Measurement of residual stress

- **The uniform macro strain causes a shift of the diffraction lines to new 2θ positions.**
- **The non-uniform micro-strain causes a broadening of the corresponding diffraction line.**

Both kinds of strain are usually superimposed in plastically deformed metals and diffraction lines are both shifted and broadened.

X-ray diffraction can be used as a method of residual stress measurement in metals and alloys.

Applied stress and residual stress

Consider a metal bar deformed elastically, for example in uniform tension.

If the external force is removed, the stress disappears and the bar regains its initial stress-free dimensions.

The stress, which persists in the absence of external forces, is called residual stress.

The x-ray method is nondestructive for the measurement of surface stress.

Uniaxial stress:

Consider a cylindrical rod of cross-section area A stressed elastically in tension by a force F (see Fig .1)

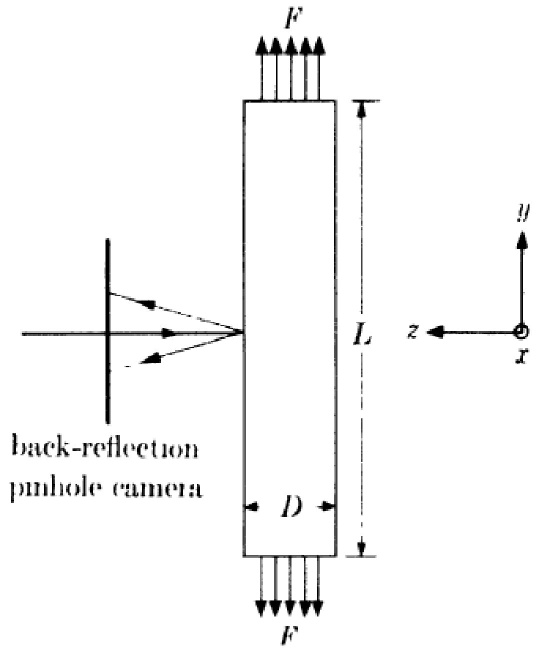

Fig. 1 Bar in pure tension

There is a stress $\sigma_y = F/A$ in the y direction.

The stress σ_y produces a strain ϵ_y in the y direction given by

$$\epsilon_y = \frac{\Delta L}{L} = \frac{L_f - L_0}{L_0}$$

where L_0 and L_f are the original and final length of the bar. This strain is related to the stress by

$$\sigma_y = E\epsilon_y$$

where E is elasticity constant. The elongation of the bar is accompanied by a decrease in its diameter D. The strain in the x and z direction are therefore given by

$$\epsilon_x = \epsilon_z = \frac{D_f - D_0}{D_0}$$

where D_0 and D_f are the original and final diameters of the bar.

If the material of the bar is isotropic, these strains are related by the equation

$$\epsilon_x = \epsilon_z = -\nu\epsilon_y$$

where ν is *poison's* ratio for the material of the bar.

To measure ϵ_y by x-rays → diffraction from planes perpendicular to the axis of bar

Since this is usually physically **impossible**, we use instead reflecting planes which are parallel to the axis of the bar by making the back reflection x-ray measurement

In this way, the obtained strain in z direction:

$$\epsilon_z = \frac{d_n - d_0}{d_0}$$

where d_n is the spacing of the planes parallel to the bar axis under stress, and d_0 is the spacing of the same planes in the absence of stress.

$$\sigma_y = -\frac{E}{\nu}\left(\frac{d_n - d_0}{d_0}\right)$$

Biaxial stress:

In a bar subject to pure tension the normal stress acts only in a single direction

→ there will be stress components in two or three directions at right angles to one another, forming so-called **biaxial** or triaxial stress systems.

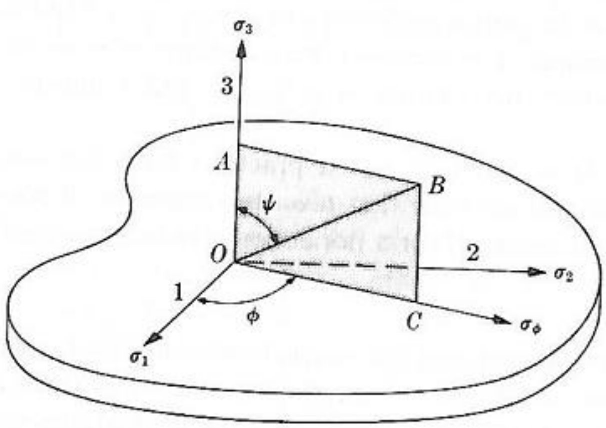

Fig. 2 Stress at the surface of a stressed body. $\sigma_3 = 0$. The stress to be measured is σ_ϕ (B.D. CULLITY 1977)

Principal stresses σ_1 and σ_2 are parallel to the surface, and σ_3 is zero. However ϵ_3 the strain normal to the surface is not zero. It has a finite value, given by the Poisson contraction due to $\sigma_1\ and\ \sigma_2$.

$$\epsilon_3 = -\nu(\epsilon_1 + \epsilon_2) = -\frac{\nu}{E}(\sigma_1 + \sigma_2)$$

<u>What we want to measure</u> → single stress σ_ϕ acting in some chosen direction in the surface, direction OB, making an angle of ϕ with the principal direction 1.

We do this by making two measurements: strain ϵ_3 along the surface normal, and one of the strain ϵ_ψ along OB. The direction OB lies in the vertical plane OABC through OC at an angle ψ, usually chosen to be 45°, to the surface normal (see Fig. 2).

- ➔ ϵ_3 is driven from the spacing d_n of planes parallel to the surface, and
- ➔ ϵ_ψ from spacing d_i of planes whose normal is inclined along OB.

If the specimen were unstressed, the d_0 plane spacing is independent of plane orientation.

This not true when stress is present → if the stress is tensile, d_i increases with ψ .

Elasticity theory for isotropic solid shows that the strain along the inclined line OB is:

$$\epsilon_\psi = 1/E[\sigma_\phi(1+\nu)sin^2\psi - \nu(\sigma_1 + \sigma_2)]$$

$$\Rightarrow \epsilon_\psi - \epsilon_3 = \frac{\sigma_\phi}{E}(1+\nu)sin^2\psi$$

This Eq. is the basis for the x-ray measurement of stress.

- ➔ **The difference between two strains in a stressed specimen depends only on the stress acting in the plane of those strains.**
- ➔ Expressing the strain in terms of plane spacing (unknown $d_0$can be replaced by d_n or d_i with negligible error):

$$\sigma_\phi = \frac{E}{(1+\nu)sin^2\psi}\left(\frac{d_i - d_n}{d_n}\right)$$

Diffractometry method:

Because the angular position 2θ of the diffracted beam is measured directly with a diffractometer, it is convenient to write the stress equation in terms of 2θ rather than plane spacing.

$$\sigma_{\phi} = k_1(2\theta_n - 2\theta_i) = k_1(\Delta 2\theta)$$
$$k_1 = \frac{E cot\ \theta\ )}{2(1+v)sin\ ^2\psi}$$

where $2\theta_n$ is the observed value (in radian) of the diffraction angle in the "normal" measurement ($\psi = 0$) and $2\theta_i$ its value in the inclined measurement ($\psi = \psi$).

The constant k_1 is called the stress constant. For greatest sensitivity k_1 should be as small as possible, which is why θ should be as large as possible.

- But instrument misalignment can introduce small errors which cause a change in 2θ even for a stress-free specimen, when ψ is changed from 0 to 45°.

If $(\Delta 2\theta)_0$ is the line shift for a stress-free specimen and $(\Delta 2\theta)_m$ the measured shift for a stressed specimen, then the line shift due to stress is

$$\Delta 2\theta = (\Delta 2\theta)_m - (\Delta 2\theta)_0$$

Measurement of line position:

Line position cannot be measured with sufficient precision →

The standard method of finding the center of a diffraction line, broad or narrow, is to fit a parabola to the top of the line and take the axis of the parabola as the line center.

Three-point method

A simpler method for locating the parabola axis was suggested by Koistinen and Marburger:

Only three points on the line profile need be measured but they must be separated by the same angular interval c, as shown in Fig. 3.

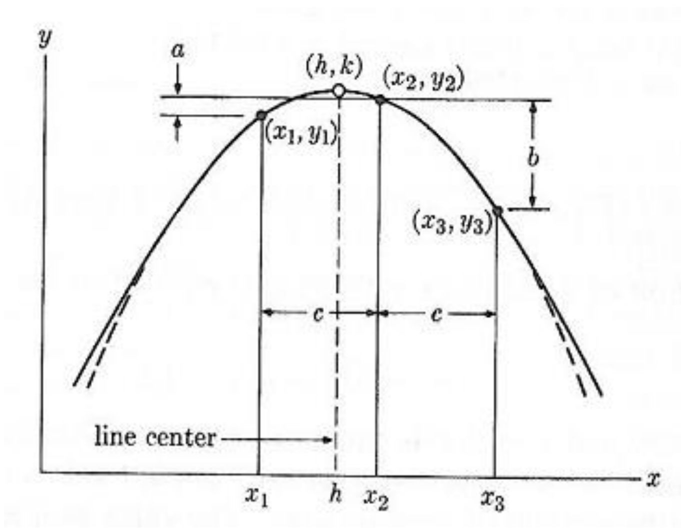

Fig. 3 Three-point method for fitting a parabola. X=2θ, y=I (B.D. CULLITY 1977)

→The center point should be near the maximum and the other two have intensities of about 85 percent of the maximum. Once the intensity differences $a=y_2-y_1$ and $b=y_2-y_3$ are found, the center of the line is given by

$$h = x_1 + \frac{c}{2}\left(\frac{3a+b}{a+b}\right)$$

the y coordinate may be intensity *I* in counts/sec, counts n for a fixed time, or reciprocal time 1/t for a fixed count.

The breath of an x-ray line often correlates well with the **hardness** of the specimen. Marburger and Koistinen also showed that the **hardness** of certain quenched and tempered steel is related to the breath B_p of the parabola used to fit the x-ray line for stress measurements. B_p may be determined solely from the data obtained in the stress measurements, if the following more complex expression is used:

$$B_p = c\left[\frac{(3a+b)^2 + 8y_1(a+b)}{2(a+b)^2}\right]^{1/2}$$

- When the lines are broad, certain corrections should be applied to the intensity data before finding the line center.

Lorenz-polarization (L-P) factor can be modified as:

$$(1 + cos^2 2\theta)/(sin^2\theta).$$

The variation of this factor with 2θ makes a high-angle line asymmetrical about its center.

Absorption on the specimen has a similar effect when ψ Is not zero, because the absorption factor is then

$$(1\text{-tan } \psi \text{ cot } \theta).$$

➔ Combine these two factors into one and call it LPA factor, then

LPA=(modified L-P factor)(absorption factor)= $(\frac{1+cos^2 2\theta}{sin^2\theta})(1 - tan\psi\ cot\theta)$

Measured intensities are to be divided by LPA in order to make the lines more nearly symmetrical, before determining the line center by the least-squares or three-point method.