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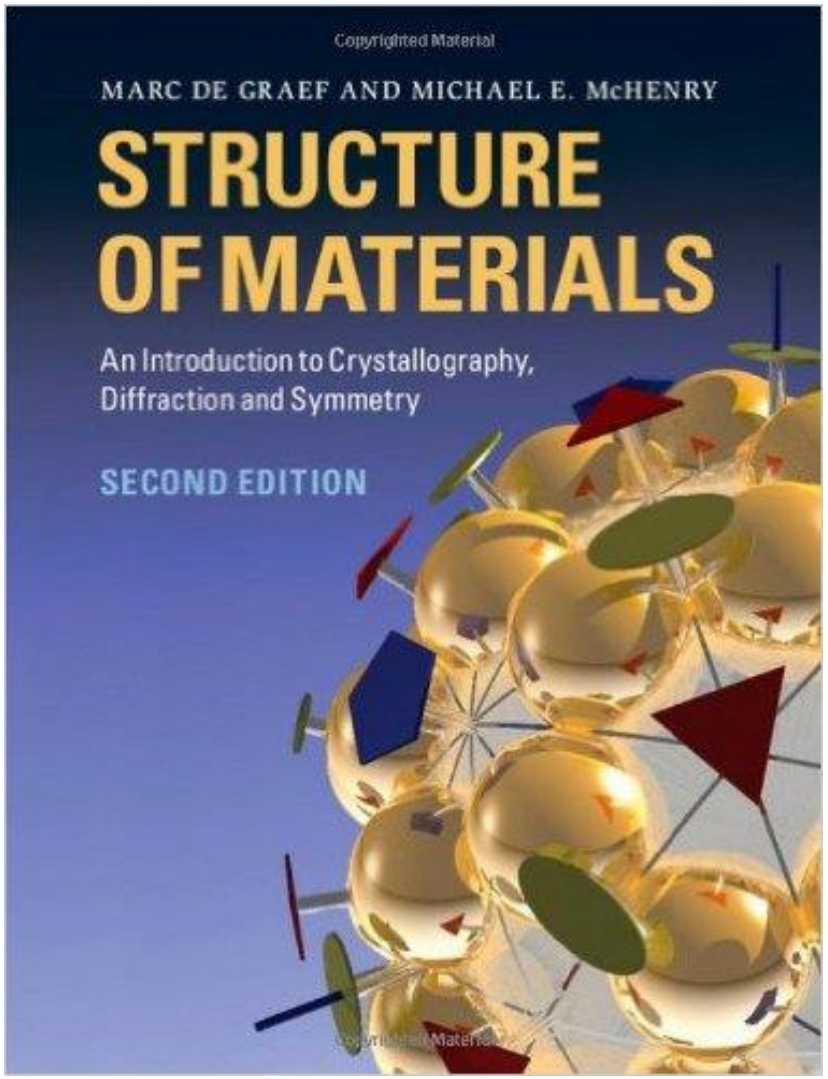
Lecture course on crystallography, 2015

Lecture 7: 32 Point Groups

NB! This lecture uses the images from the book



http://materials.cmu.edu/degraef/pg/pg.html

Most important messages from the previous lectures

Crystal structures are 3D periodic and are set up by specifying the *UNIT CELL (what to translate)* and CRYSTAL LATTICE *(how to translate)*

Symmetry of a crystal is the OVERLAP of the symmetries of the UNIT CELL and CRYSTAL LATTICE

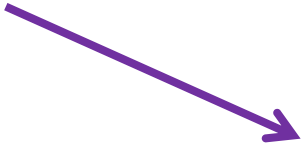
A CRYSTAL LATTICE may have only restricted symmetry elements (mirror planes, inversion and 2,3,4,6 fold axes). Because of that crystals have the same restriction on the possible symmetry

According to the symmetry of a lattice all crystals are subdivided into crystal systems (4 2D crystal systems and 7 3D crystal systems) . The lattices are splitted into 5 (2D) or 14(3D) Bravais lattice types.

Point symmetry groups (crystal classes)

CRYSTAL SYSTEM is the symmetry of a lattice. The symmetry of a crystal can be only lower (i.e. crystal may have the same or fewer symmetry operations then its own lattice). Considering that there is a restricted number of crystal systems, we can list all possible combinations of point symmetries of a crystal. Each symmetry is a referred to as a point symmetry group

CRYSTAL SYSTEM (THE SYMMETRY GROUP OF A CRYSTAL LATTICE , HOLOHEDRY)



TAKING OUT A SYMMETRY ELEMENT

Getting a symmetry group for a crystal

What is a symmetry group?

Symmetry group is a GROUP (in the mathematical sense) of movements each representing the particular symmetry operation (rotation, reflection, inversion, rotoinversion and translation). The elements of the GROUP (G_1, G_2,...,G_n) should satisfy the following conditions:

1. Closure

the combination of any pair of element gives another element of the group, i.e. $G_i * G_j$ belongs to the same group

2. Existence of identity element, I

the element which has the following property $I * G_i = G_i$

3. Associativity

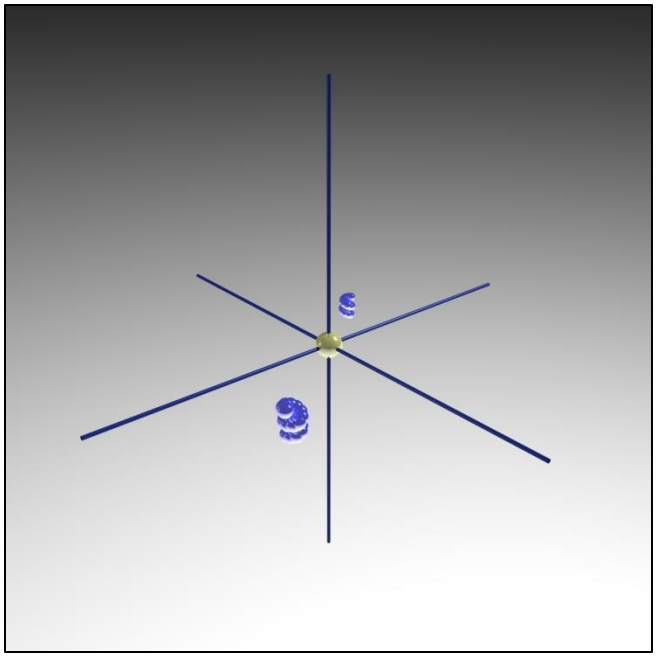
for any three elements of the group: $(G_i * G_j) * G_k = G_i * (G_j * G_k)$ *holds*

4. Invertibility of the elements

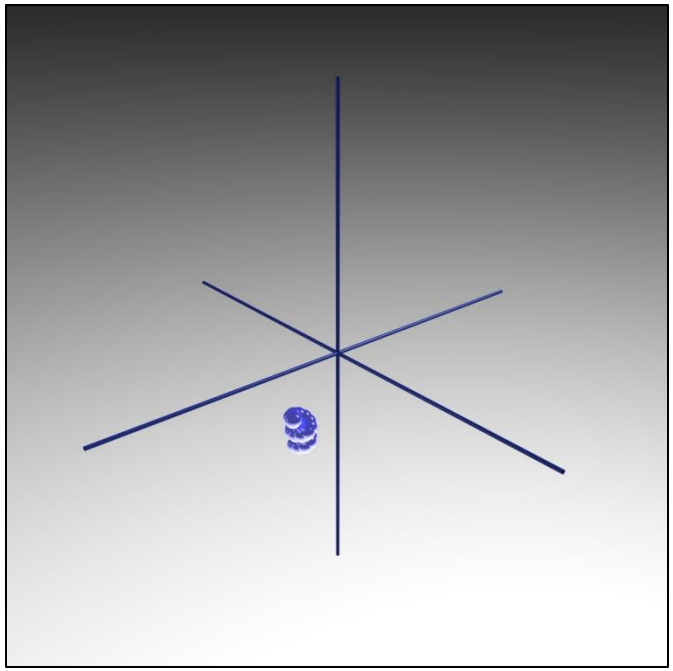
each element has the inversion element, i.e. $G_i G_i^{-1} = I$

TRICLINIC SYSTEM

HOLOHEDRY GROUP



Taking out
the inversion



The group has the centre of inversion only.
The group name is

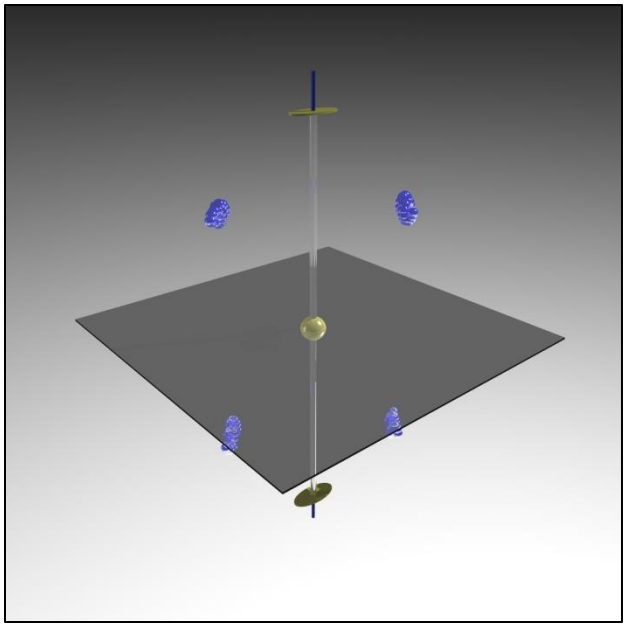
$\bar{1}$

The group does not have any symmetry elements

1

MONOCLINIC SYSTEM

HOLOHEDRY GROUP

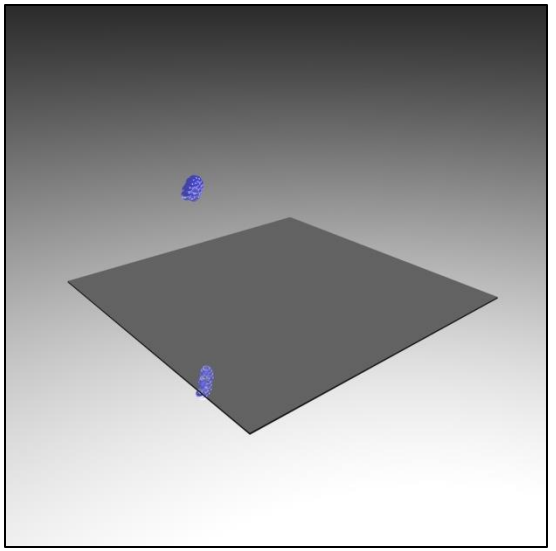


The group has the mirror plane and two fold axis.

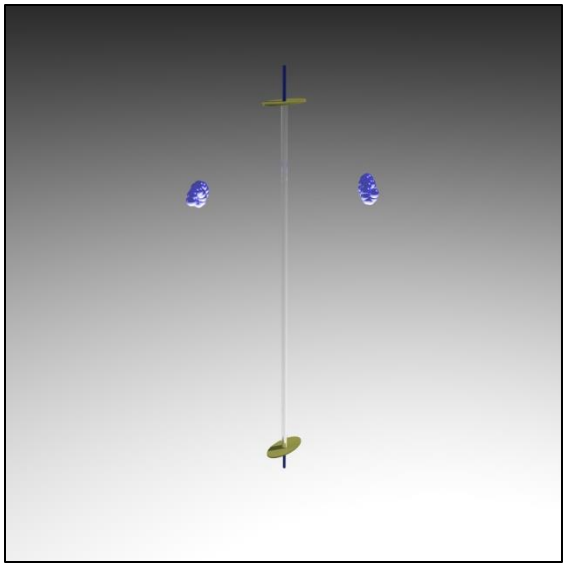
$2/m$

Taking out *2*

Taking out *m*



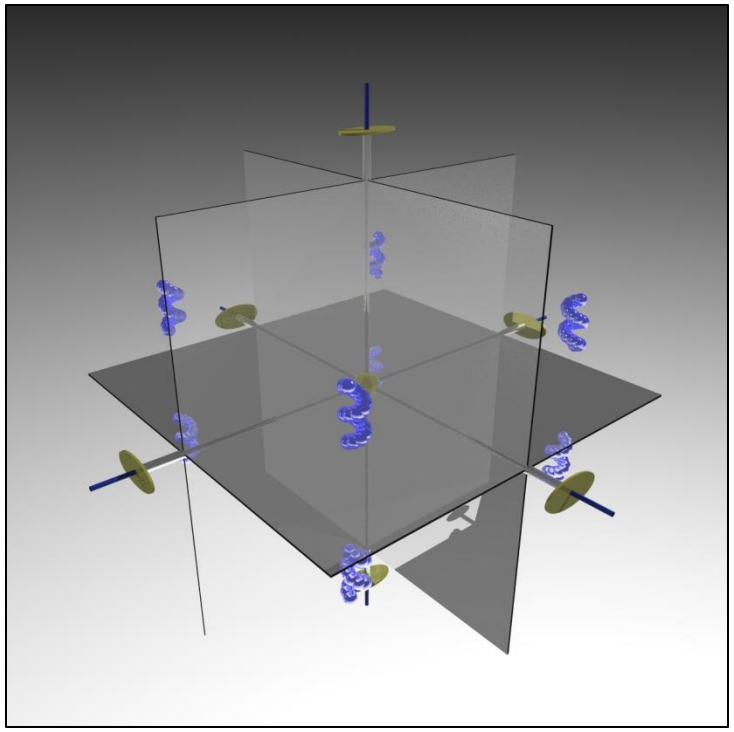
m



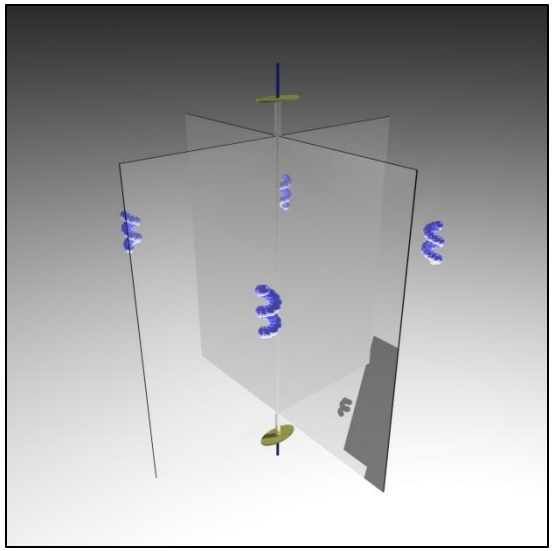
2

ORTHORHOMBIC SYSTEM

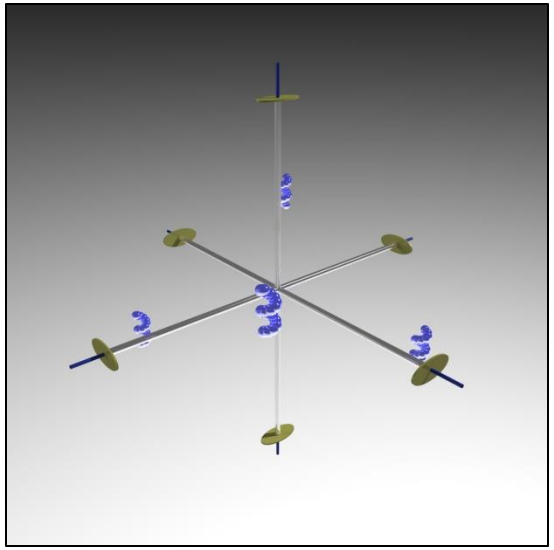
HOLOHEDRY GROUP



mmm



mm2



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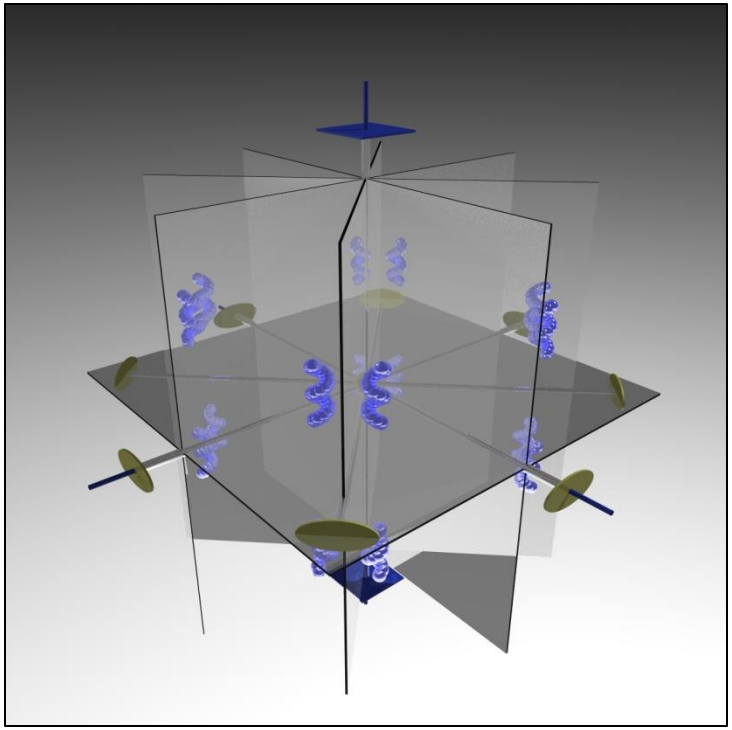
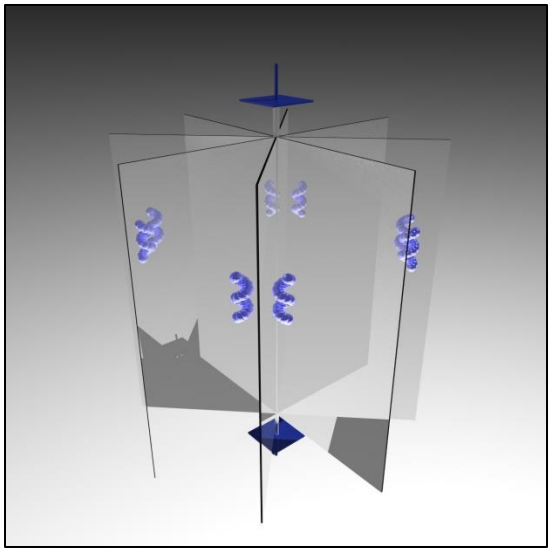
TETRAGONAL

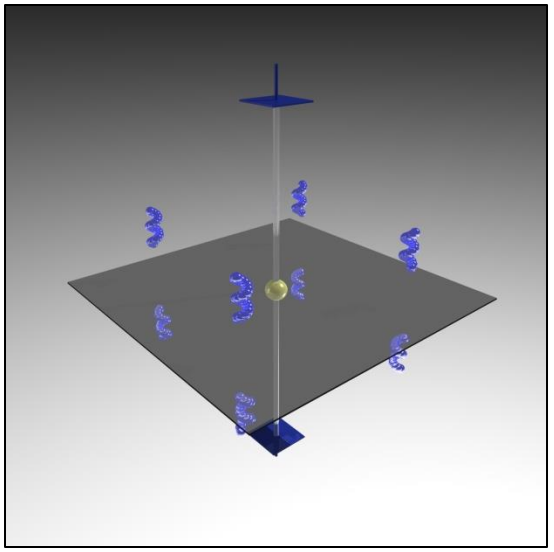
HOLOHEDRY GROUP



$4/mmm$



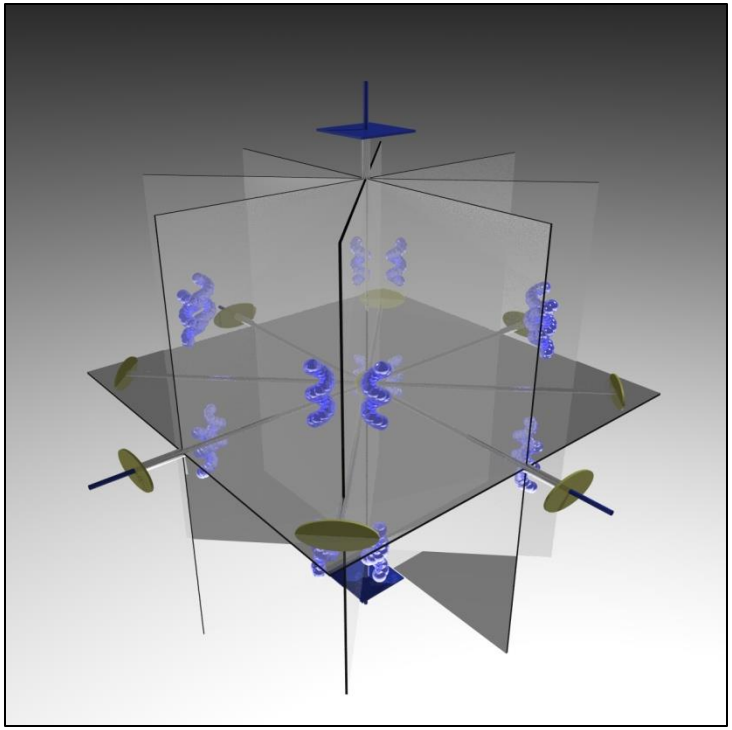
$4mm$



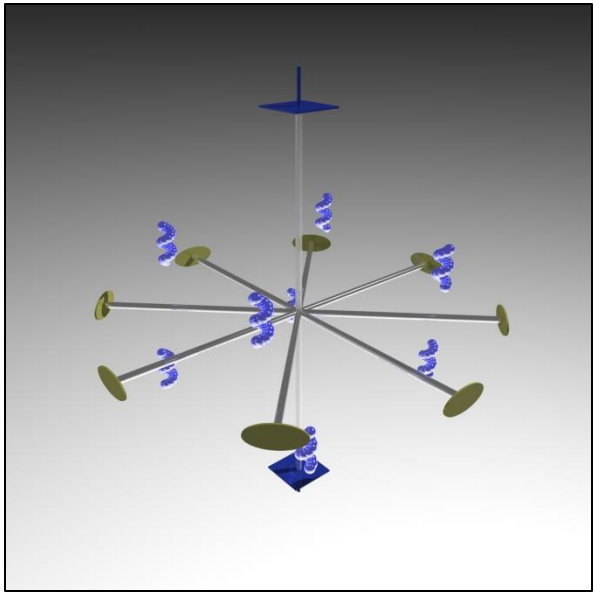
$4/m$

TETRAGONAL SYSTEM

HOLOHEDRY GROUP



$4/mmm$



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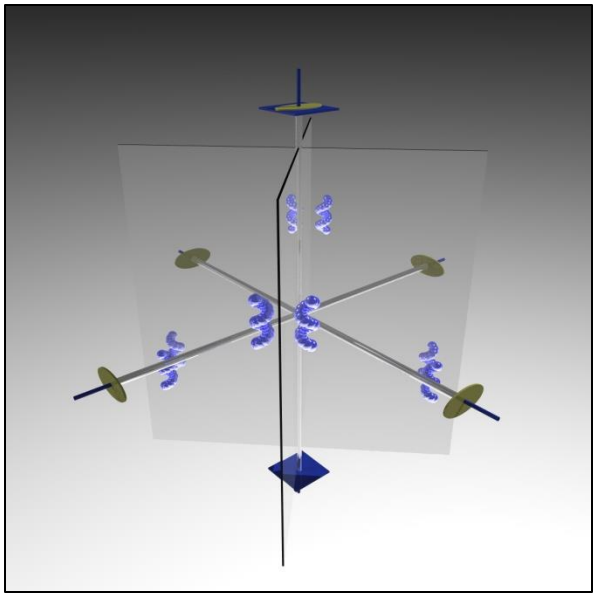
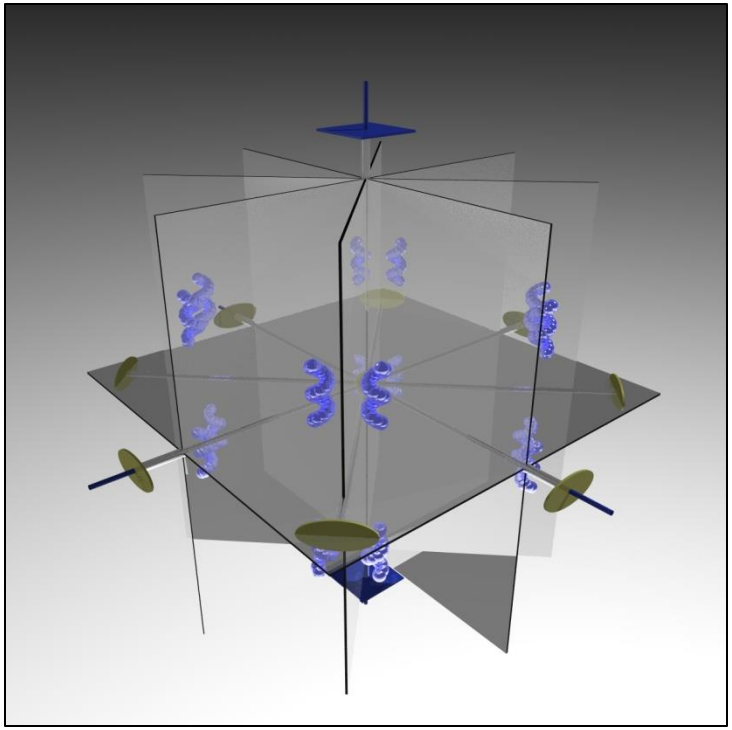


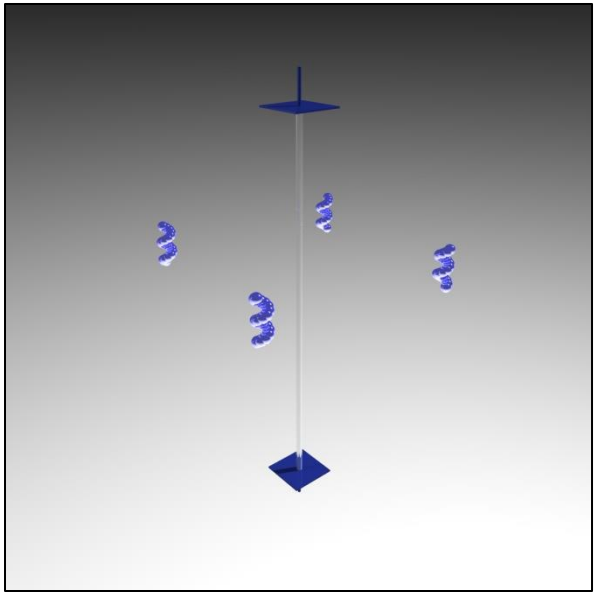
$\bar{4}2m$

TETRAGONAL SYSTEM

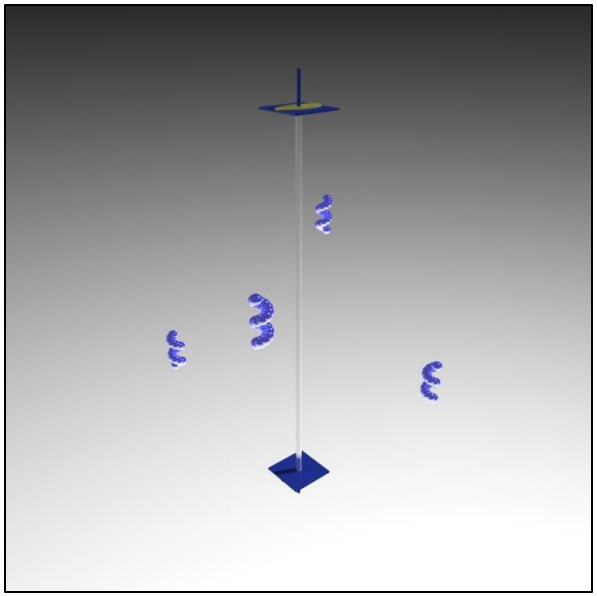
HOLOHEDRY GROUP



$4/mmm$



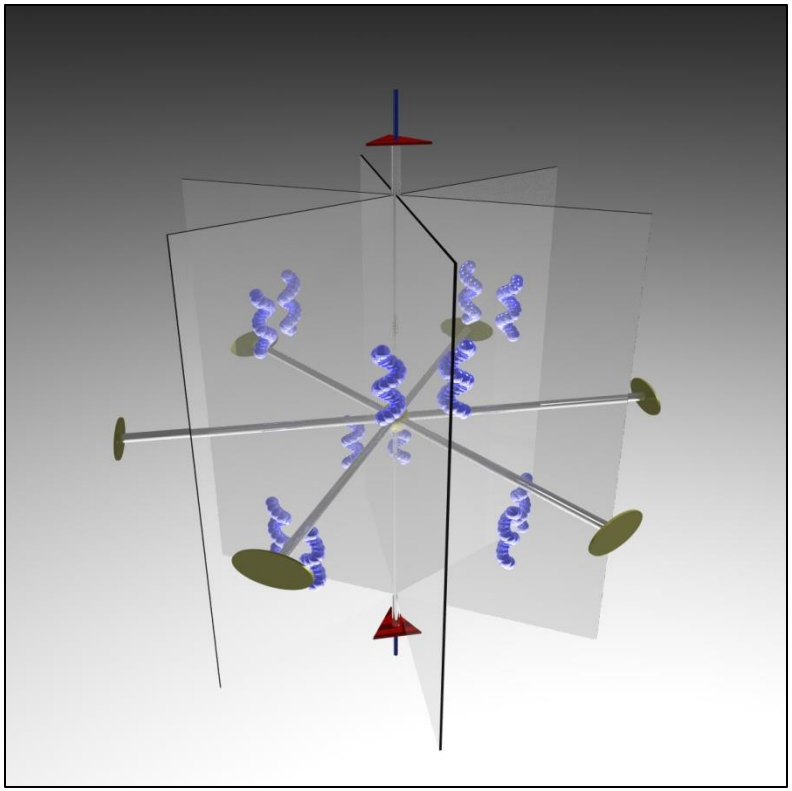
4



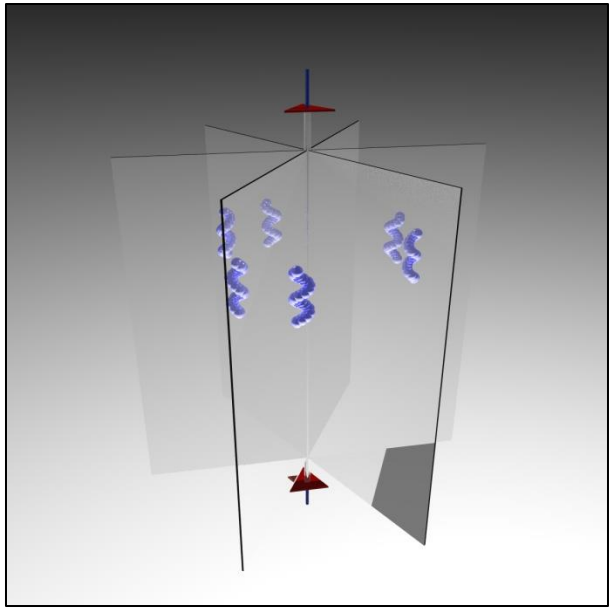
$\bar{4}$

TRIGONAL (RHOMBOHEDRAL) SYSTEM

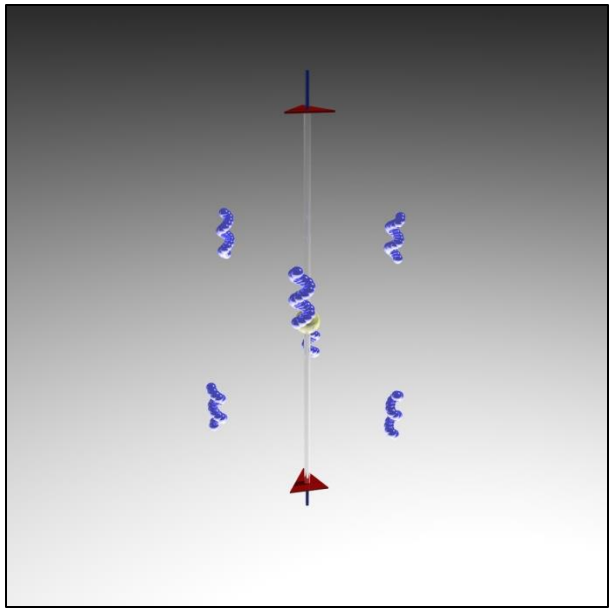
HOLOHEDRY GROUP



$\bar{3}m$



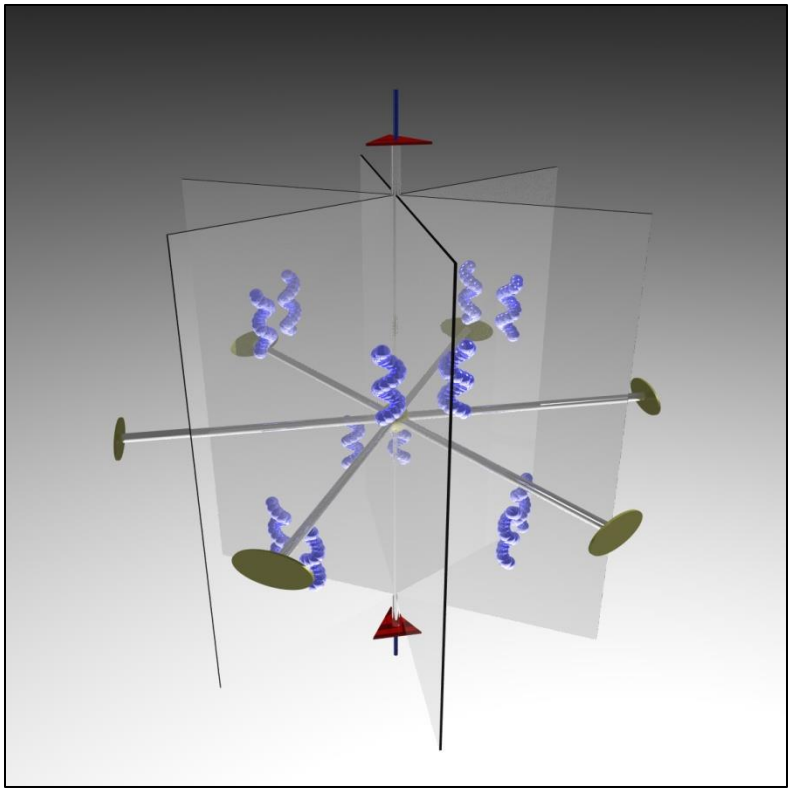
$3m$



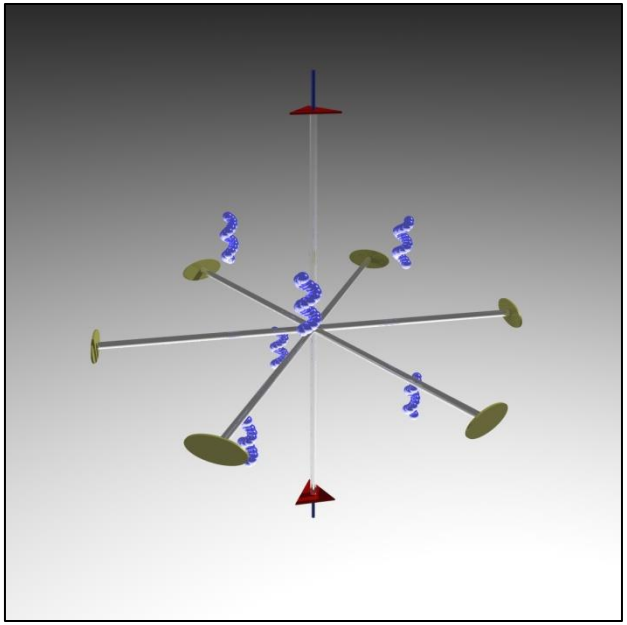
$\bar{3}$

TRIGONAL (RHOMBOHEDRAL) SYSTEM

HOLOHEDRY GROUP



$\bar{3}m$



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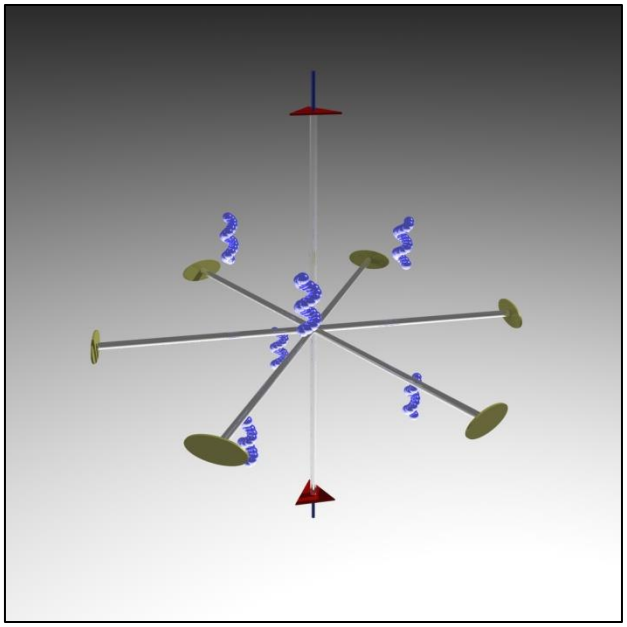
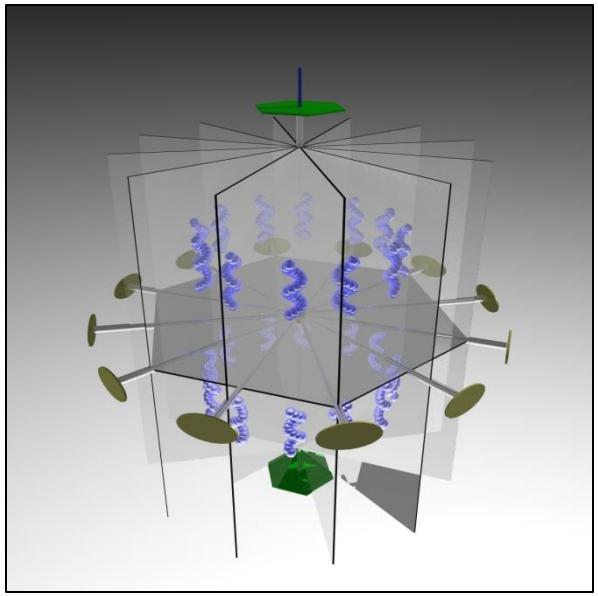


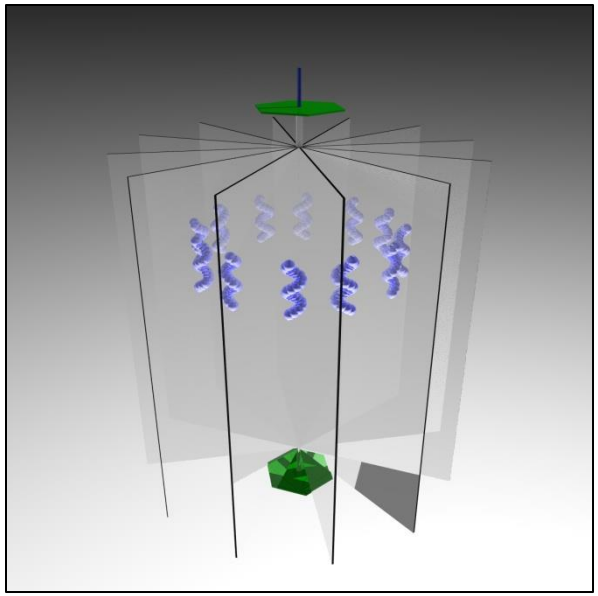
3

HEXAGONAL SYSTEM

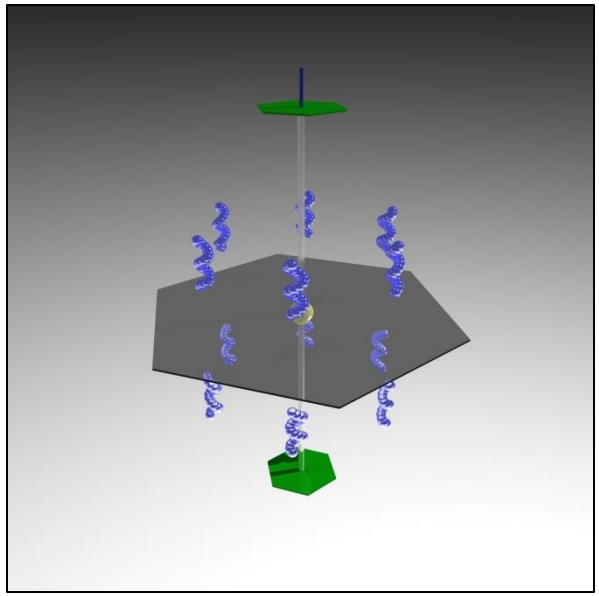
HOLOHEDRY GROUP



$6/mmm$



$6mm$



$6/m$

HEXAGONAL SYSTEM

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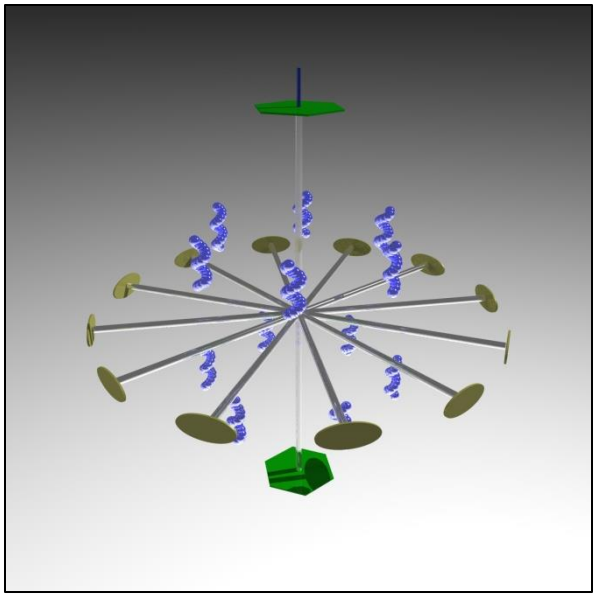
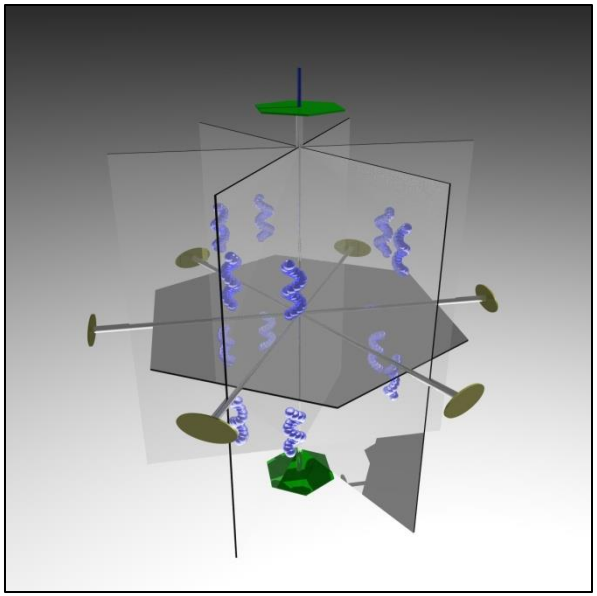
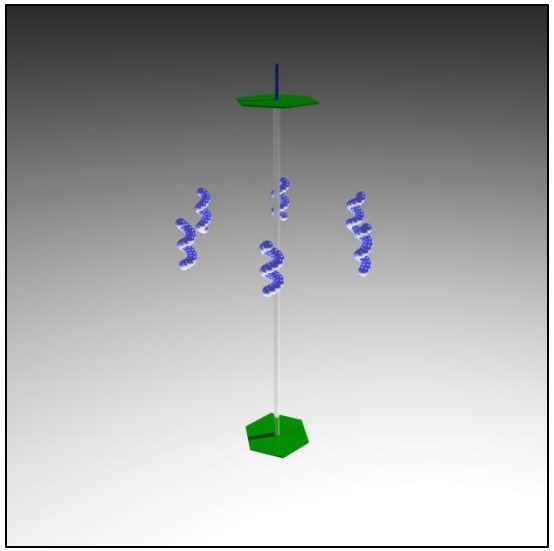
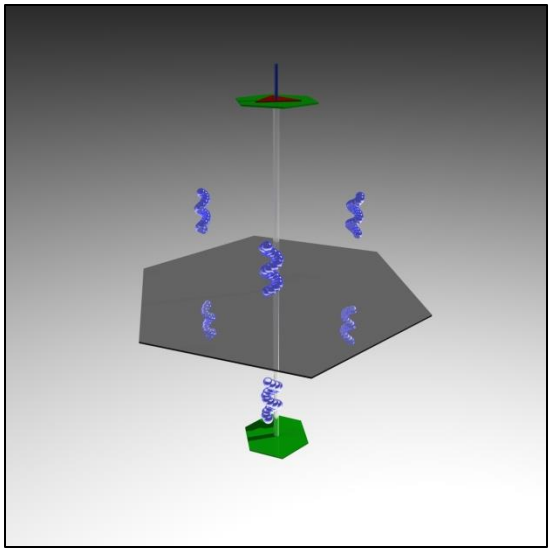




$\bar{6}m2$

6



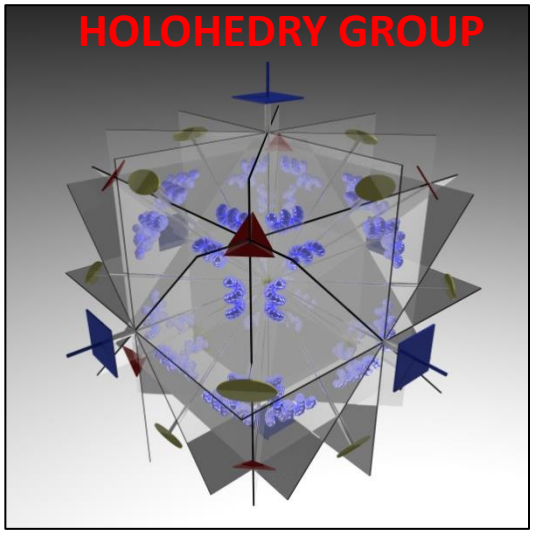


$\bar{6}$

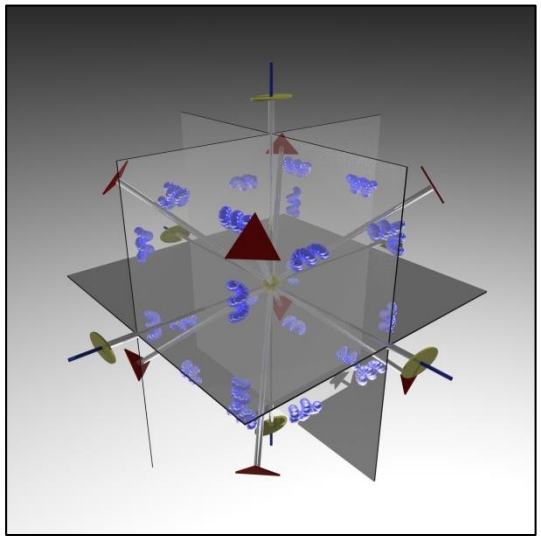
CUBIC SYSTEM

$m\bar{3}m$

HOLOHEDRY GROUP



$m\bar{3}$



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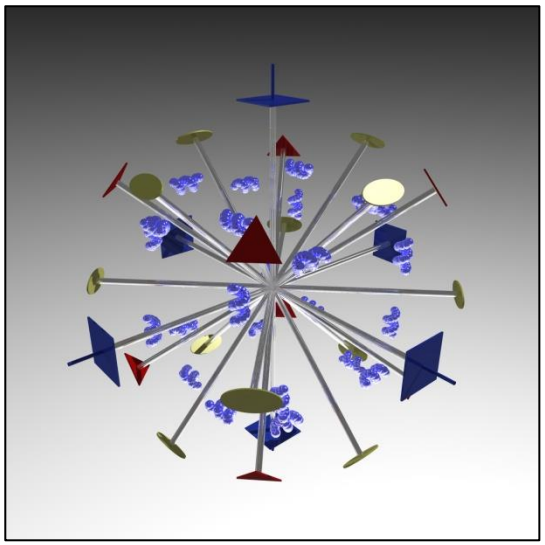
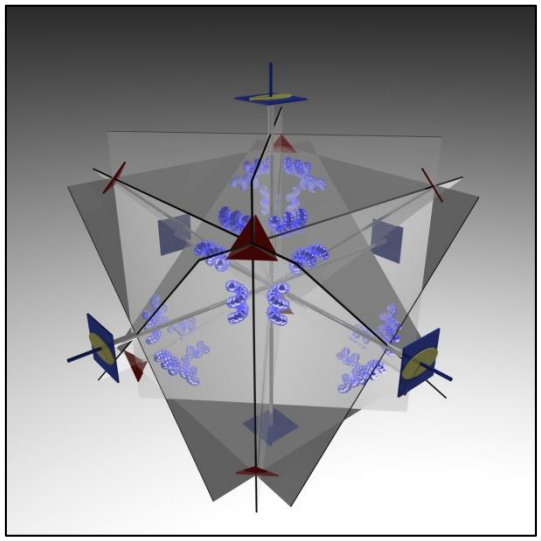
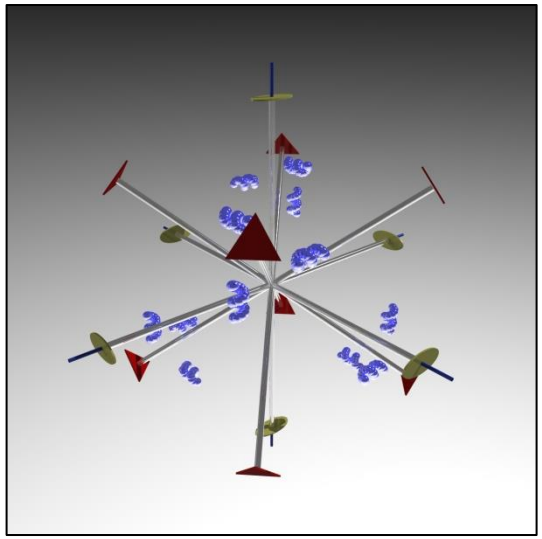


$\bar{4}3m$





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SUMMARY, 32 POINT SYMMETRY GROUPS IN CRYSTALS

Triclinic	Monoclinic	Orthorhombic	Tetragonal	Rhombohedral	Hexagonal	Cubic
$\bar{1}$	$2/m$	mmm	$4/mmm$	$\bar{3}m$	$6/mmm$	$m\bar{3}m$
		$mm2$	$4mm$	$3m$	$6mm$	
			$4/m$	$\bar{3}$	$6/m$	$m\bar{3}$
1	2	222	422	32	622	432
	m		$\bar{4}2m$		$\bar{6}2m$	$\bar{4}3m$
			4	3	6	23
			$\bar{4}$		$\bar{6}$	