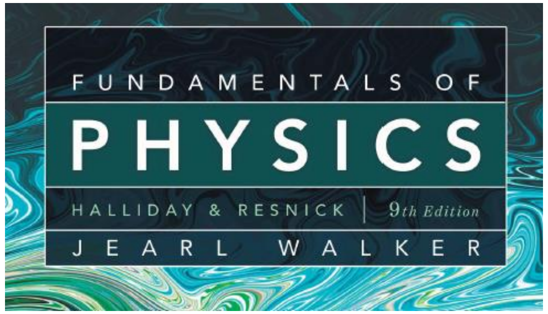
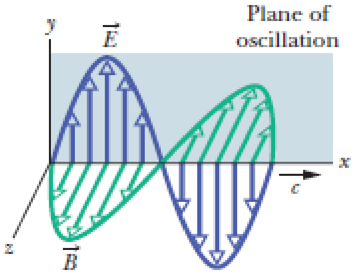
Physics 1

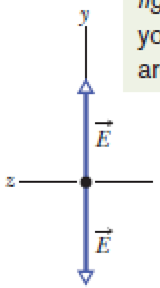
Lecture 9: Polarization, Reflection, Refraction of Light

Prof. Dr. U. Pietsch



Polarization

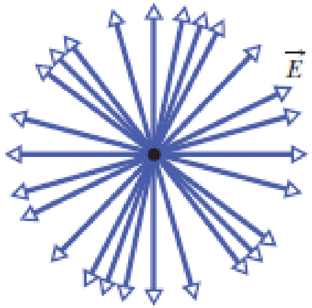


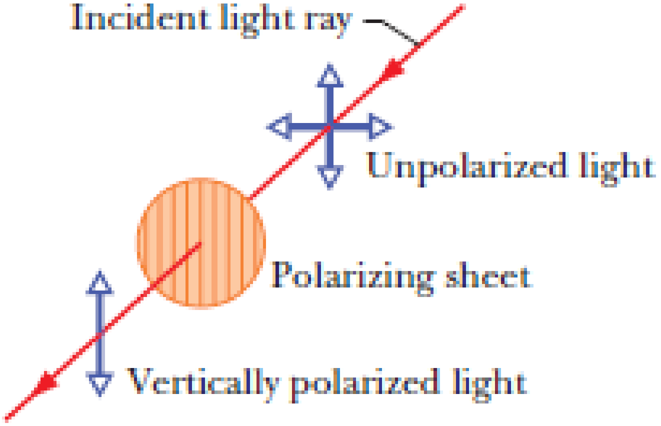


Vertically polarized light headed toward you–the electric fields are all vertical.

An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

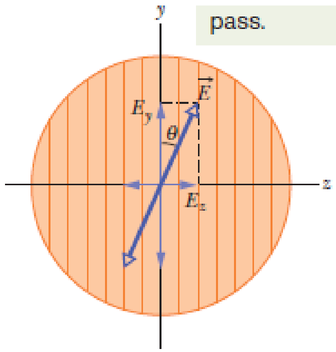
Unpolarized light headed toward you–the electric fields are in all directions in the plane.





$$E_y = E \cos \theta.$$

$$I = I_0 \cos^2 \theta.$$



pass.

Figure 33-15*a*, drawn in perspective, shows a system of three polarizing sheets in the path of initially unpolarized light. The polarizing direction of the first sheet is parallel to the y axis, that of the second sheet is at an angle of 60° counterclockwise from the y axis, and that of the third sheet is parallel to the x axis. What fraction of the initial intensity I_0 of the light emerges from the three-sheet system, and in which direction is that emerging light polarized?

KEY IDEAS

1. We work through the system sheet by sheet, from the first one encountered by the light to the last one.
2. To find the intensity transmitted by any sheet, we apply either the one-half rule or the cosine-squared rule, depending on whether the light reaching the sheet is unpolarized or already polarized.
3. The light that is transmitted by a polarizing sheet is always polarized parallel to the polarizing direction of the sheet.

First sheet: The original light wave is represented in Fig. 33-15*b*, using the head-on, double-arrow representation of Fig. 33-10*b*. Because the light is initially unpolarized, the intensity I_1 of the light transmitted by the first sheet is given by the one-half rule (Eq. 33-36):

$$I_1 = \tfrac{1}{2} I_0.$$

Because the polarizing direction of the first sheet is parallel to the y axis, the polarization of the light transmitted by it is also, as shown in the head-on view of Fig. 33-15*c*.

Second sheet: Because the light reaching the second sheet is polarized, the intensity I_2 of the light transmitted by that sheet is given by the cosine-squared rule (Eq. 33-38). The angle θ in the rule is the angle between the polarization direction of the entering light (parallel to the y axis) and the polarizing direction of the second sheet (60° counterclockwise from the y axis), and so θ is 60°. (The larger angle between the two directions, namely 120°, can also be used.) We have

$$I_2 = I_1 \cos^2 60°.$$

The polarization of this transmitted light is parallel to the polarizing direction of the sheet transmitting it—that is, 60° counterclockwise from the y axis, as shown in the head-on view of Fig. 33-15*d*.

Third sheet: Because the light reaching the third sheet is polarized, the intensity I_3 of the light transmitted by that sheet is given by the cosine-squared rule. The angle θ is now the angle between the polarization direction of the entering light (Fig. 33-15*d*) and the polarizing direction of the third sheet (parallel to the x axis), and so $\theta = 30°$. Thus,

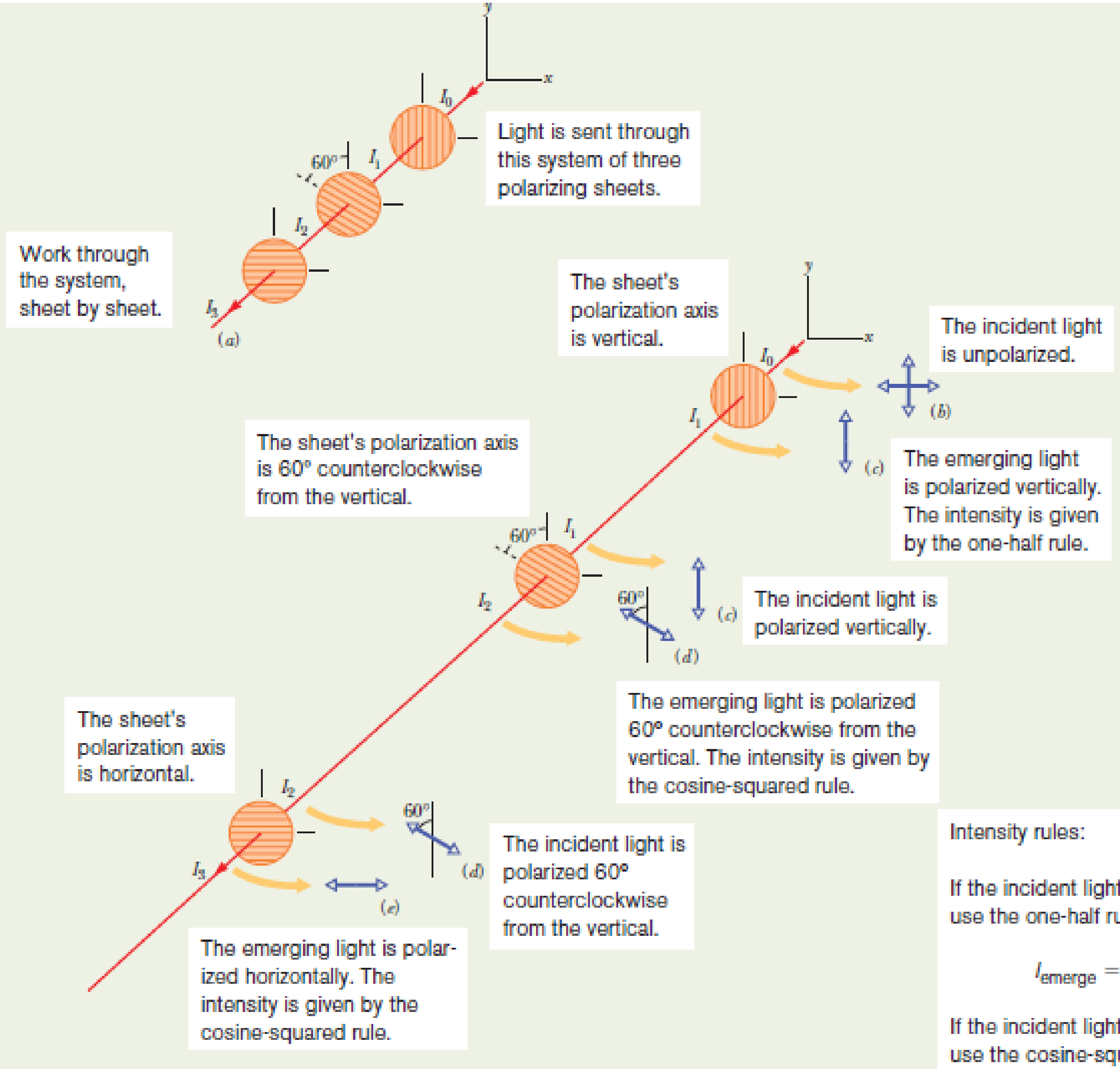
$$I_3 = I_2 \cos^2 30°.$$

This final transmitted light is polarized parallel to the x axis (Fig. 33-15*e*). We find its intensity by substituting first for I_2 and then for I_1 in the equation above:

$$I_3 = I_2 \cos^2 30° = (I_1 \cos^2 60°) \cos^2 30°$$
$$= (\tfrac{1}{2} I_0) \cos^2 60° \cos^2 30° = 0.094 I_0.$$

Thus, $$\frac{I_3}{I_0} = 0.094. \qquad \text{(Answer)}$$

That is to say, 9.4% of the initial intensity emerges from the three-sheet system. (If we now remove the second sheet, what fraction of the initial intensity emerges from the system?)



Intensity rules:

If the incident light is unpolarized, use the one-half rule:

$$I_{emerge} = 0.5 I_{incident}.$$

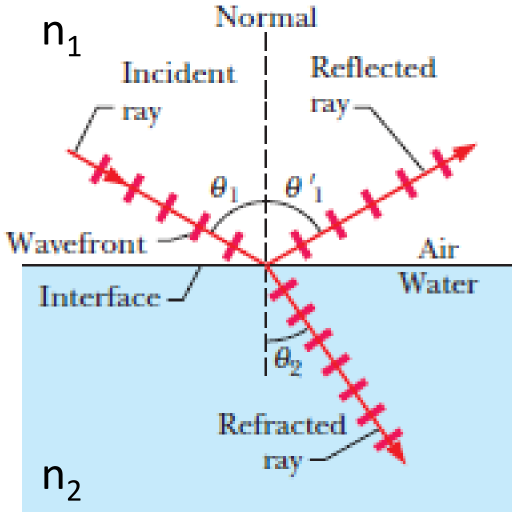
If the incident light is already polarized, use the cosine-square rule:

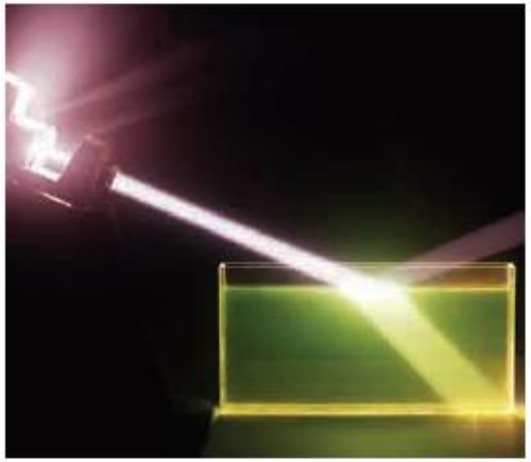
$$I_{emerge} = I_{incident}(\cos\theta)^2,$$

but be sure to insert the angle between the polarization of the incident light and the polarization axis of the sheet.

33-15 (*a*) Initially unpolarized light of intensity I_0 is sent into a system of three rizing sheets. The intensities I_1, I_2, and I_3 of the light transmitted by the sheets are ied. Shown also are the polarizations, from head-on views, of (*b*) the initial light and ight transmitted by (*c*) the first sheet, (*d*) the second sheet, and (*e*) the third sheet.

Reflection and refraction





Law of reflection

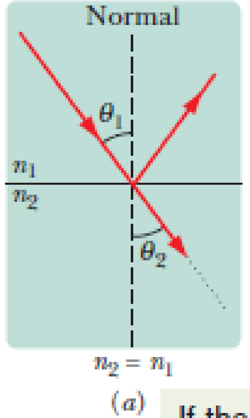
$$\theta'_1 = \theta_1 \qquad \text{(reflection).}$$

Law of refraction

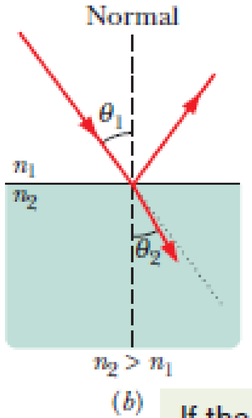
$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \qquad \text{(refraction).}$$

Some Indexes of Refraction[a]

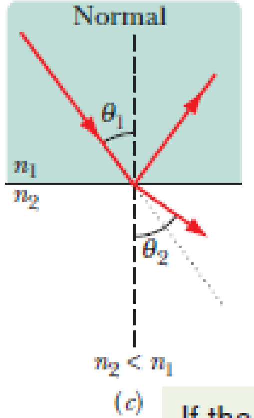
Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42



If the indexes match, there is no direction change.

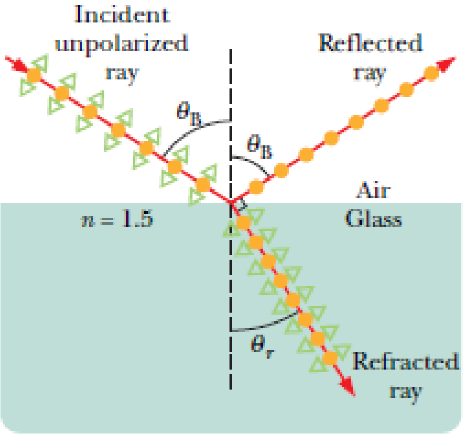


If the next index is greater, the ray is bent *toward* the normal.



If the next index is less, the ray is bent *away from* the normal.

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$



Polarization by Reflection , Brewster´s law

$$\theta_B + \theta_r = 90°.$$

$$n_1 \sin \theta_B = n_2 \sin(90° - \theta_B) = n_2 \cos \theta_B,$$

$$\theta_B = \tan^{-1} \frac{n_2}{n_1} \qquad \text{(Brewster angle).}$$

Biochimica et Biophysica Acta 1859 (2017) 1749–1766

Applications of Brewster angle microscopy from biological materials to biological systems

Weiam Daear, Mark Mahadeo, Elmar J. Prenner *

Department of Biological Sciences, Faculty of Science, University of Calgary, 2500 University Drive NW, Calgary, T2N 1N4, Alberta, Canada

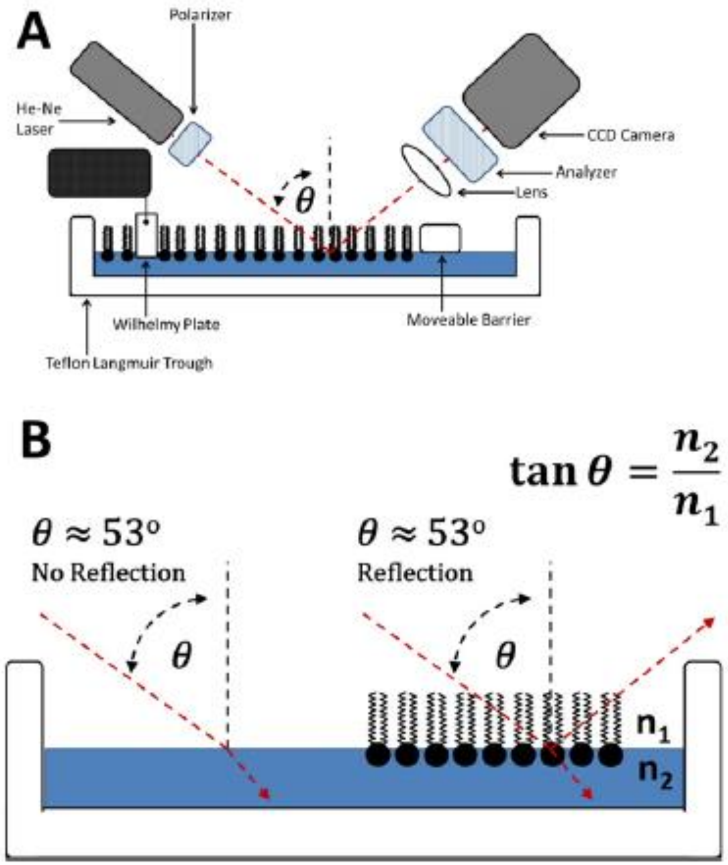


Fig. 1. Schematic of the experimental set up of Brewster angle microscopy (A). Schematic diagram of the principle of Brewster angle microscopy (B).

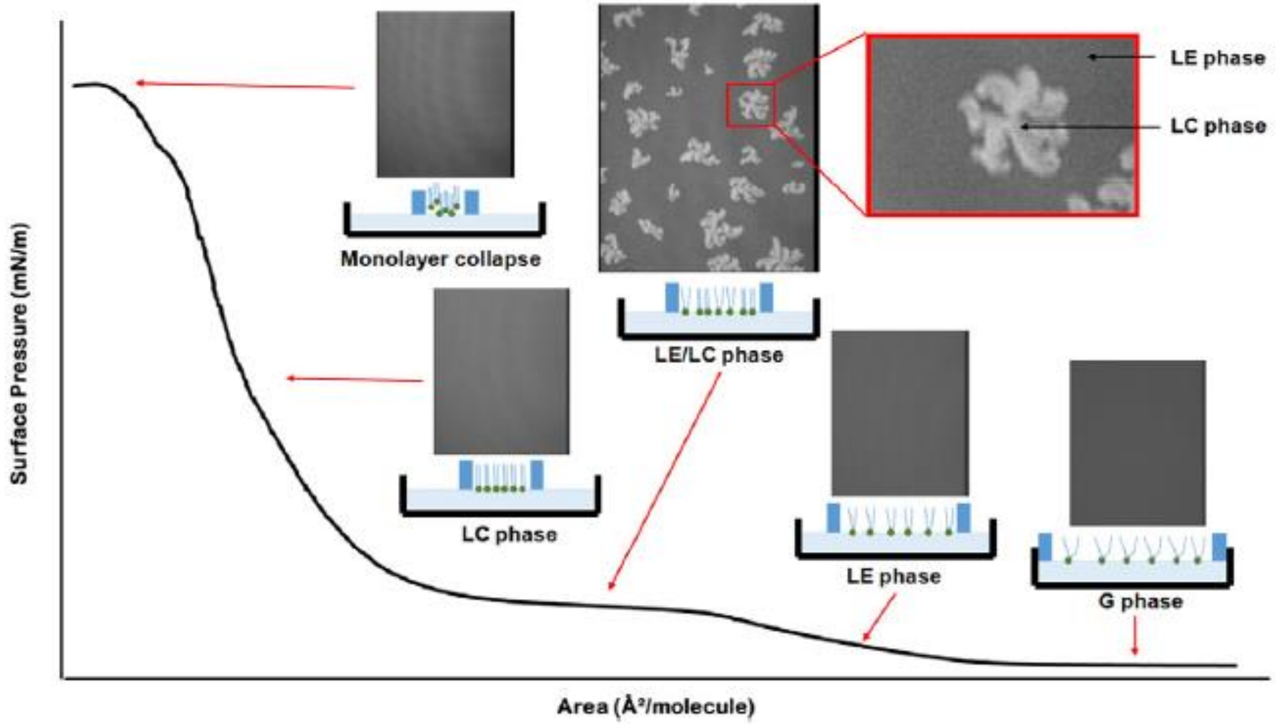
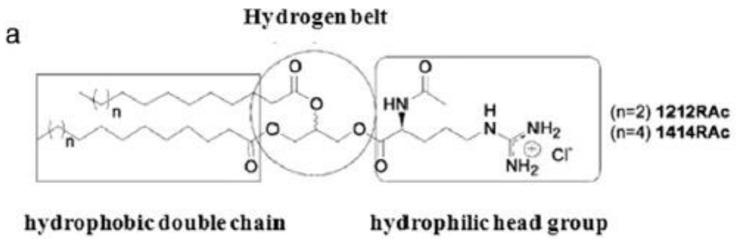
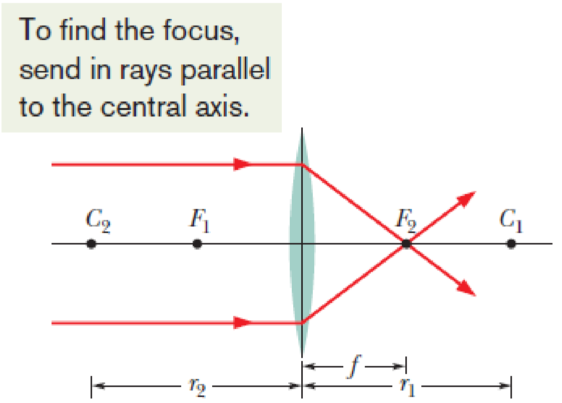


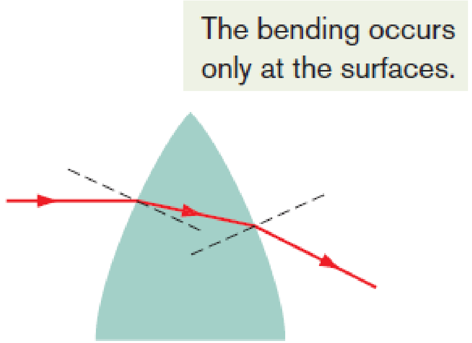
Fig. 2. DPPC isotherm and BAM images at selected stages of the compression.



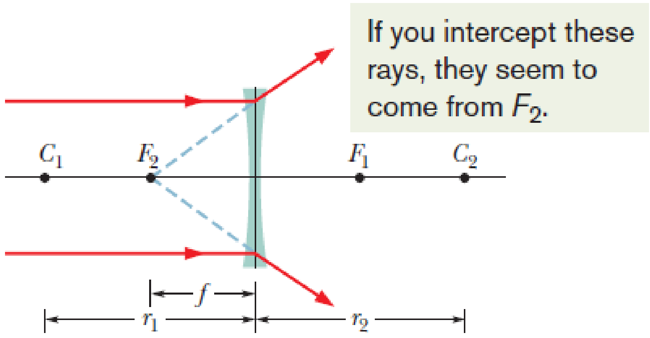
Refraction applied: lenses



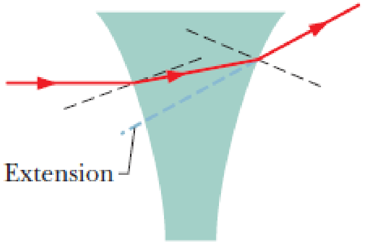
(a)



A lens can produce an image of an object only because the lens can bend light rays, but it can bend light rays only if its index of refraction differs from that of the surrounding medium.



(c)

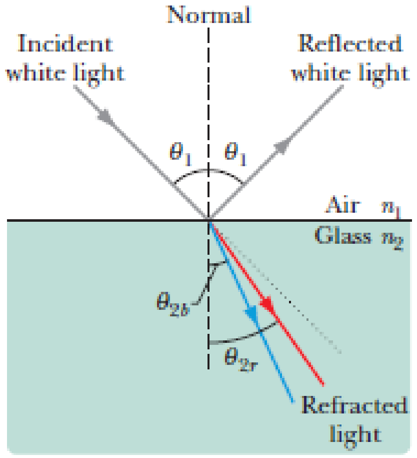


(d)

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \text{(thin lens in air)},$$

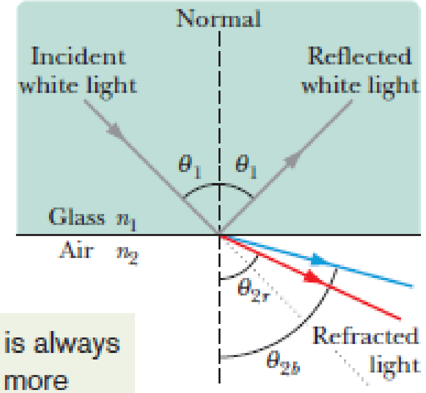
Lense equation

Chromatic dispersion

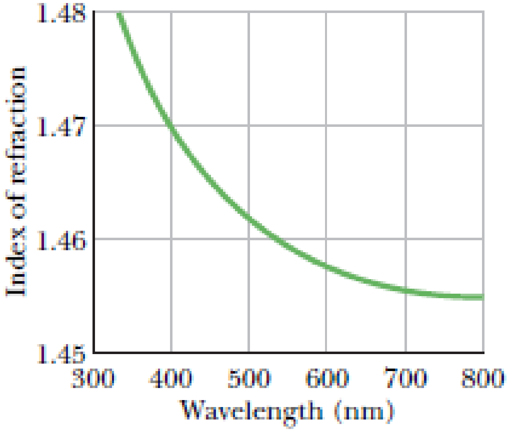


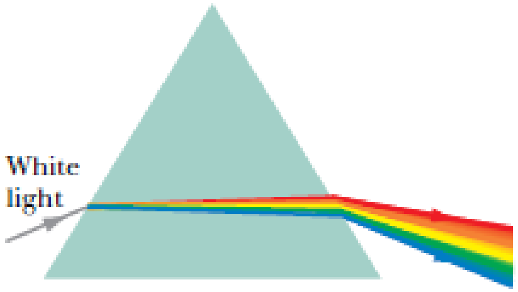
(a)

Blue is always bent more than red.



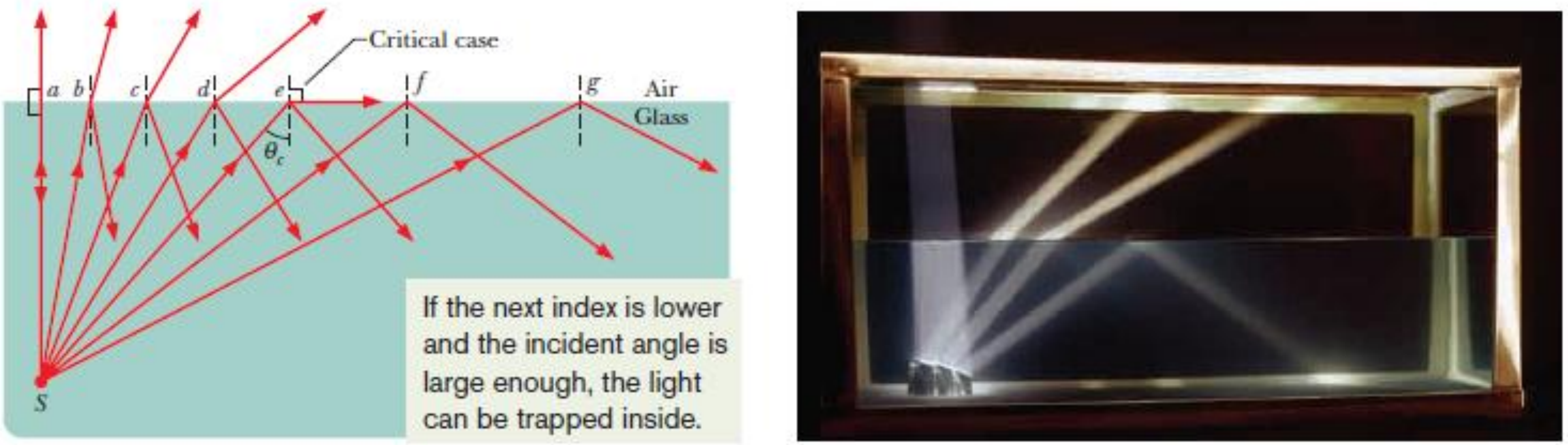
(b)





(b)

Total internal reflection



(a) (b)

$$n_1 \sin \theta_c = n_2 \sin 90°,$$

$$\theta_c = \sin^{-1} \frac{n_2}{n_1} \qquad \text{(critical angle).}$$

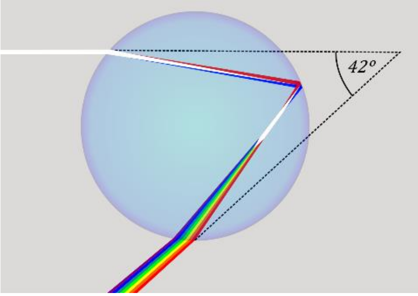
Rain bow

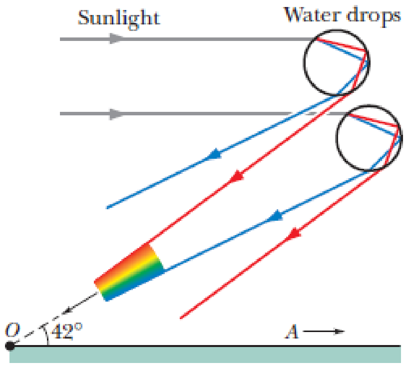
Rain bow

Refraction at water droplet surface and reflection at the inner part of the droplet

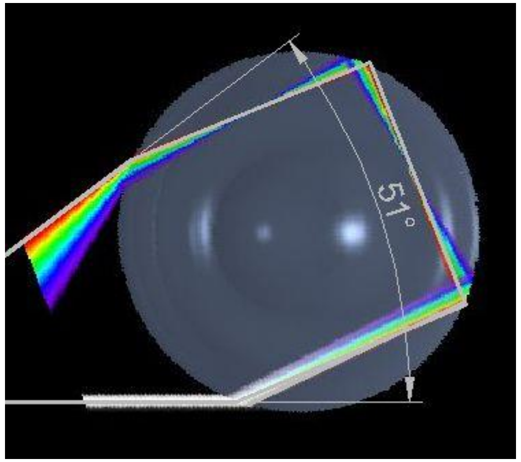
1. refraction disperses the white light into colors
2. Reflection at the inner drop´s surface increases dispersion
3. Refreaction towards outside

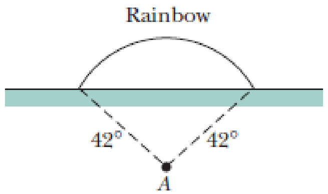
Rainbow appears always on an arc of 42°
Visible under an angle of 52° with respect to the surface of earth



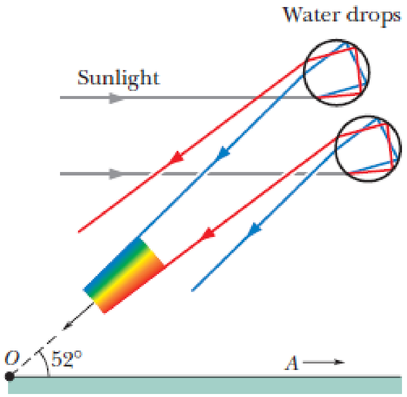


(a)





(c)



(d)

(a) In Fig. 33-22*a*, a beam of monochromatic light reflects and refracts at point *A* on the interface between material 1 with index of refraction $n_1 = 1.33$ and material 2 with index of refraction $n_2 = 1.77$. The incident beam makes an angle of 50° with the interface. What is the angle of reflection at point *A*? What is the angle of refraction there?

KEY IDEAS

(1) The angle of reflection is equal to the angle of incidence, and both angles are measured relative to the normal to the surface at the point of reflection. (2) When light reaches the interface between two materials with different indexes of refraction (call them n_1 and n_2), part of the light can be refracted by the interface according to Snell's law, Eq. 33-40:

$$n_2 \sin \theta_2 = n_1 \sin \theta_1, \tag{33-42}$$

where both angles are measured relative to the normal at the point of refraction.

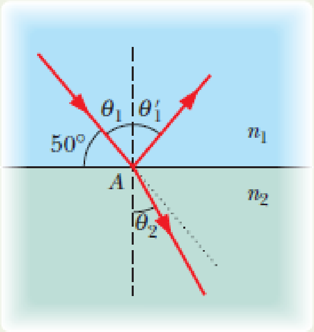
Calculations: In Fig. 33-22*a*, the normal at point *A* is drawn as a dashed line through the point. Note that the angle of incidence θ_1 is not the given 50° but is 90° − 50° = 40°. Thus, the angle of reflection is

$$\theta_1' = \theta_1 = 40°. \qquad \text{(Answer)}$$

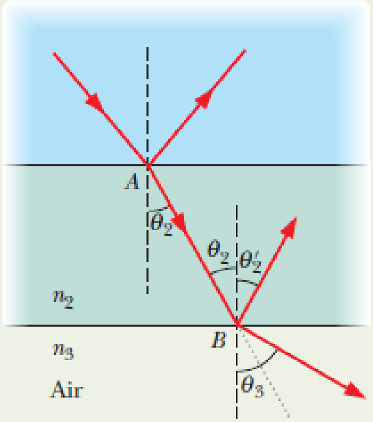
The light that passes from material 1 into material 2 undergoes refraction at point *A* on the interface between the two materials. Again we measure angles between light rays and a normal, here at the point of refraction. Thus, in Fig. 33-22*a*, the angle of refraction is the angle marked θ_2. Solving Eq. 33-42 for θ_2 gives us

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.77}\sin 40°\right)$$

$$= 28.88° \approx 29°. \qquad \text{(Answer)}$$



(*a*)

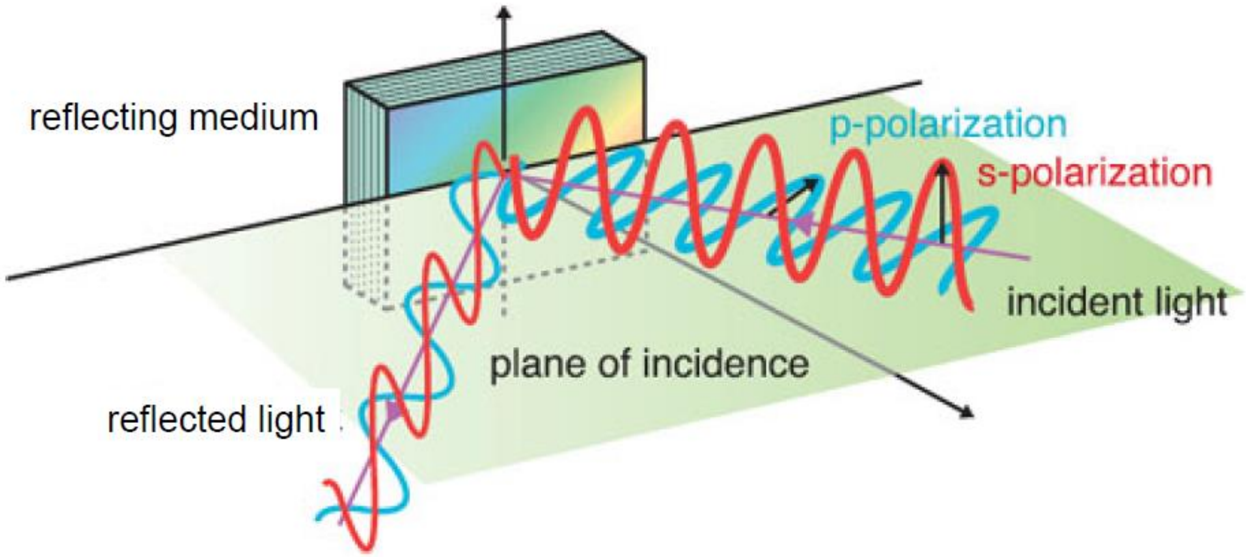


(*b*)

Fig. 33-22 (*a*) Light reflects and refracts at point *A* on the interface between materials 1 and 2. (*b*) The light that passes through material 2 reflects and refracts at point *B* on the interface between materials 2 and 3 (air). Each dashed line is a normal. Each dotted line gives the incident direction of travel.

This result means that the beam swings toward the normal (it was at 40° to the normal and is now at 29°). The reason is that when the light travels across the interface, it moves into a material with a greater index of refraction. *Caution:* Note that the beam does *not* swing through the normal so that it appears on the left side of Fig. 33-22*a*.

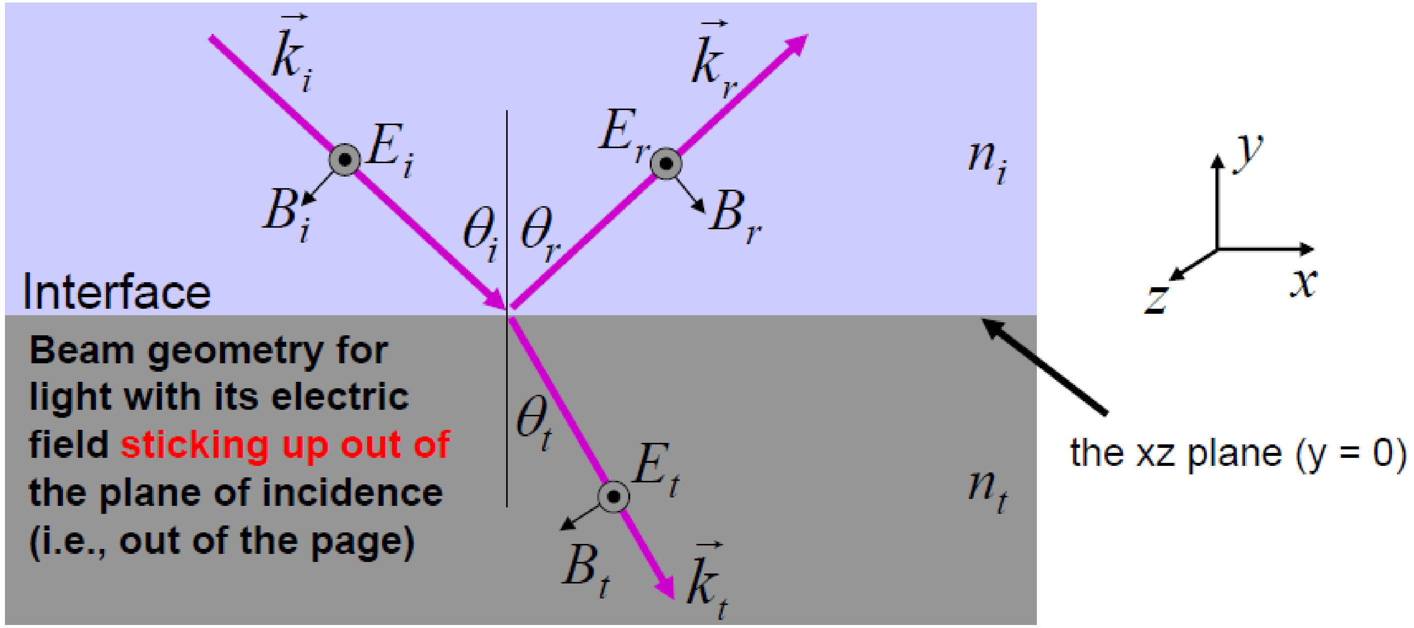
Fresnel's Equations for Reflection and Transmission



The amount of reflected (and transmitted) light is different for the two different incident polarizations.

Augustin Fresnel
1788-1827

Fresnel Equations—Perpendicular E field



The Tangential Electric Field is Continuous

$$E_i(y=0) + E_r(y=0) = E_t(y=0)$$

The Tangential Magnetic Field is Continuous*

$$-B_i(y=0)\cos\theta_i + B_r(y=0)\cos\theta_r = -B_t(y=0)\cos\theta_t$$

Ignoring the rapidly varying parts of the light wave and keeping only the complex amplitudes:

$$E_{0i} + E_{0r} = E_{0t}$$

$$-B_{0i}\cos(\theta_i) + B_{0r}\cos(\theta_r) = -B_{0t}\cos(\theta_t)$$

But $B = E/(c_0/n) = nE/c_0$ and $\theta_i = \theta_r$.

Substituting into the second equation:

$$n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t E_{0t}\cos(\theta_t)$$

Substituting for E_{0t} using $E_{0i} + E_{0r} = E_{0t}$:

$$n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t(E_{0r} + E_{0i})\cos(\theta_t)$$

Reflection & Transmission Coefficients for Perpendicularly Polarized Light

Rearranging $n_i(E_{0r} - E_{0i})\cos(\theta_i) = -n_t(E_{0r} + E_{0i})\cos(\theta_t)$ yields:

$$E_{0r}\left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right] = E_{0i}\left[n_i\cos(\theta_i) - n_t\cos(\theta_t)\right]$$

Solving for E_{0r} / E_{0i} yields the reflection coefficient :

$$r_\perp = E_{0r} / E_{0i} = \left[n_i\cos(\theta_i) - n_t\cos(\theta_t)\right] / \left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right]$$

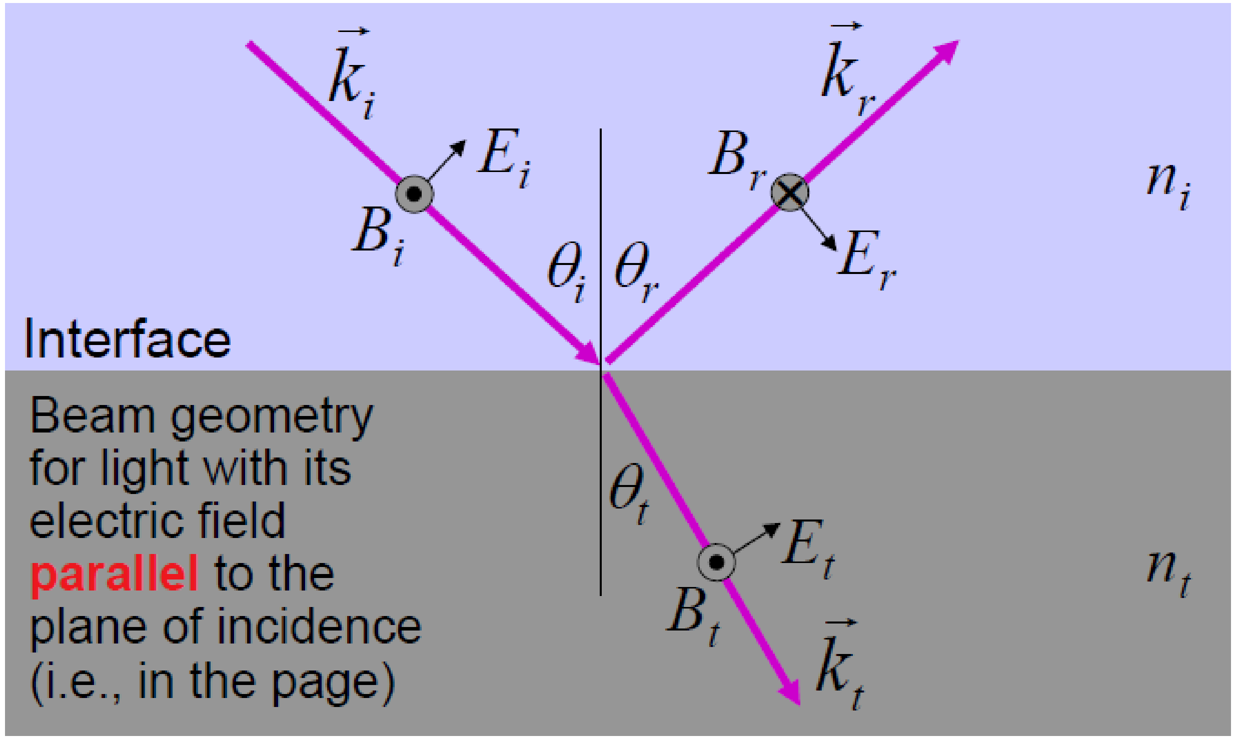
Analogously, the transmission coefficient, E_{0t} / E_{0i}, is

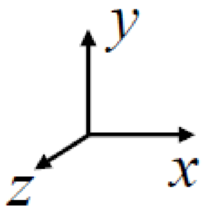
$$t_\perp = E_{0t} / E_{0i} = 2n_i\cos(\theta_i) / \left[n_i\cos(\theta_i) + n_t\cos(\theta_t)\right]$$

These equations are called the Fresnel Equations for perpendicularly polarized (s-polarized) light.

Fresnel Equations—Parallel electric field

Now, the case of P polarization:





Note that Hecht uses a different notation for the reflected field, which is confusing!

Ours is better!

This leads to a difference in the signs of some equations...

Note that the reflected magnetic field must point into the screen to achieve $\vec{E} \times \vec{B} \propto \vec{k}$ for the reflected wave. The x with a circle around it means "into the screen."

Reflection & Transmission Coefficients for Parallel Polarized Light

For parallel polarized light, $B_{0i} - B_{0r} = B_{0t}$ $\quad B = E/(c_0/n) = nE/c_0$

and $E_{0i}\cos(\theta_i) + E_{0r}\cos(\theta_r) = E_{0t}\cos(\theta_t)$

Solving for E_{0r}/E_{0i} yields the reflection coefficient, $r_{||}$:

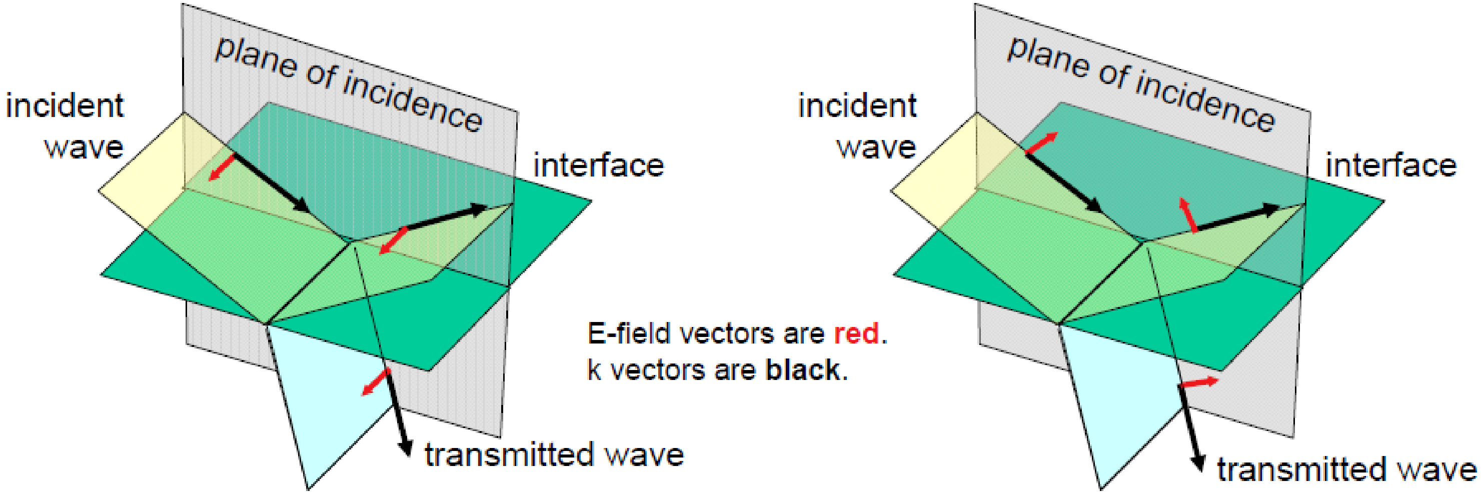
$$r_{||} = E_{0r}/E_{0i} = \left[n_i\cos(\theta_t) - n_t\cos(\theta_i)\right]/\left[n_i\cos(\theta_t) + n_t\cos(\theta_i)\right]$$

Analogously, the transmission coefficient, $t_{||} = E_{0t}/E_{0i}$, is

$$t_{||} = E_{0t}/E_{0i} = 2n_i\cos(\theta_i)/\left[n_i\cos(\theta_t) + n_t\cos(\theta_i)\right]$$

These equations are called the Fresnel Equations for parallel polarized (p-polarized) light.

To summarize…



s-polarized light:

$$r_{\perp} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_{\perp} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

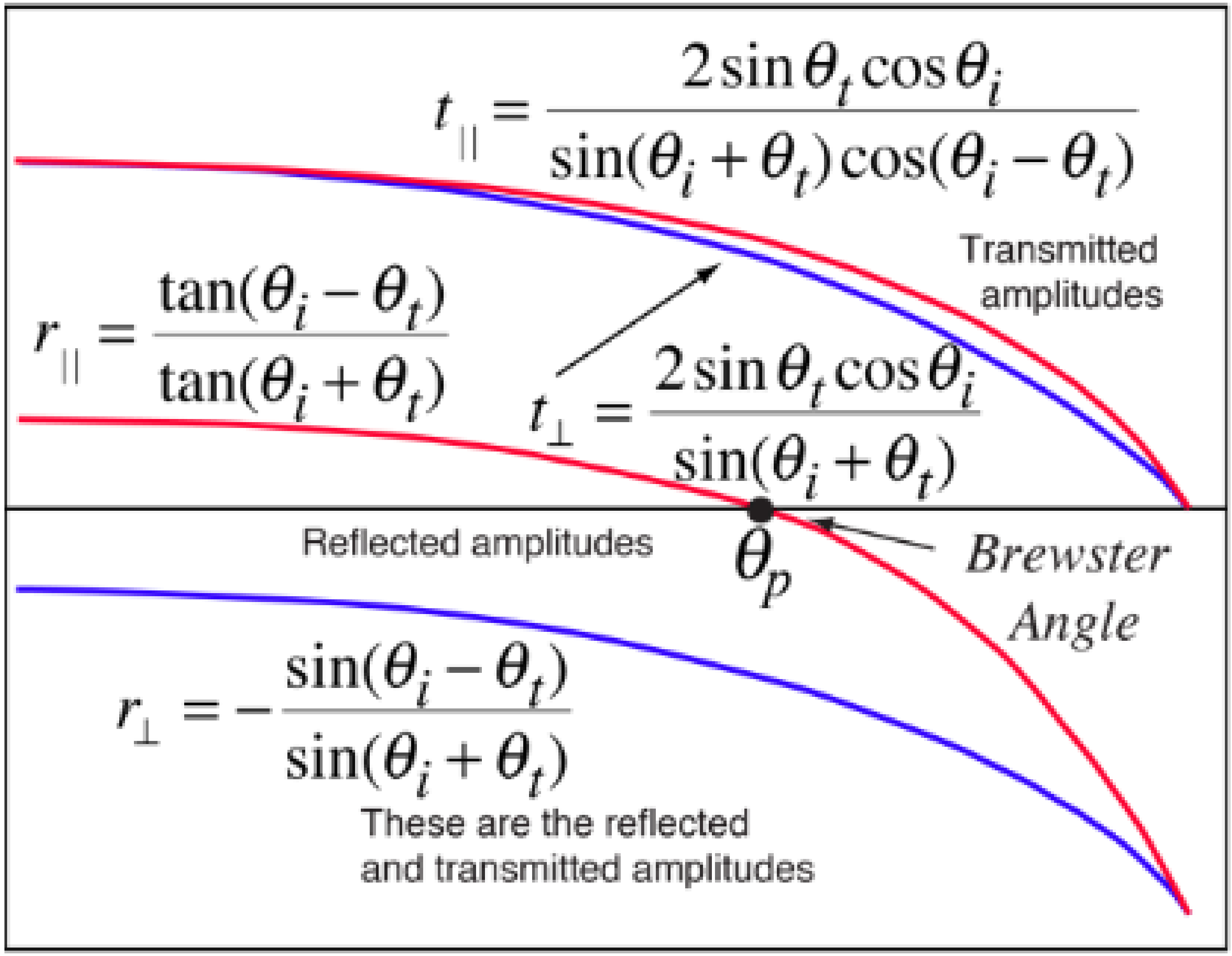
p-polarized light:

$$r_{\parallel} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

$$t_{\parallel} = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

And, for both polarizations: $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

Functional dependence of reflection and transmission amplitudes



Reflection Coefficients for an Air-to-Glass Interface

The two polarizations are indistinguishable at $\theta = 0°$

Total reflection at $\theta = 90°$ for both polarizations.

Zero reflection for parallel polarization at:

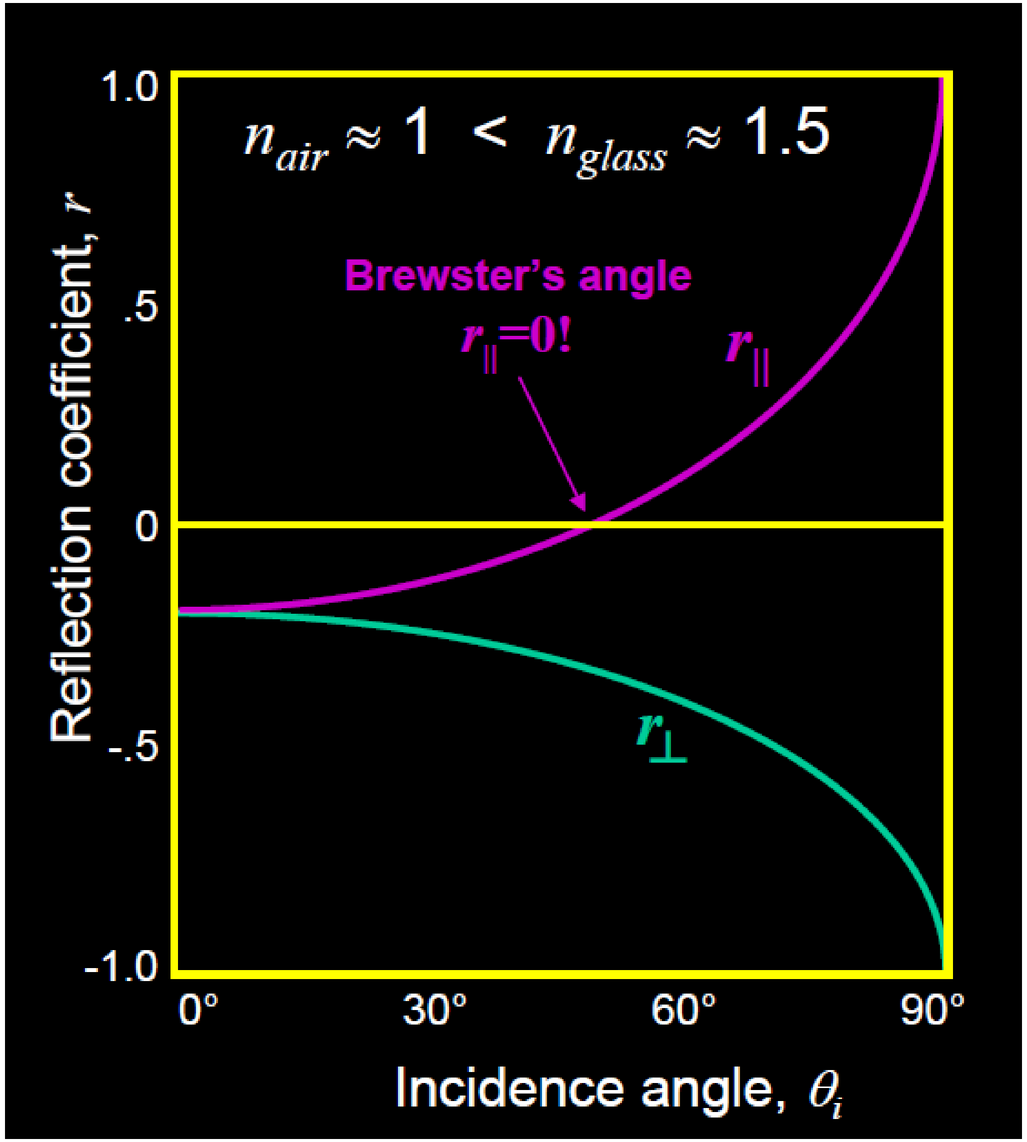
"Brewster's angle"

The value of this angle depends on the value of the ratio n_i/n_t:

$$\theta_{\text{Brewster}} = \tan^{-1}(n_t/n_i)$$

For air to glass ($n_{glass} = 1.5$), this is 56.3°.

Sir David Brewster
1781 - 1868



Reflection Coefficients for a Glass-to-Air Interface

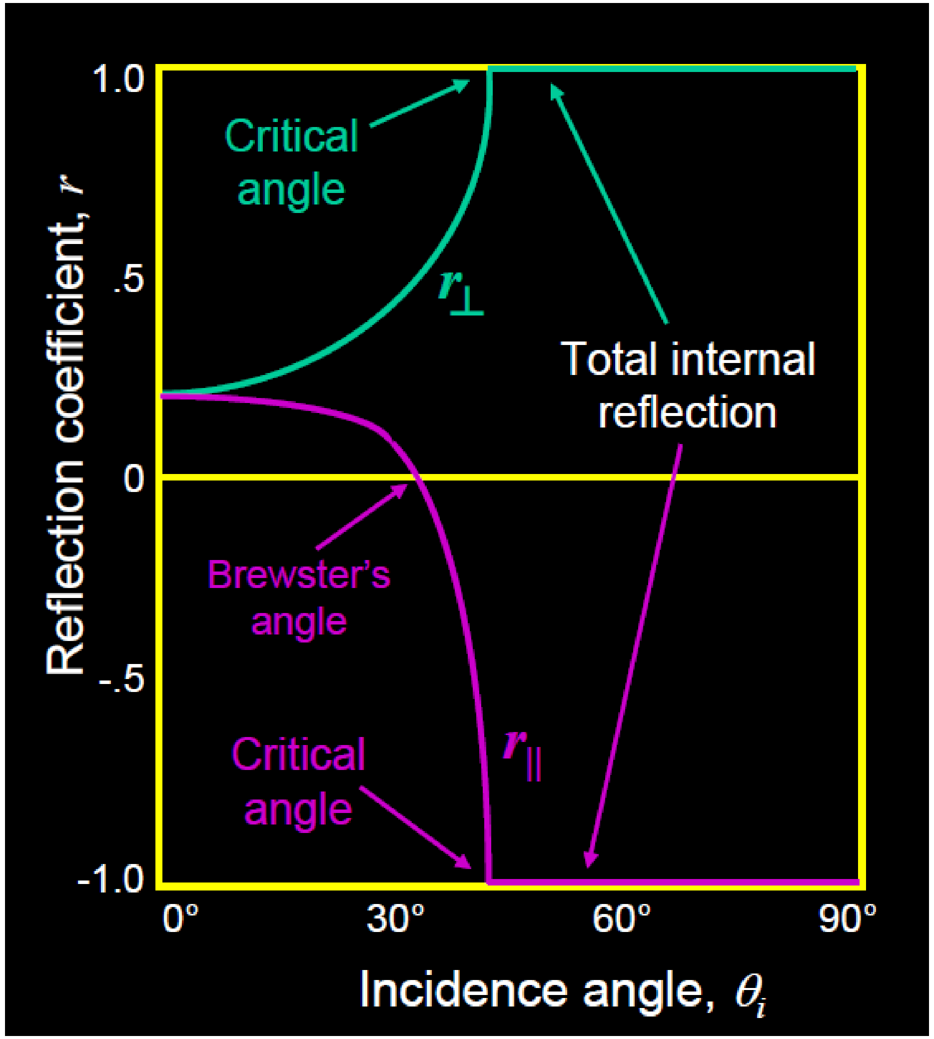
$n_{glass} > n_{air}$

Total internal reflection above the **"critical angle"**

$\theta_{crit} \equiv \sin^{-1}(n_t / n_i)$

$\approx 41.8°$ for glass-to-air

(The sine in Snell's Law can't be greater than one!)

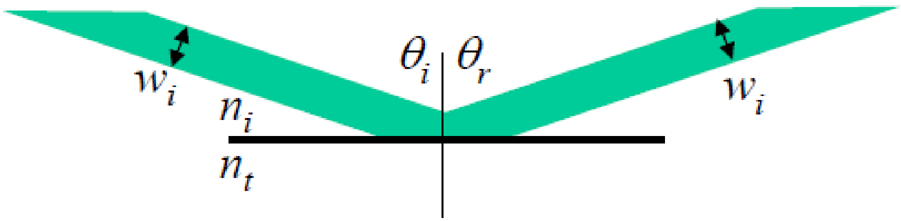


Reflectance (R)

$$I = \left(n \frac{\varepsilon_0 c_0}{2} \right) |E_0|^2$$

$$R \equiv \text{Reflected Power / Incident Power} = \frac{I_r A_r}{I_i A_i}$$

A = Area



Because the angle of incidence = the angle of reflection, the beam's area doesn't change on reflection.

Also, n is the same for both incident and reflected beams.

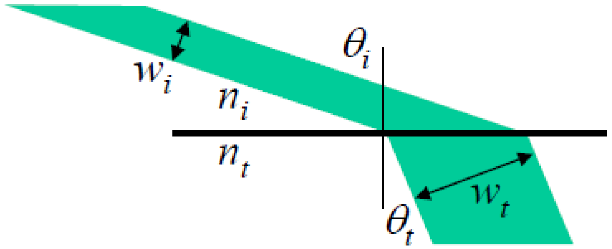
So: $R = r^2$ since $\dfrac{|E_{0r}|^2}{|E_{0i}|^2} = r^2$

Transmittance (T)

$$I = \left(n \frac{\varepsilon_0 c_0}{2} \right) |E_0|^2$$

$T \equiv$ Transmitted Power / Incident Power $= \dfrac{I_t A_t}{I_i A_i}$ ← A = Area

If the beam has width w_i:



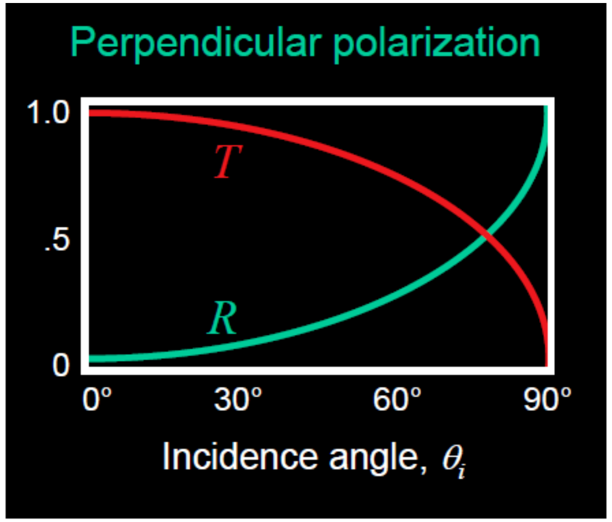
$$\frac{A_t}{A_i} = \frac{w_t}{w_i} = \frac{\cos(\theta_t)}{\cos(\theta_i)}$$

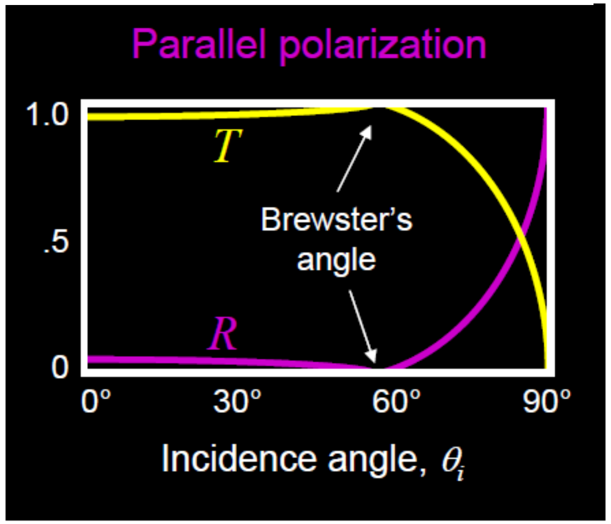
The beam expands (or contracts) in one dimension on refraction.

$$T = \frac{I_t A_t}{I_i A_i} = \frac{\left(n_t \frac{\varepsilon_0 c_0}{2} \right) |E_{0t}|^2}{\left(n_i \frac{\varepsilon_0 c_0}{2} \right) |E_{0i}|^2} \left[\frac{w_t}{w_i} \right] = \frac{n_t |E_{0t}|^2 w_t}{n_i |E_{0i}|^2 w_i} = \frac{n_t w_t}{n_i w_i} t^2 \quad \text{since } \frac{|E_{0t}|^2}{|E_{0i}|^2} = t^2$$

$$\Rightarrow \boxed{T = \left[\frac{(n_t \cos(\theta_t))}{(n_i \cos(\theta_i))} \right] t^2}$$

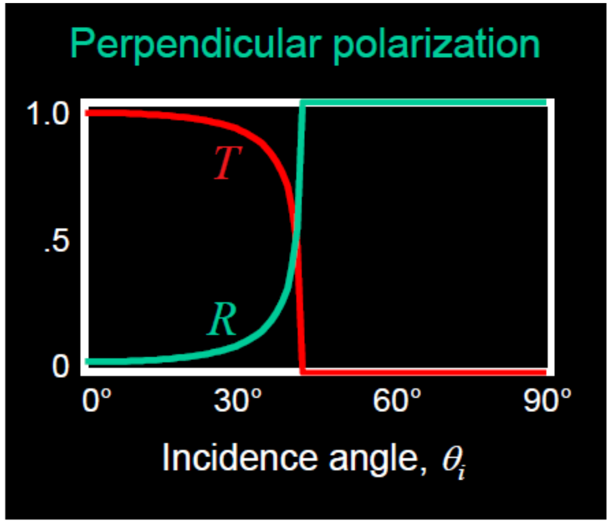
Reflectance and Transmittance for an Air-to-Glass Interface

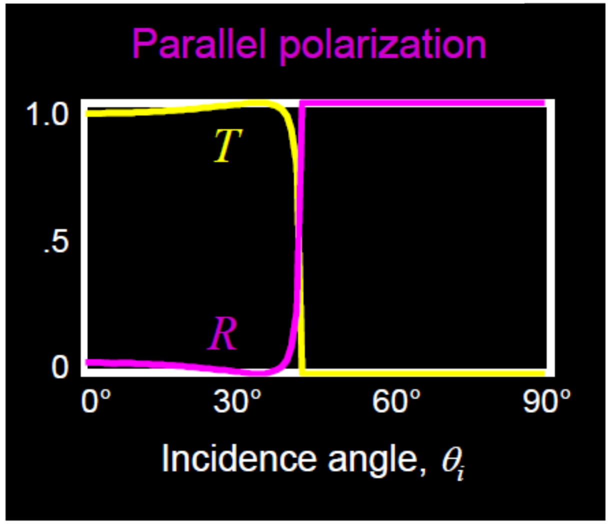




R + T = 1

Reflectance and Transmittance for a Glass-to-Air Interface





Reflection at normal incidence, $\theta_i = 0$

Taken from: https://www.brown.edu/research/labs/mittleman/sites/brown.edu.research.labs.mittleman/files/uploads/lecture13_0.pdf

When $\theta_i = 0$, the Fresnel equations reduce to:

$$R = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2 \qquad T = \frac{4\,n_t n_i}{(n_t + n_i)^2}$$

For an air-glass interface ($n_i = 1$ and $n_t = 1.5$),

R = 4% and T = 96%

Reflectance for absorbing media, θ_i =0

$$R = \frac{(n_t - n_i)^2 + \kappa_t^2}{(n_t + n_i)^2 + \kappa_t^2}$$

R (Au) = 0.87 (87%) R(Au, κ=0) = 0.29 (29%)

Metalloptik Tabelle: Optische Konstanten ausgewählter Metalle bei der Wellenlänge λ=563,6 nm (n Realteil, $\varkappa = n\varkappa'$ Imaginärteil des Brechungsindexes; ρ aus n und κ berechneter Reflexionsgrad).

Metall	n	$\varkappa$	ρ
Osmium	4,58	1,62	0,457
Wolfram	3,49	2,75	0,496
Molybdän	3,76	3,41	0,561
Nickel	1,80	3,33	0,620
Platin	2,17	3,77	0,642
Kupfer	0,826	2,60	0,673
Iridium	2,29	4,38	0,695
Rhodium	2,00	5,11	0,772
Gold	0,306	2,88	0,878
Silber	0,120	3,45	0,964

spectral dependence of reflectance