Semiconductors

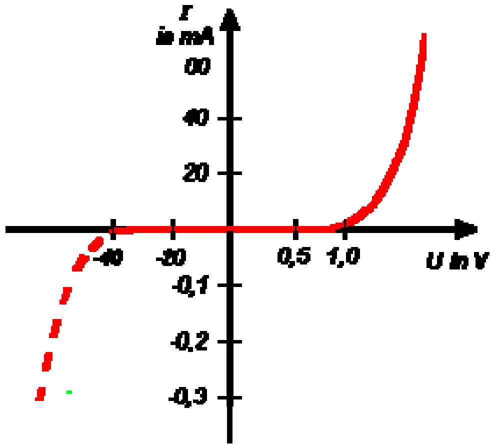
In semiconductors the upper filled band (valence band) is separated by a energy gap, E_g, from the next empty band (conduction band). In real space the valence band states can be associated with atomic like states whereas the conduction band states built a common energy level throughout the whole crystal. Considering the k-space and setting the upper valence band energy to zero the conduction band can be described by k-dispersion function as $E_{CB} = E_g + \frac{\hbar^2}{2m_e} k^2$ where m_e is the electron effective mass. Considering that in a pure semiconductor each electron excited into the CB will create a “hole” state into the VB the energy dispersion of holes can be described by a function $E_{VB} = -\frac{\hbar^2}{2m_h} k^2$ where m_h is the hole effective mass. For most semiconductors $m_h > m_e$, therefore the opening of the parabola is larger for holes states compared to electron states. In semiconductors like GaAs, InAs, InP … the minimum of CB and the maximum of VB appear at same value, k=0 (direct gap semiconductors). Here electron holepairs can be created by exciting light with photon energy $E_{photon} = h\upsilon \geq E_g$. In the semiconductors Silicon and Germanium the CB minimum appears at k>0 but the VB maximum still at k=0 (indirect gap semiconductor). In order to fill the CB minimum the excited electron needs interaction with the lattice (phonon). The requested photon energy is $E_{photon} = h\upsilon \geq E_g \pm \hbar\omega$ .where $\hbar\omega$ is the phonon energy.

Considering the density of states of electrons $\boxed{D(E) = \frac{V}{2\pi^2}(\frac{2m_e}{\hbar^2})^{3/2}(E - E_g)^{1/2}}$ And the Fermi-Dirac distribution f(E) $\boxed{f(E,T) \approx \exp(-(E - E_F / k_B T))}$. The number of electrons in conduction band is $n = 2(\frac{2\pi m_e k_B T)}{\hbar^2})^{3/2} \exp((E_F - E\ )/k_B T)$. The Fermi-Dirac function for holes is $f_h = 1 - f_e \approx \exp((E\ - E_F)/k_B T)$ and the hole density of states is fnumber of created holes $\boxed{D_h(E) = \frac{V}{2\pi^2}(\frac{2m_h}{\hbar^2})^{3/2}(-E)^{1/2}}$. Thus the number of holes is $p = 2(\frac{2\pi m_h k_B T)}{\hbar^2})^{3/2} \exp(-E_F / k_B T)$. Using the condition n=p , valid for pure semiconductors, the Fermi energy results to $E_F = \frac{E_g}{2} - \frac{3}{4} kT \ln\frac{m_e}{m_h}$. IN mist cases the second term is very small and can be neglected. Because of $E_F = \frac{E_g}{2}$ the number of excited charge carriers is $n_i = p_i = 2(\frac{kT}{2\pi\hbar^2})^{3/2}(m_e m_h)^{3/4} \exp(\frac{E_g}{2k_B T})$. At 300K n_i is 2.5 10^{13} $cm^{-3}$for Ge and 1.4 $10^{10}$$cm^{-3}$ for Si. The conductivity $\sigma = ne\mu_e + pe\mu_h$ has to consider both the mobility of electrons μ_e and holes, μ_h.

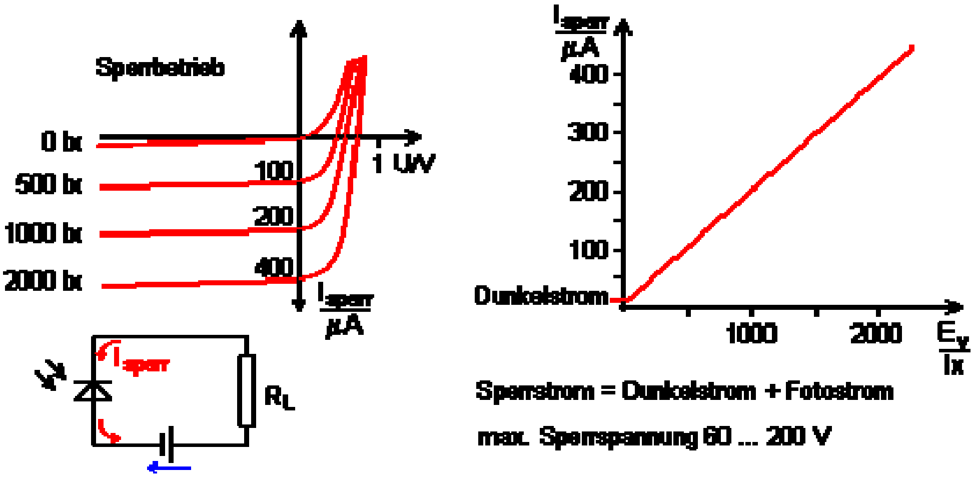
The number of charge carriers can be significantly increased by doping, substituting host atoms by atoms of higher/lower electron number compared to host atoms. For silicon doping by group 5elements increases the number of electrons (donators) and by group 3 elements it increases the number of holes (acceptors). The interaction between the additional electron/hole and the ionized host atom can be described similar to H-atom replacing $e^2 \rightarrow e^2/\varepsilon$ and $m \rightarrow m_{e,h}$.where ε is the statistic dielectric constant of host material. The bonding energy $E_d = 13.6eV \frac{m_e}{m\varepsilon^2}$ which in case of doping by Phosphorous is 0.045 eV below the bottom of CB in silicon. Similar for holes, replaceing m_e by m_h resulting in 0.045 eV above the upper level of VB in case of doping by boron. The radius of revolution of electrons/holes around the ion is described by Bohr radius $r_d = 0.0529nm \frac{\varepsilon m}{m_e}$ which is 8nm in Ge and about 3nm in Silicon. The number of electrons in conduction band becomes $n = 2(\frac{m_e kT}{2\pi\hbar^2})^{3/2} \sqrt{N_d} \exp(-E_d / 2kT)$ where N_d ist the number of doping atoms. Therefore the conductivity is depends on E_d at low temperature but on E_g at very high temperatures. E_F is pinned between E_d and the CB at low temperature and moved towards $E_g/2$ at high temperature.

Semiconductor diode: p-type semiconductor and n-type semiconductor are combined together to a semiconductor diode. Majority charge carriers in p-type semiconductors are the holes but also electrons exist as minority charge carriers (due to impurities and thermal excitation). Majority charge carriers in n-type semiconductors are electrons. At the interface between n- and p-types SC electrons will diffuse from n-type into the p-type part filling the holes. This diffusion current is competed by the creating electric field due to the remaining positive donors in n-type and the now negatively charged acceptors in p-type (space charges) creating an drift current opposite to the diffusion current. The equilibrium of both currents results in a charge carrier free zone at the interface where neither free electrons nor free holes exist. The interface zone has a finite width of $Z_n = \sqrt{\frac{2V_0\varepsilon_0\varepsilon}{e}\frac{n_a}{n_d}\frac{1}{n_a + n_d}}$ and $Z_p = \frac{n_d}{n_a} Z_n$ where n_d is the electron concentration on n-side and n_a the electron concentration on p-side and $V_0 = \frac{kT}{e}\ln\frac{n_n}{n_p}$. Within the interface the electric field varies as $E(z) = -\frac{en_d}{\varepsilon_0\varepsilon} Z_n$ (all quantities can be expresses in similar way based on p_n/p_p).
applying an external voltage with positive contact at p side and negative contact at n-side the remaining electrons of n-side (holes of the p-side) will move towards the positive (negative) contact throughout the interface reducing the widths of the charge carrier free zone (conducting direction). In contrast to this contacting the n-side with positive voltage and the p-side with negative voltage the electrons (hole) will move towards these external contacts increasing the widths of charge carrier free zone (blocking direction).

The I-U characteristic shows exponential increase of current for positive voltage but very low current (due to minority charge carriers) for negative voltage. This behaviour is used as electrical **rectifier** to rectify AC voltage into DC voltage.



I-U characteristic of a SC diode (from wikipedia)



Created photocurrent at a photodiode (from Wikipedia)

The same diode can be used as **photodiode**. Here the diode is used in blocking direction only. Incident photons create electron-hole (e-h) pairs inside the charge carrier free zone. . Applying an external blocking voltage these electrons and holes will create a certain blocking current which can be used as measure of photon current.(see figure). Because the formation energy for a single e-h pair is E_{np}=3.6eV in silicon a photon of higher energy will create a distinct number of such e-h pair. Therefore the photon energy is proportional to $h\nu = N * E_{np}$ The number N of electrons/holes can charge a capacitor and can be transformed into a pulse of certain voltage which is proportional to the photon energy.

Modern electronics is realized in planar technology and is based on miniaturized field effect transistors (FET) where the electrical current between *Source* and *Drain* contact is controlled via the space charge region in between the both contacts. The decay length L_D (Debye length) of the influenced space charges is controlled by an external potential applied to the *Gate* contact on top of the space charge region.

$L_D = \sqrt{\dfrac{\varepsilon\varepsilon_0 kT}{2e^2 N_i}}$ for intrinsic SC and $L_D = \sqrt{\dfrac{\varepsilon\varepsilon_0 kT}{e^2 N_{n,p}}}$ for doped SC

Solar cells: Energy density of 1367±7 W/m² arrives the earth in front of the atmosphere. Total energy that reaches the earth from sun is 1.8 10^{17}W/year, this is 10^4 times the yearly energy consumption. In Germany the mean energy density is 844 W/m² considering the different incidence angle in summer and winter.

A solar cell is a pn-junction: the induced current is

$$I = I_s \left[\exp\left(\frac{eV}{kT} \right) - 1 \right] - I_L \,,$$

where I_L is the leakage current. Solar cell operates in a range between -1 < I < 0A and 0 < V < 1V. In this curve the open-circuit voltage V_{oc} is that for I=0. At V=0 the current $I=I_L$ is the short-circuit current. Only a part of the rectangle I_{sc} x V_{oc} can be used for power conversion. The use of the load resistance R_L set the working point at I_m and V_m defining the filling factor $F = \frac{I_m V_m}{I_{sc} V_{oc}} < 1$. The open circuit voltage is

$V_{oc} = \frac{KT}{e} \ln(\frac{I_L}{I_s} + 1)$ which increases with the increasing light power and decreasing dark current. The output power is

$$P = IV = I_s V \left[\exp\left(\frac{eV}{kT} \right) - 1 \right] - I_L V$$

The condition dP/dV=0 defines the working point

$$V_m = V_{oc} - \frac{kT}{e} \ln(1 - \frac{kT}{e} V_m) \quad \text{and} \quad I_m = I_L (1 - \frac{kT}{eV_m})$$

The energy that is delivered per photon at the load resistor at power maximum at

$$P_m = I_L E_m / e$$

is

$$E_m = e \left[V_{oc} - \frac{kT}{e} \ln(1 + \frac{kT}{e} V_m) - \frac{kT}{e} \right]$$

The ideal solar cell has a (power conversion efficiency $\eta = P_m / P_{in}$. It is about 31% for single junctions following the Shockley-Queisser limit. For optimum devices the conversion factor can reach a maximum of 43%