4. **Critical current of a wire.** Problem 3, Ashcroft & Mermin, Ch. 34.

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**Solution:** Solving Maxwell's equations and the London equation self-consistently for the current *distribution* in the wire is a nontrivial problem. However Ampère tells us

$$\oint \mathbf{B} \cdot d\vec{\ell} = \frac{4\pi}{c} I_{encl}, \tag{26}$$

so exactly at the surface of the wire the total current $I$ is enclosed; therefore if $R$ is the radius of the wire the relation between $B$ and $R$ is

$$B = \frac{2I}{cR} \tag{27}$$

Type-I superconductivity will be destroyed when this field is equal to the critical field $H_c$, so the critical current is

$$\boxed{I_c = \frac{cRH_c}{2}} \tag{28}$$

Unfortunately this is in Gaussian units, and A& M want you to measure the current in Amps ($H_c$ in Gauss, $r$ in cm). The current in Gaussian units is in esu/s, where

$$1 \text{ esu/s} = \frac{\text{esu}}{\text{s}} \frac{1 \text{ Coulomb}}{3 \times 10^9 \text{esu}} = \frac{1 \text{ Amp}}{3 \times 10^9} \tag{29}$$

So to get $I(Amps)$ we need to divide by $3 \times 10^9$. Doing this on both sides we get ($c = 3 \times 10^{10}$ cgs)

$$I(Amp) = 5RH_c \tag{30}$$