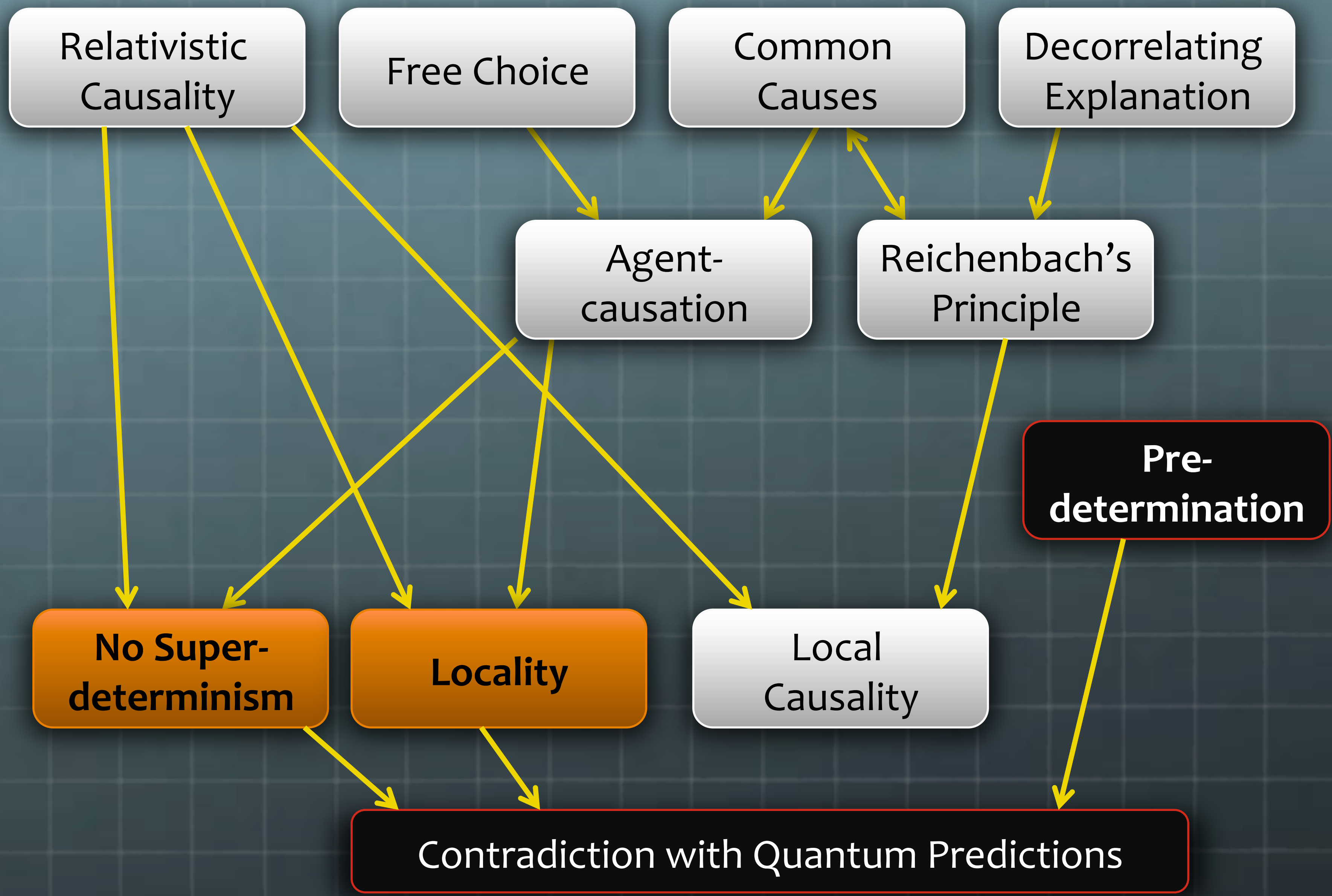
- **POSTULATE 3: COMMON CAUSES.**
  If two sets of STVs, $\mathcal{A}$ and $\mathcal{B}$, are correlated, and no STV in either is a cause of any STV in the other, then they have a set of common causes $\mathcal{C}$ that explains the correlation.

- **POSTULATE 4: DECORRELATING EXPLANATION.**
  A set of causes $\mathcal{C}$, common to $\mathcal{A}$ and $\mathcal{B}$, explains a correlation between $\mathcal{A}$ and $\mathcal{B}$ only if conditioning on the value of $\mathcal{C}$ always eliminates the correlation.

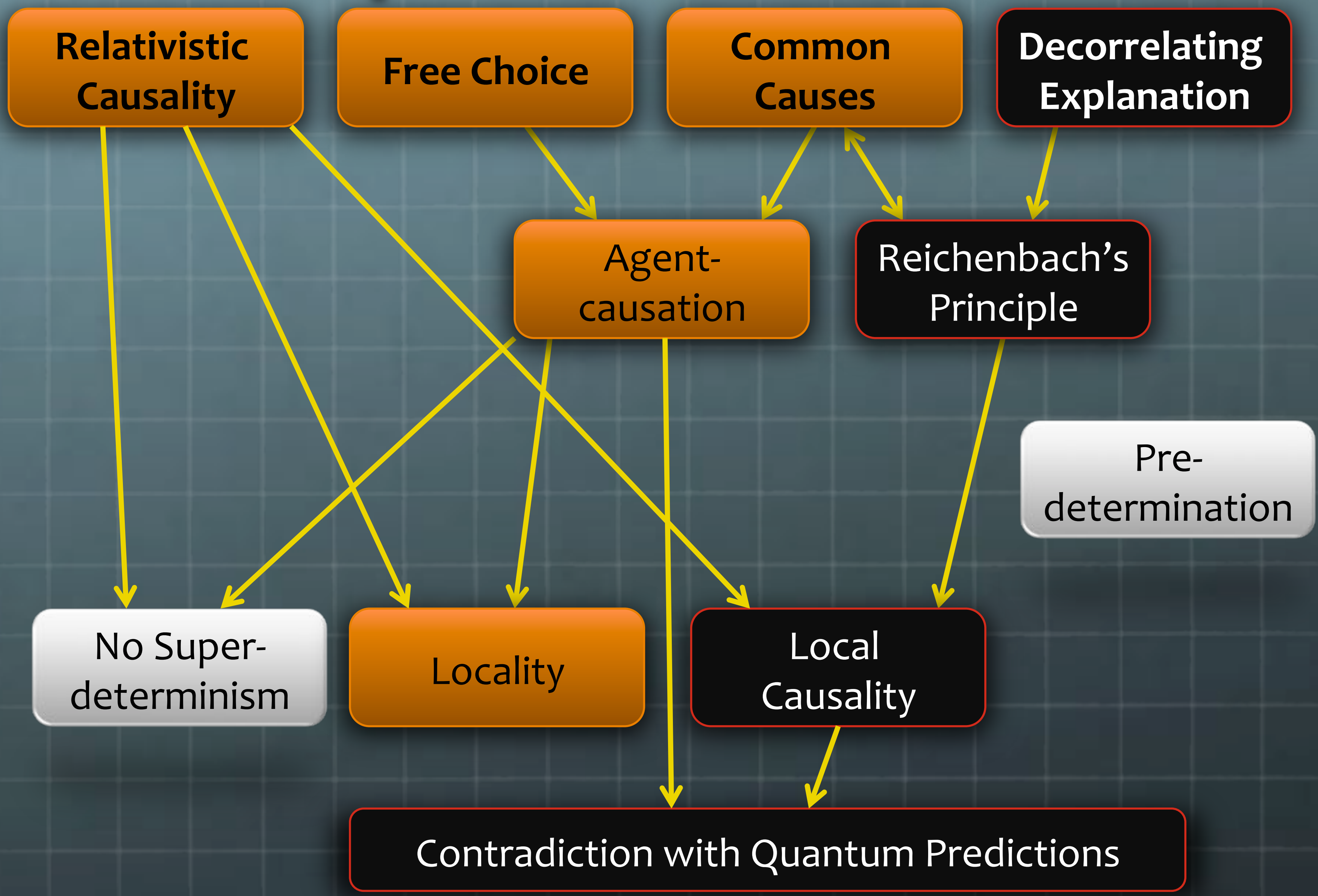
Commonality in a Common Cause



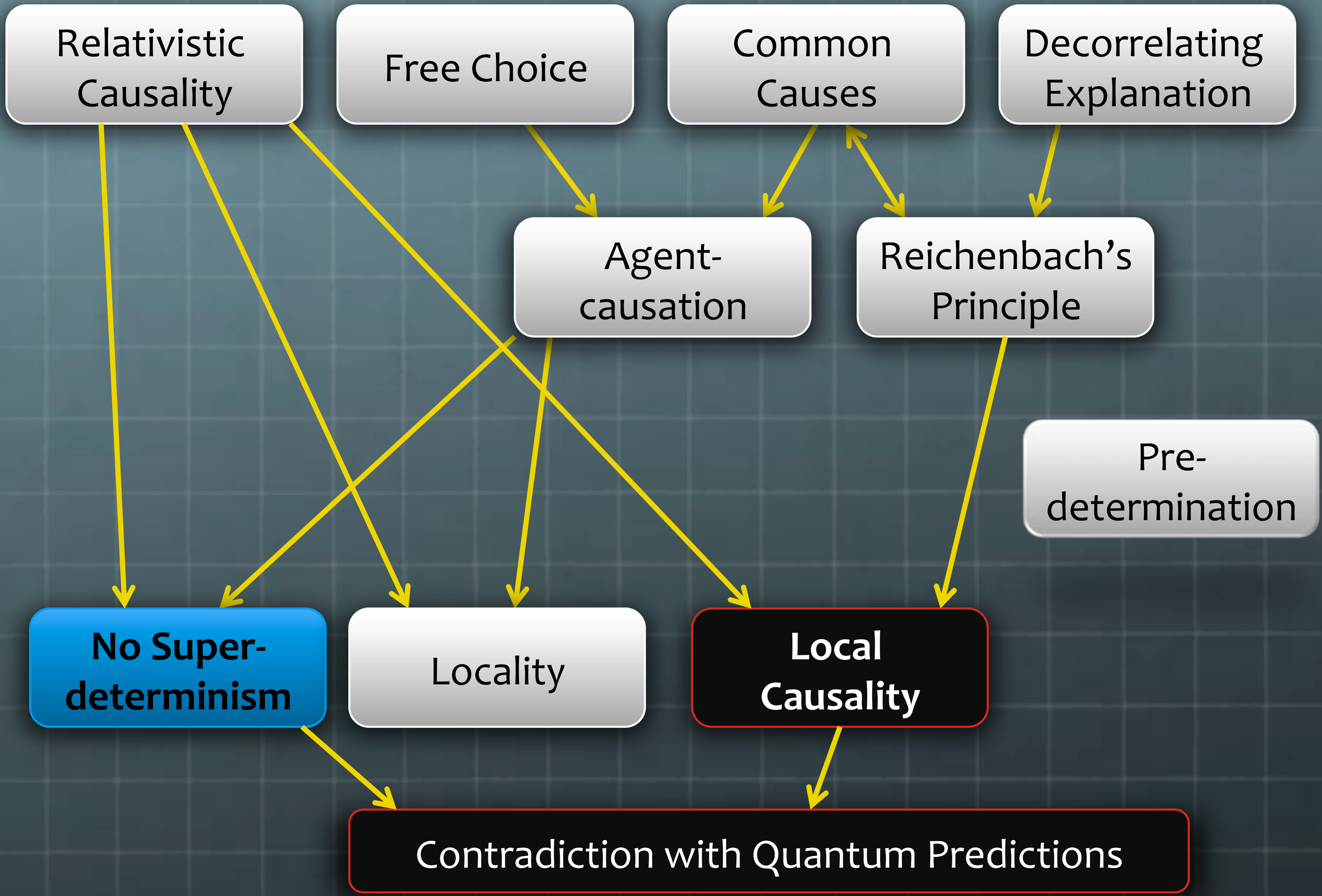
Original Operationalist Version (1964)



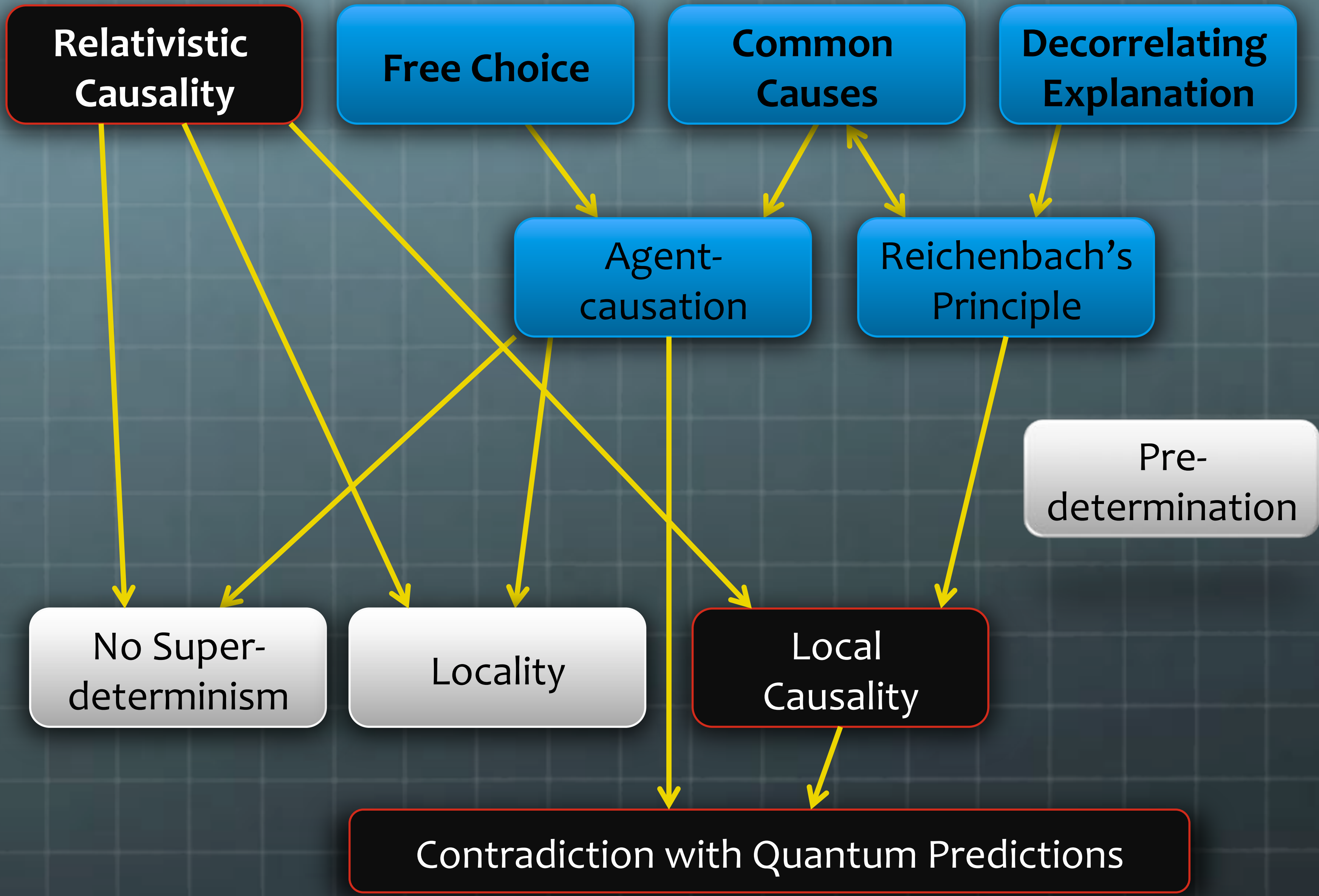
Final Operationalist Version



Original Realist Version (1976)



Final Realist Version



Summary

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- There are actually two Bell's theorems (1964 and 1976), with essentially the same proof, but with different assumptions:
  - Bell 1964, favoured by operationalists
  - Bell 1976, favoured by realists
- By considering notions of causation we can:
  - Understand why the two camps disagree;
  - Find a form of Bell's theorem both can agree upon; …
- Still, we have to give up something important:
  - Macroreality; Minkowski ST; temporal order; causal arrow;
  - Freedom of choice; relativistic causality; common cause principle; or "decorrelating explanation" principle.
- ➔ The ramifications of Bell's theorem are still most profound.