**Solid state physics (*winter term 2015/2016*)**

Lecturer: Prof. Dr. Ullrich. Pietsch Exercise tutor: Dr. Ali Abboud (ENC B-024)

# Exercise sheet 9

## Superconductivity

### Exercise 1: Critical Magnetic Field

Lead (Pb) has a critical magnetic field equal to 0.0803 T at absolute zero (0K) and its critical temperature is 7.193 K. Given a Lead wire of R=4.00 mm and is at a temperature of 4.15 K. Find (a) the critical magnetic field in Lead at this temperature and (b) maximum current the wire can carry at this temperature and still remain superconducting.

Hint: For part (b) use Ampere's Law relating a current a wire to the magnitude of the induced magnetic field.

### Exercise 2: Super Conducting Solenoid

A solenoid is constructed from a wire made of the alloy $Nb_3Al$, which has an upper critical field of 32.0 T zero Kelvin and $T_c = 18.0$ K.

The wire has a radius of 1.00 mm, the solenoid is wound on a hollow cylinder of diameter 8.00 cm and length of 90.0 cm, and there are 150 turns of wire per centimeter of length.

How much current is required to produce a magnetic field of 5.00T at the center of the solenoid?

**Exercise 3: Critical Current in a Cylindrical Wire**

A current of I amperes flows in a cylindrical superconducting wire of radius r cm. Show that when the field produced by the current immediately outside the wire is $H_c$ (in gauss), then

$$I=5rH_c$$

**Exercise 4: The London equation for a superconducting Slab**

Find the minimum frequency of a photon that can be absorbed by lead at T=0 K to break apart a Cooper pair. The energy gap for lead is $2.73 \times 10^{-3}$ eV.

**Exercise 5: The Meissner Effect**

Consider an infinite superconducting slab bounded by two parallel planes perpendicular to the y-axis at $y=\pm d$. Let a uniform magnetic field of strength Ho be applied along the z-axis.

(a) Taking as a boundary condition that the parallel component of B be continuous at the surface, deduce from the London equation $\nabla \times j = -\frac{n_s e^2}{mc} B$ and the Maxwell equation $\nabla \times B = \frac{4\pi}{c} j$ that within the superconductor

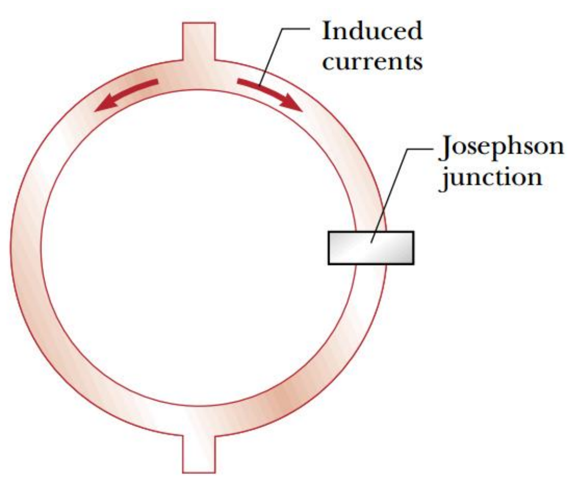
$$\boldsymbol{B} = B(y)\hat{z}\,, \qquad b(y) = H_0 \frac{\cosh(\frac{y}{\Delta})}{\cosh(\frac{d}{\Delta})}, \qquad \Delta \textit{ is penetration depth}$$

(b) Show that the diamagnetic current density flowing in equilibrium is

$$\boldsymbol{j} = j(y)\hat{x}\,, \qquad j(y) = \frac{c}{4\pi\Delta} H_0 \frac{\sinh(\frac{y}{\Delta})}{\sinh(\frac{d}{\Delta})},$$

**Exercise 6: A superconducting quantum interference device (SQUID).**

If a magnetic flux of $1.0\text{x}10^{-4}$ $\phi_o$ (where $\phi_o$ is the flux quantum $=2.067\text{x}10^{-15}$T.m$^2$) can be measured with a SQUID (see figure below), what is the smallest magnetic field change ΔB that can be detected with this device for a ring having a radius of 2.0 mm?

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