Superconductance

In 1911 Kammerling Omnes discovered the appearance of vanishing electrical resistance in Hg at temperature of liquid hydrogen. The resistance is so small that an induced electrical current in a superconducting ring will decay with time constant of years. As found for other metals as Nb (9.5K), Pb (7.2K), or alloys as Nb_3Ge (23.3K)or Nb_3Al (17.5K) SC appears below a jump temperature (T_c) In 1986 Müller and Bednorz discovered the appearance of SC in ceramics as $(LaSr)_2CuO_4$ (Tc=35K). Later additional compounds have been synthesized with Tc above the temperature of liquid nitrogen which are very challenging for future technical application.

Besides loss free current SC show several remarkable properties:

1) : Under cooping below Tc the magnetic flux applied to a SC becomes completely crowded out of the SC specimen – Meissner –Ochsenfeld neffect. SC are ideal diamagnets with χ=-1. As long the external field is smaller than a critical field strength H_c the inner field is

$$B = 0 = \mu_0 (H + M)$$

Above H_c the magnetic flux penetrates into the sample and destroys the SC property. Hc depends on temperature and decreases for increasing T up to T_c. SC and normal conductor (NC) state can be assumed to be two different phases with phase transition of second order..

2)Another phenomenon is that the SC current is limited up to critical current density, jc. Exceeding jc Sc also breaks down. Jc is larger for metals and metal alloys compared to high temperature SC.

3) Third phenomenon is the appearance of a gap energy 2Δ of electronic ground state with respect to the Fermi energy E_F of the NC phase in the order of few meV. The energy is again a function of temperature and decays as

$$\frac{2\Delta(T)}{2\Delta(0)} \approx \sqrt{\cos(\frac{\pi}{2}(\frac{T}{T_c})^2}$$

4) In contrast to the typical T^2 dependence of specific head for NC, c_V of SC below Tc shows a different behavior. It increases like ~exp(-2Δ/kT) up to Tc and drops down to the curve known from NC. Such behavior is typical for 2 level systems which means that the phase transition between SC and NC state is continuous,(i.e. of 2. Order). At T=0 are all electrons SC, but for 0<T<Tc there is a mixture of SC and NC electrons, where the number of NC increases with T.

Most of the phenomena can be described by the BCS theorie (Bardeen, Cooper and Schrieffer 1957). They postulate the existence of “Cooper pairs” . Whereas the electrons in NC are fermions carrying a spin, in SC phase two electrons are pairing such as their common spin S=0, i.e. they haves as bosons. These Cooper pairs interact with crystal lattice inducing local distortions which explains the isotopy effect and the contribution of electron –phonon interaction to specific heat. Because of bosonic properties all Cooper pairs can occupy the same ground state. Their energy is 2Δ below E_F of the one electron band structure. On the other hand, 2Δ is necessary to break a Cooper pair into 2 individual electrons.

Based on BCS the Cooper pair wave function is $\Psi(r) = \sqrt{\rho_s}\exp(i\varphi(r)$, where $\rho_s = |\Psi|^2$ is the density of Cooper pair. This density again depends on temperature and vanishes at T=Tc.

The current carried by Cooper pairs is described by $j_s = q\Psi^*(\frac{p}{m})\Psi = -\rho_s\frac{e}{m}(\hbar\nabla\varphi - qA)$

where only the phase of the wave varies because of the constant pair density. The equation for j_s replaces Ohm´s law (j=σE) valid in NC. Applying rot() at both sides of j_s equation yields.

$$\nabla x j_s = -\frac{2e^2\rho_s}{m_e}B \quad \text{(London equation)}$$

Solution of London equation starts from Maxwell equ. $\mu_0 j = \nabla x B$ followed by

$$\mu_0\nabla x j = \nabla x(\nabla x B) = -\nabla^2 B$$

Compared with London equation provides and equation for determining the spatial variation of the magnetic field.

$$\nabla^2 B = -\frac{2e^2\rho_s\mu_0}{m_e}B = \frac{1}{\lambda^2}B$$

This solution shows that B cannot be constant inside SC except the case B=0. The external field acting at the surface of the SC decays in normal direction with the London penetration length λ towards the bulk. i.e. $B_z = B_0\exp(-x/\lambda)$, Note also λ is a function of T and diverge at T=T_c as

$$\frac{\lambda(T)}{\lambda(0)} = \frac{1}{\sqrt{1+(\frac{T_c}{T})^4}}.$$

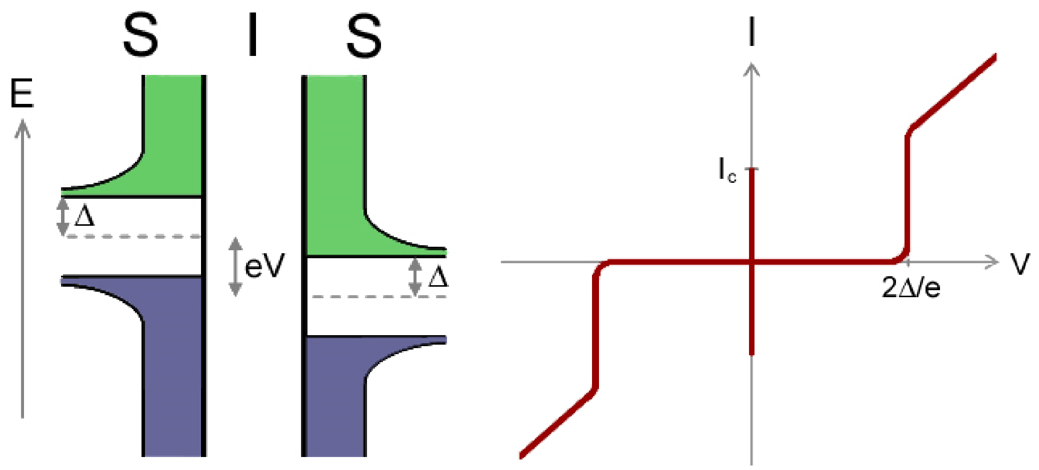
From BCS theory follows that $\lambda = \sqrt{\frac{\varepsilon_0 mc^2}{\rho_s q^2}}$ and is in the order of 40nm.

One remarkable phenomenon is the quantization of magnetic flux through a SC ring. Suppose a SC ring is exposed to an external magnetic field. All current vanishes inside the ring, i.e. j_s=0. Therefore the phase is to change along the ring. What means: $\hbar\nabla\varphi = qA$. However, after full revolution the wave function has to be the same as in beginning, i.e. $\Delta\varphi = s2\pi$ Δϕ= s 2π, where s integer. One can show that $s2\pi = \frac{q}{\hbar}\Phi$ which provides $\Phi = s\frac{\pi\hbar}{e} = s\Phi_0$ with $\Phi_0 = \frac{h}{2e} = 2.0678x10^{-17}Tm^2$

This property is applied in a SQUID (super conducting quantum interference device). Here two SC half rings are applied to a sample carrying an unknown magnetic moment. Whereas for B=0 the difference of the two phases are zero , the difference $\varphi_2 - \varphi_1 = \frac{2e}{\hbar}\Phi$ therefore the total current through both half rings is $j_{total} = j_0(\sin(\varphi_a + \frac{e}{\hbar}\Phi) + \sin(\varphi_b - \frac{e}{\hbar}\Phi) = 2j_0\sin(\varphi_0)\cos(\frac{e\Phi}{\hbar})$ using $\varphi_a = \varphi_b = \varphi_0$

js becomes a function of magnetic flux. In experiment one measures the oscillation of current as function of B . j becomes maximum always if $\frac{e}{\hbar}\Phi = s\pi$. SQUID is sensitive for small magnetic fields down to 10^{-10} T.

In a tunnel junction two metals are separated by a thin isolating barrier. If the barrier thin enough the electronic wave function of electrons can tunnel throughout the isolating barrier. In NC phase the tunnel current I is linear proportional to the applied Voltage U. Is the metal in SC phase then the electron first has to exceed the SC gap before it can penetrate into the barrier which increases the requested voltage by $V = \frac{2\Delta}{2e}$. The value 2Δ represent the energy request to break a Cooper pair into Fermions. In SC tunnel junction there is a minimum energy eV=Δ is required before a current can be measured.



Josephson effect: IN JD two SC are coupled via a thin isolator layer, due to tunnel effect both SC wave function have certain probability in the respective other SC. The coupling between SC is given by

$$i\hbar\frac{\partial\psi_L(t)}{\partial t} = E_L\psi_L(t) + K\psi_R(t)$$
$$i\hbar\frac{\partial\psi_R(t)}{\partial t} = E_L\psi_R(t) + K\psi_L(t)$$

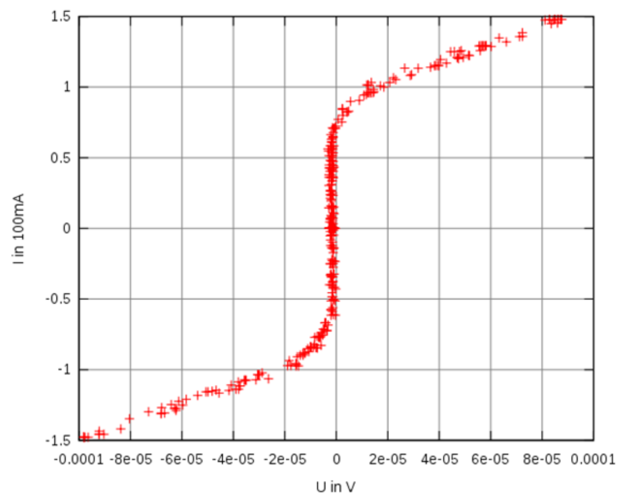
where K is coupling constant.

Applying an external DC voltage the difference of total energy is $\Delta_0 = E_R - E_L = 2eU$. The respective current to voltage relation shows operator behavior: It can be shown that already without u there is a Josephson current: $i_s(t) = I_0\sin(\frac{2e}{\hbar}\int_0^t u(t)dt + \varphi_0)$. Applying u=U>0 the current becomes $i_s(t) = I_0 \sin(\frac{2e}{\hbar} U \cdot t + \varphi_0)$, a constant AC voltage results in a sinus shape SC AC current of amplitude I0 with frequency $\omega = (\frac{2eU}{\hbar})$ or $f = (\frac{2eU}{h})$ → Josephson Voltage-frequency relation.

In time average and for U>0 the AC current is *<i_s(t)>=0*. For U=0 there is a SC DC of $i_s(t) = I_0 \sin(\varphi_0)$ → DC Josephson effect.

Applying an addition AC voltage of $u(t) = U + v \cos \omega_M t)$, the current changes to $i_s(t) = I_0 \sin(\omega_J t + \frac{2e}{\hbar} \frac{v}{\omega_M} \sin \omega_M t + \varphi_0)$. The current can be expanded in Bessel-functions J_n, providing discrete plateaus of voltage U_n $u(t) = U_n = \frac{h}{2e} f_M n$, with n=0,1,2….--> Zero current plateaus, with no-zero current of $\Delta I_{s,n} = 2I_0 \, | J_n(\frac{2e}{\hbar} \frac{v}{\omega_M}) |$.

This effect allows for a precise definition of voltage and is used as voltage normal in BAM.



Current – voltage relation of Josephson contact