What is and to which end does one study Bohmian Mechanics?

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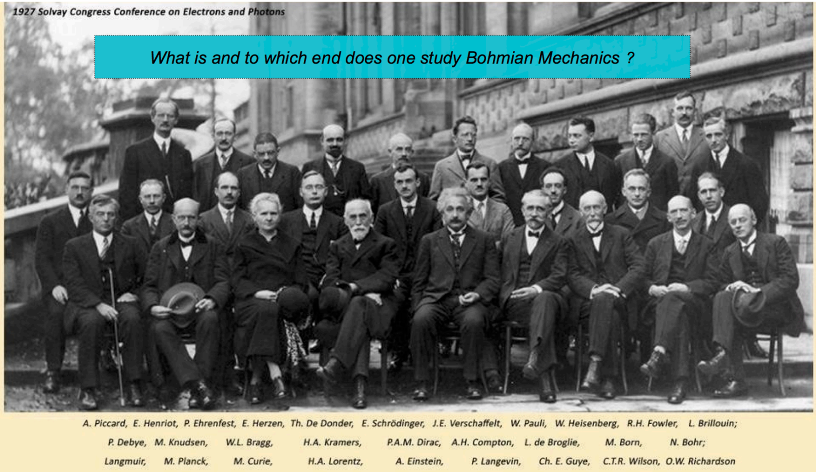
The 1927 Solvay conference could have made the question obsolete



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- particles move

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- respects Galilean symmetry but is non-Newtonian. It is a mathematically consistent simplification of the Hamilton Jacobi idea of mechanics:

$$Q(t) = (\mathbf{Q}_1(t), ..., \mathbf{Q}_N(t)), \quad \nabla = \frac{\partial}{\partial q} \quad \text{configuration}$$

obeys (time reversal invariance in "first order" theory achieved by complex conjugation)

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = v^{\Psi}(Q(t), t) = \alpha \mathrm{Im} \frac{\Psi^* \nabla \Psi}{\Psi^* \Psi}(Q(t), t) \quad \text{guiding equation}$$

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- the "universal" wave function

$$\Psi : \mathbb{R}^{3N} \times \mathbb{R} \mapsto \mathbb{C}^{(n)} \quad (q = (\mathbf{q}_1, \ldots, \mathbf{q}_N), t) \mapsto \Psi(q, t)$$

ψ

Ψ

- solves the Schrödinger equation

$$\mathrm{i}\frac{\partial \Psi}{\partial t}(q,t) = H\Psi(q,t) \quad \text{“Schrödinger“ equation}$$

$H = -\sum_{k=1}^{n} \frac{\alpha}{2}\Delta_k + W$ (Galilean invariant operator)

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- $v^{\Psi} = \frac{\alpha}{\hbar}\nabla S \Longrightarrow$ identify $\alpha = \frac{\hbar}{m}$ and $\frac{W}{\hbar} =: V$ as the "Newtonian potential" (de Broglie 1927)
- Newtonian Bohmian motion for "Quantum Potential" $\frac{\hbar^2}{2m}\frac{\Delta R}{R} \approx 0$

Bohmian mechanics with Newtonian identification of parameters

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = v^{\Psi}(Q(t), t) = \hbar m^{-1} \mathrm{Im} \frac{\Psi^* \nabla \Psi}{\Psi^* \Psi}(Q(t), t)$$

where m is a diagonal matrix with mass entries m_k

$$\mathrm{i}\hbar \frac{\partial \Psi}{\partial t}(q, t) = \left(-\sum_{k=1}^{n} \frac{\hbar^2}{2m_k} \Delta_k + V(q) \right) \Psi(q, t)$$

Analogy: Boltzmann's constant k_B relates thermodynamics to Newtonian mechanics, $\hbar$ relates Newtonian mechanics to Bohmian Mechanics

simplest way to Bohmian mechanics

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- $v^\Psi = \frac{j^\Psi}{|\Psi|^2}$ (Pauli 1927, J.S. Bell 1964)
- read the above as $v^\Psi \sim j^\Psi$ for relativistic generalization

''Bohmian Mechanics agrees with Quantum Predictions''

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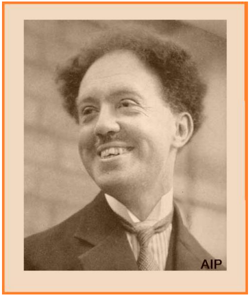
- quantum flux equation means $\rho(t) = |\Psi(t)|^2$ is equivariant: Assume Q is distributed according to $\rho = |\Psi|^2$ then $Q(t)$ at any other time is distributed according to $\rho(t) = |\Psi(t)|^2$

History: The 1927 Solvay Conference



The Harmonizer: de Broglie 1892- 1987: wave and particle

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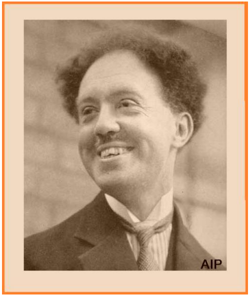

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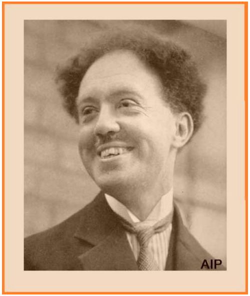

The Harmonizer: de Broglie 1892- 1987: wave and particle

de Broglie proposed that the wave guides the particles: (Q, ψ)

BUT he was ridiculed at the Solvay conference in 1927. WHY?

One man did not ridicule him, Hendrik A. Lorentz, who at the 1927 Solvay conference said:

I imagine that, in the new theory, one still has electrons. It is of course possible that in the new theory, once it is well-developed, one will have to suppose that the electrons undergo transformations. I happily concede that the electron may dissolve into a cloud. But then I would try to discover on which occasion this transformation occurs. If one wished to forbid me such an enquiry by invoking a principle, that would trouble me very much. It seems to me that one may always hope one will do later that which we cannot yet do at the moment. Even if one abandons the old ideas, one may always preserve the old classifications. I should like to preserve this ideal of the past, to describe everything that happens in the world with distinct images. I am ready to accept other theories, on condition that one is able to re-express them in terms of clear and distinct images.

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- solution: standard birth and death process

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for entangled wave function influenced by all particles at $t \Longrightarrow$ manifestly not local, against the "spirit of relativity"

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Since then, it had often been claimed that Bell had disproven BM

de Broglie's reaction to the onslaught

L'onde ψ utilisé en Mécanique ondulatoire ne peut pas être une réalité physique: Sa normalsiation est arbitraire, sa propagation est censée s'effectuer en général dans un espace de configuration visiblement fictif, et, conformément aux idées de M. Born, elle n'est qu'une représentation de probabilité dépendant de l'état de nos connaissances et brusquement modifiée par les informations que nous apporte toute nouvelle mesure. One ne peut donc obtenir à l'aide de la seule théorie de l'onde-pilote une interprétation causale et objective de la mécanique ondulatoire en supposant que le corpuscule est guidé par l'onde ψ. Pour cette raison, je m'étais entièrement rallié depuis 1927 à l'interprétation purement probabiliste de MM. Born, Bohr et Heisenberg.

de Broglie nevertheless

On retrouve donc l'hypothèse de M. Born sur la signification statistique de $|\psi|^2$. Cette hypothèse présente ici comme un peu analogue à celle qu'on fait en Mécanique statistique quand on admet on s'appuyant uniquement sur la theorème de Liouville, l' égale probabilité des élements égaux d'extension-en-phase. Mais une justification plus complète parait nécessaire...

universal Ψ and subsystem's φ

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- Boltzmann's statistical analysis of BM ($\rho = |\varphi|^2$) based on typicality measure $\mathrm{d}\mathbb{P}^\Psi = |\Psi|^2 \mathrm{d}q^{3N}$ which is equivariant (cf. quantum flux equation)
  Bohmian flow $T_t^\Psi : Q \mapsto Q(t)$ commutes with Schrödinger evolution $\Psi \mapsto \Psi_t$:
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- φ is wave function of subsystem

conditional wave function φ of subsystem

$$X = (\mathbf{X}_1, \ldots, \mathbf{X}_n) \quad \text{system's particles}$$

$$Q = (X, Y) \quad \text{splitting in system and rest of universe}$$

$$\Downarrow$$

$$\varphi^Y(x) := \Psi(x, Y)/\|\Psi(Y)\|$$

normalized *conditional* wave function of subsystem guides X

The conditional wave function is the "collapsing wave function" of orthodox quantum mechanics

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crucial "conditional measure" formula

$$\mathbb{P}^{\Psi}(X \in \mathrm{d}x | \varphi^Y = \varphi) = |\varphi(x)|^2 \mathrm{d}x$$

Autonomous subsystem: effective wave function

If wave function of universe $\Psi(x, y) = \varphi(x)\Phi(y) + \Psi(x, y)^{\perp}$

where

$y\text{-supp}\Phi \cap y\text{-supp}\Psi^{\perp} = \emptyset$ macroscopically disjoint

and if $Y \in \text{supp}\Phi$ e.g. preparation of φ

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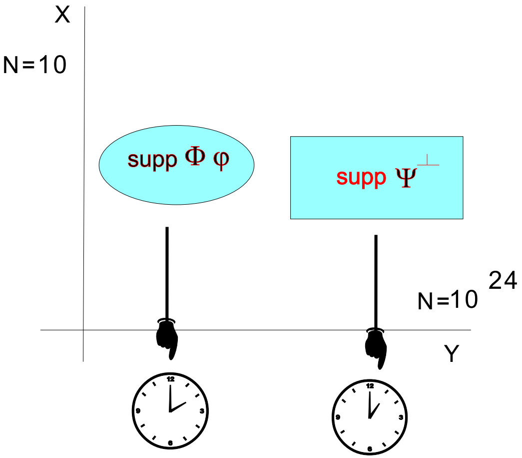
decoherence sustains disjointness of supports

$\Downarrow$

Schrödinger equation for φ for some time

$$\mathrm{i}\hbar\frac{\partial\varphi}{\partial t}(x,t) = -\sum_{k=1}^{n}\frac{\hbar^2}{2m_k}\Delta_k\varphi(x,t) + V(x)\varphi(x,t)$$

macroscopically disjoint Y- supports



Bohmian Subsystem

$$(X, \varphi) \quad \text{physical variables}$$

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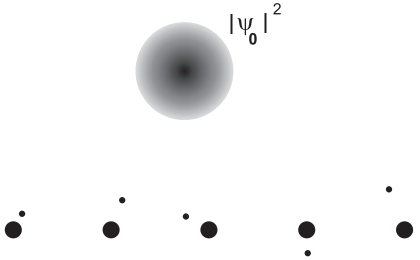
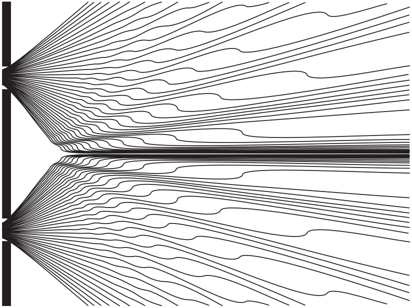
- Consider an ensemble of subsystems each having effective wave function φ
- Theorem (DGZ): $\mathbb{P}^{\Psi}$*-typically the empirical distribution* ρ *of* X*-values is* $\approx |\varphi|^2$
- In short: Quantum Equilibrium holds!

Hydrogene ground state: $\rho = |\psi_0|^2, \quad v^{\psi_0} = 0$

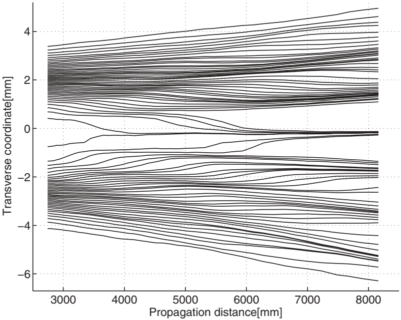


two slit experiment, computed trajectories



computer simulation of Bohmian trajectories by Chris Dewdney

two slit experiment: weak measurement of phase, trajectories reconstructed



S.Kocsis et al: Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer. Science 2011

operational analysis of BM: PVM's

system (X, φ) and apparatus (Y, Φ) with pointer positions Y_α pointing towards value α. Suppose

$$\varphi_\alpha \Phi \overset{\text{Schrödinger evolution}}{\longrightarrow} \varphi_\alpha \Phi_\alpha$$

then for $\varphi = \sum_\alpha c_\alpha \varphi_\alpha, \quad \sum_\alpha |c_\alpha|^2 = 1$

$$\varphi \Phi \overset{\text{Schrödinger evolution}}{\longrightarrow} \Psi = \sum_\alpha c_\alpha \varphi_\alpha \Phi_\alpha$$

$\Longrightarrow$

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- PVM $\Rightarrow$ self adjoint $\hat{A} = \sum \alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|$ encodes all relevant data for the experiment

operational analysis: POVMs

Suppose not $\varphi_\alpha \Phi \overset{\text{Schrödinger evolution}}{\longrightarrow} \varphi_\alpha \Phi_\alpha$

$$\text{but apparatus } (Y, \psi) \text{ with values} \quad F(Y) = \lambda \in \Lambda$$

then probability for pointer position if system's wave function is φ

$$\mathrm{Prob}^{\varphi}(A) := \mathbb{P}^{\Phi_T}(F^{-1}(A)) \, , A \subset \Lambda$$

can be written as

$$= \langle \varphi | \int_A d\lambda |\phi_\lambda\rangle\langle\phi_\lambda| |\varphi\rangle$$

where in general $\langle \phi_\lambda | \phi_\nu \rangle \neq \delta_{\lambda,\nu}$ (overcomplete set)

$$\int_A d\lambda |\phi_\lambda\rangle\langle\phi_\lambda|, \quad A \subset \Lambda$$

is called POVM or generalised observable

Heisenberg's uncertainty relation follows from BM

Equivarance of $\rho = |\varphi|^2$

$$\frac{\partial |\varphi(x,t)|^2}{\partial t} = -\mathrm{div} v^{\varphi}(x,t)|\varphi(x,t)|^2 \Longrightarrow$$

$$\mathbb{E}^{\varphi}(f(X(t)) = \mathbb{E}^{\varphi(t)}(f(X))$$

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$$\frac{m}{\hbar}V_\infty :\overset{\mathcal{L}}{=} \lim_{t\to\infty} \frac{m}{\hbar}\frac{X(t)}{t} \quad \text{is distributed according to } |\hat{\varphi}|^2$$

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$$\Downarrow$$

$$\hat{P} = \int dk k|k\rangle\langle k| \quad \text{momentum observable}$$

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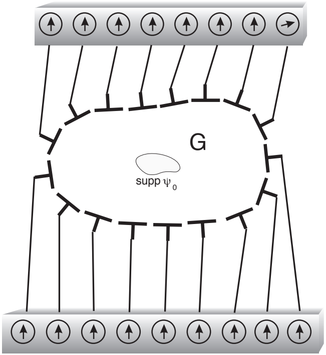
arrival time statistics



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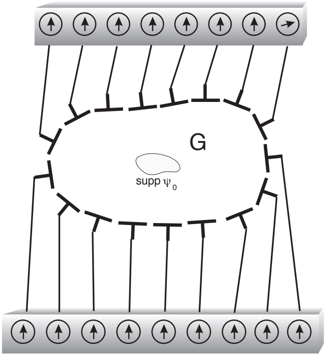
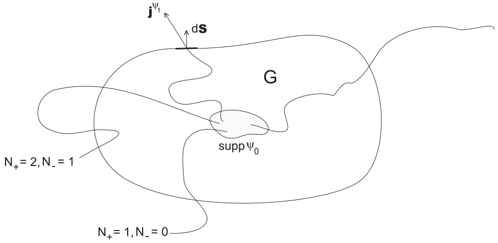


when and where does a counter click?

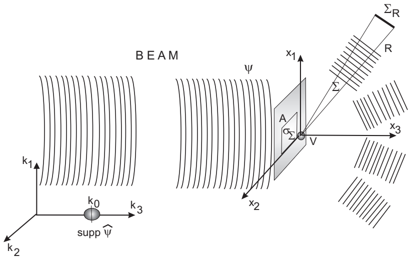
time statistics for Bohmian flow



$$\mathbb{P}^{\psi}(X(\tau) \in dS, \tau \in dt) = v^{\psi}|\psi|^2 \cdot dSdt = j^{\psi} \cdot dSdt$$

scattering formalism and scattering cross section

Born's scattering formula for single particle



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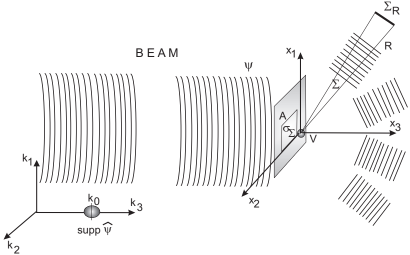
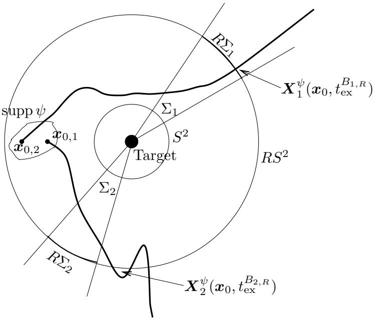


$$\mathbb{P}^{\psi}(X(\tau) \in \Sigma_R, \tau \in [0,\infty)) \overset{\text{R large}}{\approx} \int_{C_\Sigma} dk \langle k|S\psi_{\text{in}}\rangle^2$$

many particle scattering



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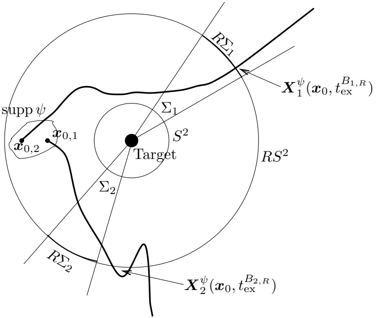

"genuine" Bohmian analysis

Gretchen Frage: Wie hältst du es mit der Relativität?

Relativistic Bohmian Theory

Weinberg's challenge

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Relativistic Bohmian Theory

Weinberg's challenge

It does not seem possible to extend Bohm's version of quantum mechanics to theories in which particles can be created and destroyed, which includes all known relativistic quantum theories. (Steven Weinberg to Shelly Goldstein, 1996)

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- technically, i.e. mathematically not possible?
- not possible in a deterministic theory of particles in motion?

Creation and Annihilation, the configuration space

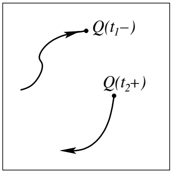
$\mathcal{Q}$: configuration space $\quad \mathcal{Q} = \bigcup_{n=0}^{\infty} \mathcal{Q}^{(n)}$ (disjoint union)

a) $\mathcal{Q}^{(0)}$ no particle b) $\mathcal{Q}^{(1)}$ one particle

c) $\mathcal{Q}^{(2)}$ two particles d) $\mathcal{Q}^{(3)}$ three particles

(a)

(b)



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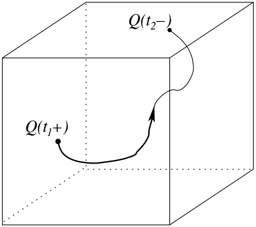
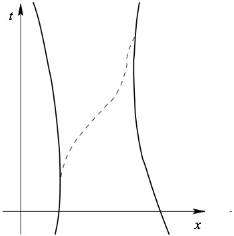


(d)



The LAW: equivariant Markov Process

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- guiding field $\Psi \in \mathcal{F}$, a Fock space

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- $P(dq)$: positive-operator-valued measure (POVM) on $\mathcal{Q}$ acting on $\mathcal{F}$ so that the probability that the systems particles in the state Ψ are in dq at time t is

$$\mathbb{P}_t(dq) = \langle \Psi_t | P(dq) | \Psi_t \rangle$$

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- For a Hamiltonian H (e.g. quantum field Hamiltonian)

$$i\hbar \frac{\partial \Psi_t}{\partial t} = H\Psi_t \longrightarrow$$

$$\frac{d\mathbb{P}_t(dq)}{dt} = \frac{2}{\hbar} \mathrm{Im} \langle \Psi_t | P(dq) H | \Psi_t \rangle \,.$$

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- Find "minimal" generator so that (rewrite left hand side, so that)

$$\frac{d\mathbb{P}_t(dq)}{dt} = \mathcal{L}_t \mathbb{P}_t(dq) .$$

(Minimal) Markovian Process: Flow, (No) Diffusion, (Only as much as necessary) Jumps

Quantum field Hamiltonians provide rates for configuration jumps

Generator for pure Jump-Process

$$(\mathcal{L}\rho)(dq) = \int\limits_{q' \in \mathcal{Q}} \Big(\sigma(dq|q')\rho(dq') - \sigma(dq'|q)\rho(dq)\Big)$$

$$H = H_0 + H_I$$

$$L = L_0 + L_I$$

H_I is often an Integral-Operator $\longrightarrow$ Jump-Generator given by rates

$$\sigma(dq|q') = \frac{[(2/\hbar)\,\mathrm{Im}\,\langle\Psi|P(dq)H_I P(dq')|\Psi\rangle]^+}{\langle\Psi|P(dq')|\Psi\rangle}\,.$$

The tension with relativity challenge: Einstein's criticism of QM

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Possible relief:(DGZ,Travis Norsen, Ward Struyve) The foliation $\mathscr{F}^{\Psi}$ is given by the wave function, e.g. defined by a time like vector field induced by the wave function. Covariance is expressed by the commutative diagram

$$\begin{array}{ccc} \Psi & \longrightarrow & \mathscr{F}^{\Psi} \\ {\scriptstyle U_g}\Big\downarrow & & \Big\downarrow{\scriptstyle \Lambda_g} \\ \Psi' & \longrightarrow & \mathscr{F}^{\Psi'}. \end{array} \tag{1}$$

Here the natural action Λ_g on the foliation is the action of Lorentzian g on any leaf Σ of the foliation $\mathscr{F}^{\Psi}$.

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  - a guiding example is Gauss-Weber-Tetrode-Fokker-Schwarzschild-Wheeler-Feynman direct interaction theory. Fully relativistic and without fields (my friends Shelly and Nino are not enthusiastic about that theory, my young friends are and the future is theirs)

the end: perhaps more on the solutions of second class difficulties in 25 years