Physics 1

Lecture 8: Maxwell´s equations and electromagnetic waves

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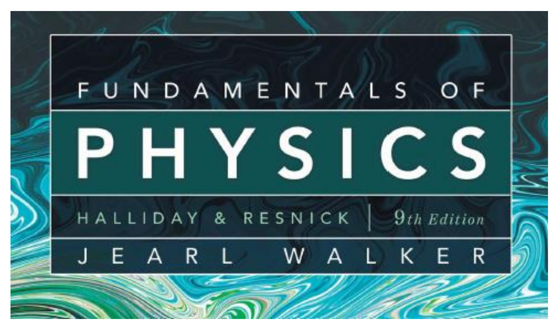
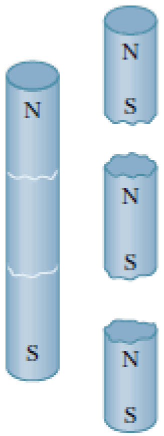


Summarizing our knowledge

Gauss´s laws

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss' law for magnetic fields).}$$

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).



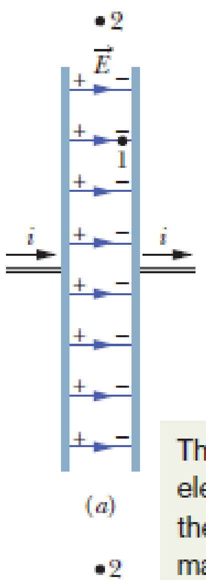
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad \text{(Gauss' law for electric fields).}$$

Flux throughout Gauss surface measures included net electric charges

Induced fields

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law of induction).}$$

a changing magnetic flux induces an electric field



The changing of the electric field between the plates creates a magnetic field.

Analogy predicted by James Clark Maxwell

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \text{(Maxwell's law of induction).}$$

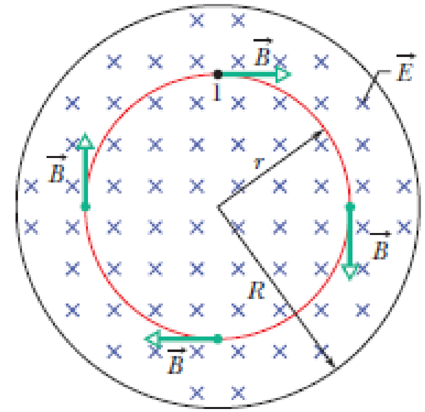
A changing electric flux induces an magnetic field

We know a similar equation :

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law)},$$

→ Combining : A current and a changing electric field give a magnetic field exactly in same form

$$\oint \vec{B} \cdot d\vec{s} = \boxed{\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}} + \mu_0 i_{enc} \qquad \text{(Ampere–Maxw}$$



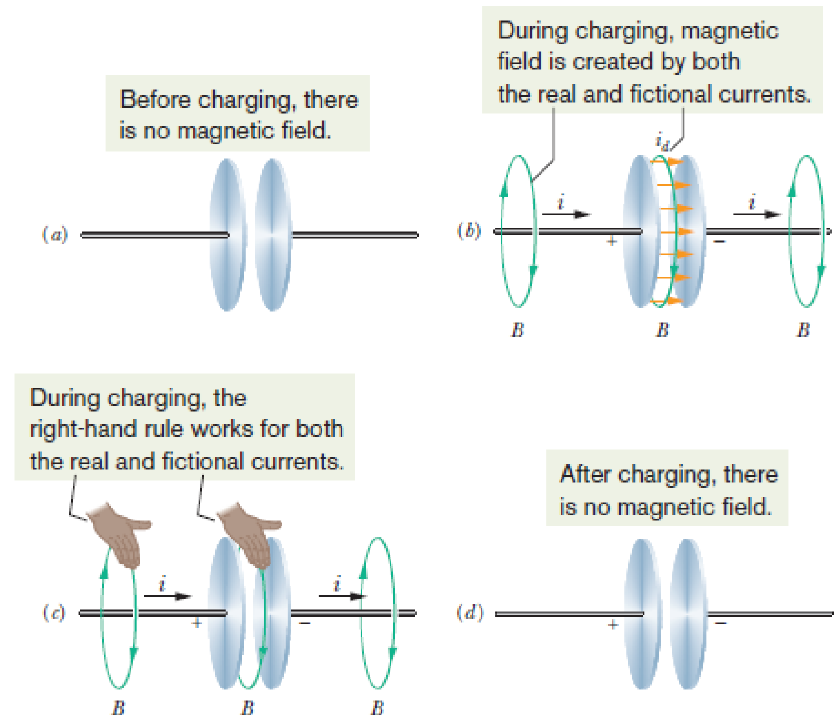
(b)

1st term: „Displacement current“

$$\boxed{i_d = \varepsilon_0 \frac{d\Phi_E}{dt}} \qquad \text{(displacement current)}.$$

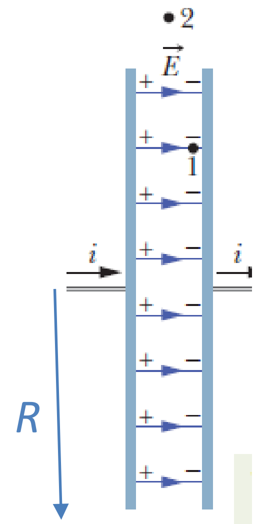
$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}.$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc} \qquad \text{(Ampere–Maxwell law)},$$



A changing electric flux induces an magnetic field

A parallel-plate capacitor with circular plates of radius R is being charged as in Fig. 32-5*a*.



(a) Derive an expression for the magnetic field at radius r for the case $r \le R$.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

i=0

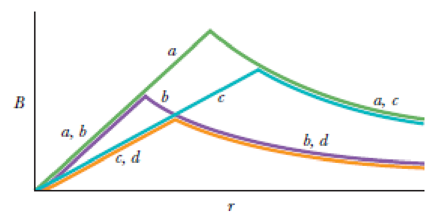
$$\oint \vec{B} \cdot d\vec{s} = \oint B\, ds \cos 0° = \oint B\, ds. = B\,(2\pi r)$$

$$(B)(2\pi r) = \mu_0 \varepsilon_0 \frac{d(EA)}{dt}.$$

$A = \pi r^2$

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt}.$$

This equation tells us that, inside the capacitor, B increases linearly with increased radial distance r, from 0 at the central axis to a maximum value at plate radius R.



(b) Evaluate the field magnitude B for $r = R/5 = 11.0$ mm and $dE/dt = 1.50 \times 10^{12}$ V/m · s.

Calculation: From the answer to (a), we have

$$\begin{aligned} B &= \frac{1}{2} \mu_0 \varepsilon_0 r \frac{dE}{dt} \\ &= \tfrac{1}{2}(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) \\ &\quad \times (11.0 \times 10^{-3}\ \text{m})(1.50 \times 10^{12}\ \text{V/m}\cdot\text{s}) \\ &= 9.18 \times 10^{-8}\ \text{T}. \end{aligned}$$

(Answer)

(c) Derive an expression for the induced magnetic field for the case $r \ge R$.

$A = \pi R^2$

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt}.$$

Outside B decreases with increased radial distance r.

Maxwell's Equations[a]

Integral form

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$	Relates induced magnetic field to changing electric flux and to current

Differential form

$$div\,\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$div\,\vec{B} = 0$$

$$rot\,\vec{E} = -\frac{d\,\vec{B}}{dt}$$

$$rot\,\vec{B} = \mu_0\,\vec{j} + \mu_0\varepsilon_0\frac{d\,\vec{E}}{dt}$$

Material equations:

$D = \varepsilon\,\varepsilon_0\,E$

$B = \mu\,\mu_0\,H$

$j = \sigma\,E$

Usage of Maxwell´s equations to derive the wave equation

Per hand

Ampere-Maxwell law $rot\,\vec{B} = \mu_0\varepsilon_0 \frac{d\,\vec{E}}{dt}$ *j=0*

2nd time derivative $\frac{d}{dt} rot\,\vec{B} = \frac{d}{dt}\mu_0\varepsilon_0 \frac{d\,\vec{E}}{dt}$ $rot\frac{d\,\vec{B}}{dt} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$

Substitute dB/dt $rot\,\vec{E} = -\frac{d\,\vec{B}}{dt}$ $rot\frac{d\,\vec{B}}{dt} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$ → $-\,rot\,rot\,\vec{E} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$

Using relation (math) $rot\,rot\,\vec{E} = -\Delta\vec{E} + grad\,div\,\vec{E}$

And consider: No charges $div\,\vec{E} = 0$ *Gauss law for electrical charges, but* $\rho = 0$

Provides 3D wave equation $\Delta\vec{E} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$ $\Delta E = (\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2})\vec{E}$

For 1D $\frac{d^2\vec{E}}{dx^2} = \mu_0\varepsilon_0 \frac{d^2\vec{E}}{dt^2}$

Or $\frac{d^2\vec{E}}{dx^2} = \frac{1}{c^2}\frac{d^2\vec{E}}{dt^2}$ *Considering* $c = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$

Analogy to Wave equation in Mechanics

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2}\frac{d^2 y}{dt^2}$$

$$\frac{d^2 E}{dx^2} = \varepsilon_0 \mu_0 \frac{d^2 E}{dt^2}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Ansatz : $y = y_m \sin(\omega t - kx)$

$E = E_0 \exp(i(\omega t - kx)$

$dy/dt = \omega\ y_m \sin(\omega t - kx)$
$d^2y/dt^2 = -\omega^2\ y_m \sin(\omega t - kx)$
$dy/dx = k\ y_m \sin(\omega t - kx)$
$d^2y/dx^2 = -k^2\ y_m \sin(\omega t - kx)$

$\mu_0\varepsilon_0 = 1/c^2$
$= 4\pi\ 10^{-7}$ N/A²*8.86 10^{-12} As/Vm
= 111.3 10^{-19}[Nm/mA² *As/Vm
→ Ws/mA²*As/Vm
→ VAs/mA² * As/Vm →s²/m²]
$c = 1/(111.3\ 10^{-19})^{1/2}$ m/s

$-\omega^2\ y_m \sin(\omega t - kx) = 1/v^2\ (-k^2\ y_m \sin(\omega t - kx))$

$-\omega^2 = 1/v^2\ (-k^2)$

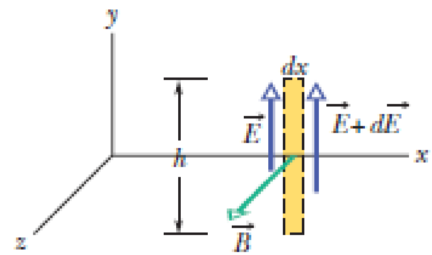
$v = \lambda f$

$c = \lambda f$

c = 299 792 458 m/s
Speed of light

Wave properties

Faraday´s law $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt},$$



Define rectangle $$\oint \vec{E} \cdot d\vec{s} = (E + dE)h - Eh = h\,dE.$$

Flux Φ_B through rectangle

$$\Phi_B = (B)(h\,dx), \qquad \frac{d\Phi_B}{dt} = h\,dx\,\frac{dB}{dt}.$$

$$h\,dE = -h\,dx\,\frac{dB}{dt} \qquad \frac{dE}{dx} = -\frac{dB}{dt}.$$

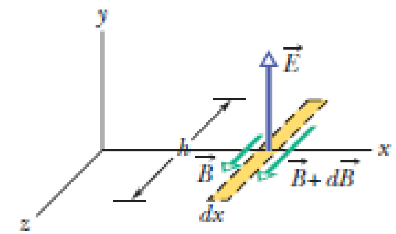
The oscillating magnetic field induces an oscillating and perpendicular electric field.

$$E = E_m \sin(kx - \omega t), \qquad \frac{\partial E}{\partial x} = kE_m \cos(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t), \qquad \frac{\partial B}{\partial t} = -\omega B_m \cos(kx - \omega t).$$

$$kE_m \cos(kx - \omega t) = \omega B_m \cos(kx - \omega t). \qquad \frac{E_m}{B_m} = \frac{\omega}{k} = c$$

Alternative approach



Maxwell´s law
$$\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt},$$

$$\oint \vec{B}\cdot d\vec{s} = -(B + dB)h + Bh = -h\,dB.$$

$$\Phi_E = (E)(h\,dx),$$

$$\frac{d\Phi_E}{dt} = h\,dx\,\frac{dE}{dt}.$$

$$-\frac{\partial B}{\partial x} = \mu_0\varepsilon_0 \frac{\partial E}{\partial t}.$$

The oscillating electric field induces an oscillating and perpendicular magnetic field.

$$-kB_m \cos(kx - \omega t) = -\mu_0\varepsilon_0\omega E_m \cos(kx - \omega t),$$

$$\frac{E_m}{B_m} = \frac{1}{\mu_0\varepsilon_0(\omega/k)} = \frac{1}{\mu_0\varepsilon_0 c}.$$

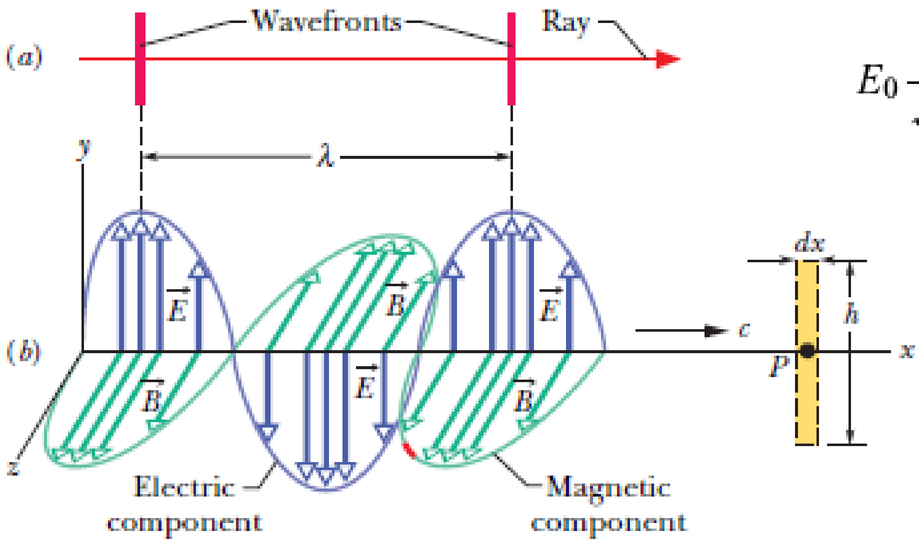
$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Wave propagation in E and B

$$\frac{d^2E}{dy^2} = \frac{1}{c^2}\frac{d^2E}{dt^2}$$

$$\frac{d^2B}{dz^2} = \frac{1}{c^2}\frac{d^2B}{dt^2}$$

Wave propagates along x- direction



Amplitudes are:

$$E_0 \frac{1}{\sqrt{\mu\mu_0\varepsilon\varepsilon_0}} B_0 = \sqrt{\frac{\mu\mu_0}{\varepsilon\varepsilon_0}} H_0 = \frac{1}{\varepsilon\varepsilon_0} D_0$$

Amplitude ratio

$$\frac{E_0}{H_0} = Z_W = \sqrt{\frac{\mu\mu_0}{\varepsilon\varepsilon_0}}$$

Wave resistance Z_W

$$Z_W = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376,7\Omega$$

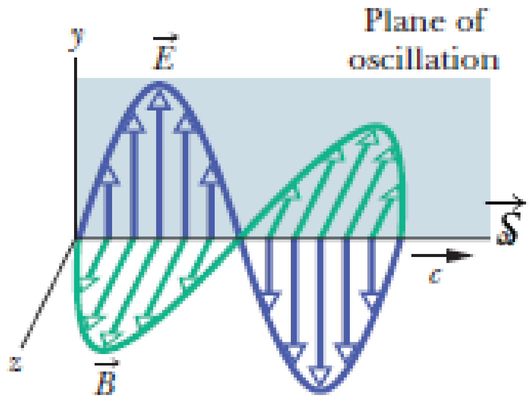
For propagation outside vacuum

$$c = \frac{1}{\sqrt{\mu\mu_0\varepsilon\varepsilon_0}}$$

Energy transport

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \, x \, \vec{B}$$

Right hand rule



$$S = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{inst}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{inst}}.$$

E and B are perpendicular to each other $S = \frac{1}{\mu_0} EB,$

Since B=E/c

$$S = \frac{1}{c\mu_0} E^2 \quad \text{(instantaneous energy flow rate).}$$

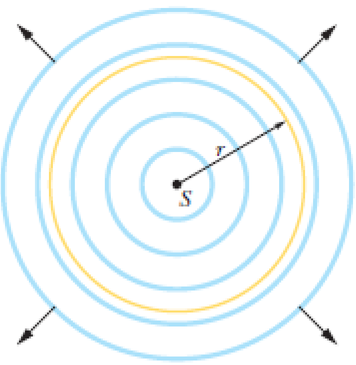
intensity

$$I = S_{\text{avg}} = \frac{1}{c\mu_0} [E^2]_{\text{avg}} = \frac{1}{c\mu_0} [E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}.$$

Root mean square value of E

$$E_{\text{rms}} = \frac{E_m}{\sqrt{2}}.$$

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2.$$



Dependence on distance

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2},$$

Light wave: rms values of the electric and magnetic fields

When you look at the North Star (Polaris), you intercept light from a star at a distance of 431 ly and emitting energy at a rate of 2.2×10^3 times that of our Sun ($P_{sun} = 3.90 \times 10^{26}$ W). Neglecting any atmospheric absorption, find the rms values of the electric and magnetic fields when the starlight reaches you.

KEY IDEAS

1. The rms value E_{rms} of the electric field in light is related to the intensity I of the light via Eq. 33-26 ($I = E^2_{rms}/c\mu_0$).
2. Because the source is so far away and emits light with equal intensity in all directions, the intensity I at any distance r from the source is related to the source's power P_s via Eq. 33-27 ($I = P_s/4\pi r^2$).
3. The magnitudes of the electric field and magnetic field of an electromagnetic wave at any instant and at any point in the wave are related by the speed of light c according to Eq. 33-5 ($E/B = c$). Thus, the rms values of those fields are also related by Eq. 33-5.

Electric field: Putting the first two ideas together gives us

$$I = \frac{P_s}{4\pi r^2} = \frac{E^2_{rms}}{c\mu_0}$$

and

$$E_{rms} = \sqrt{\frac{P_s c\mu_0}{4\pi r^2}}.$$

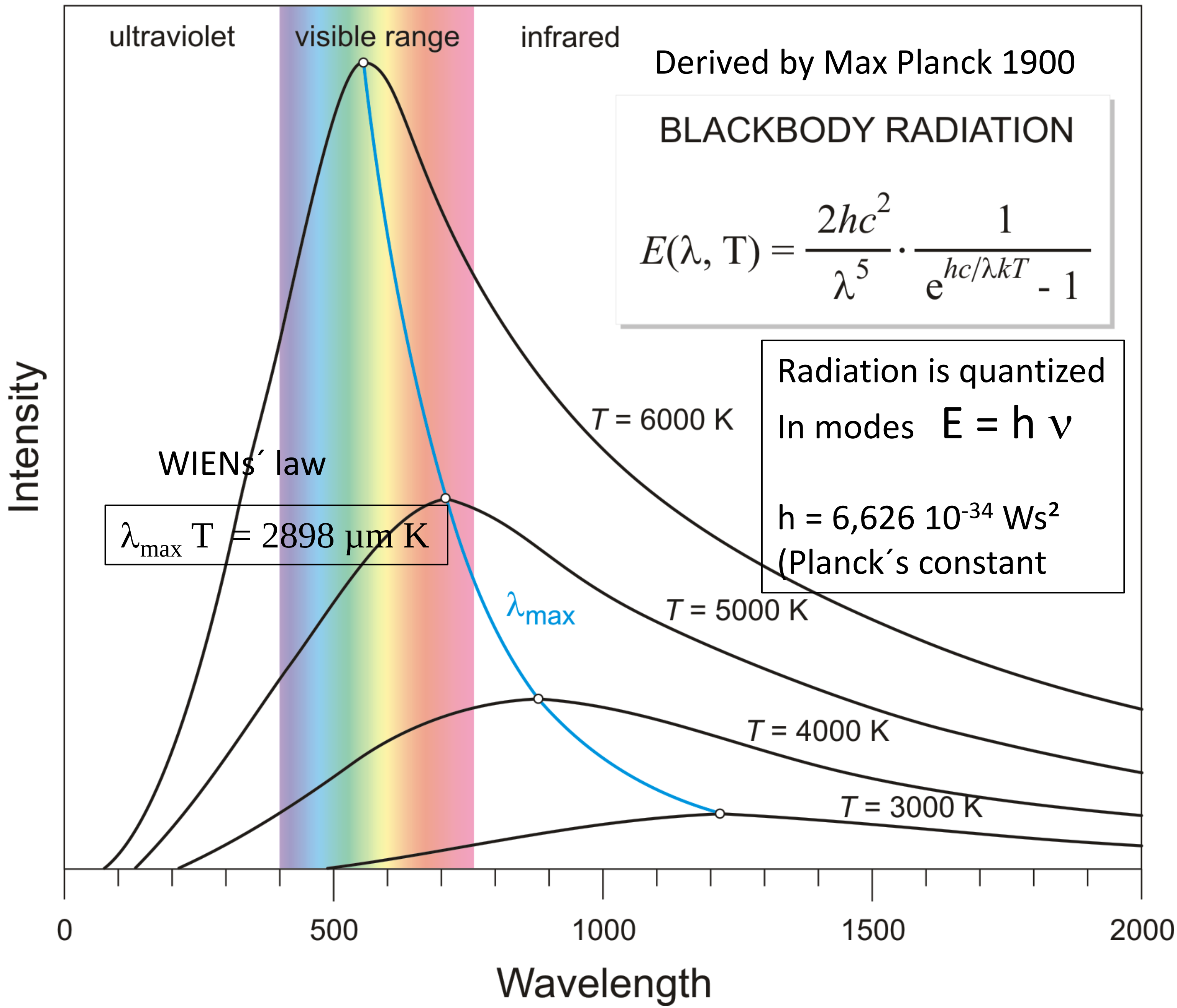
Substituting $P_s = (2.2 \times 10^3)(3.90 \times 10^{26}\ \text{W})$, $r = 431$ ly $= 4.08 \times 10^{18}$ m, and values for the constants, we find

$$E_{rms} = 1.24 \times 10^{-3}\ \text{V/m} \approx 1.2\ \text{mV/m}. \qquad \text{(Answer)}$$

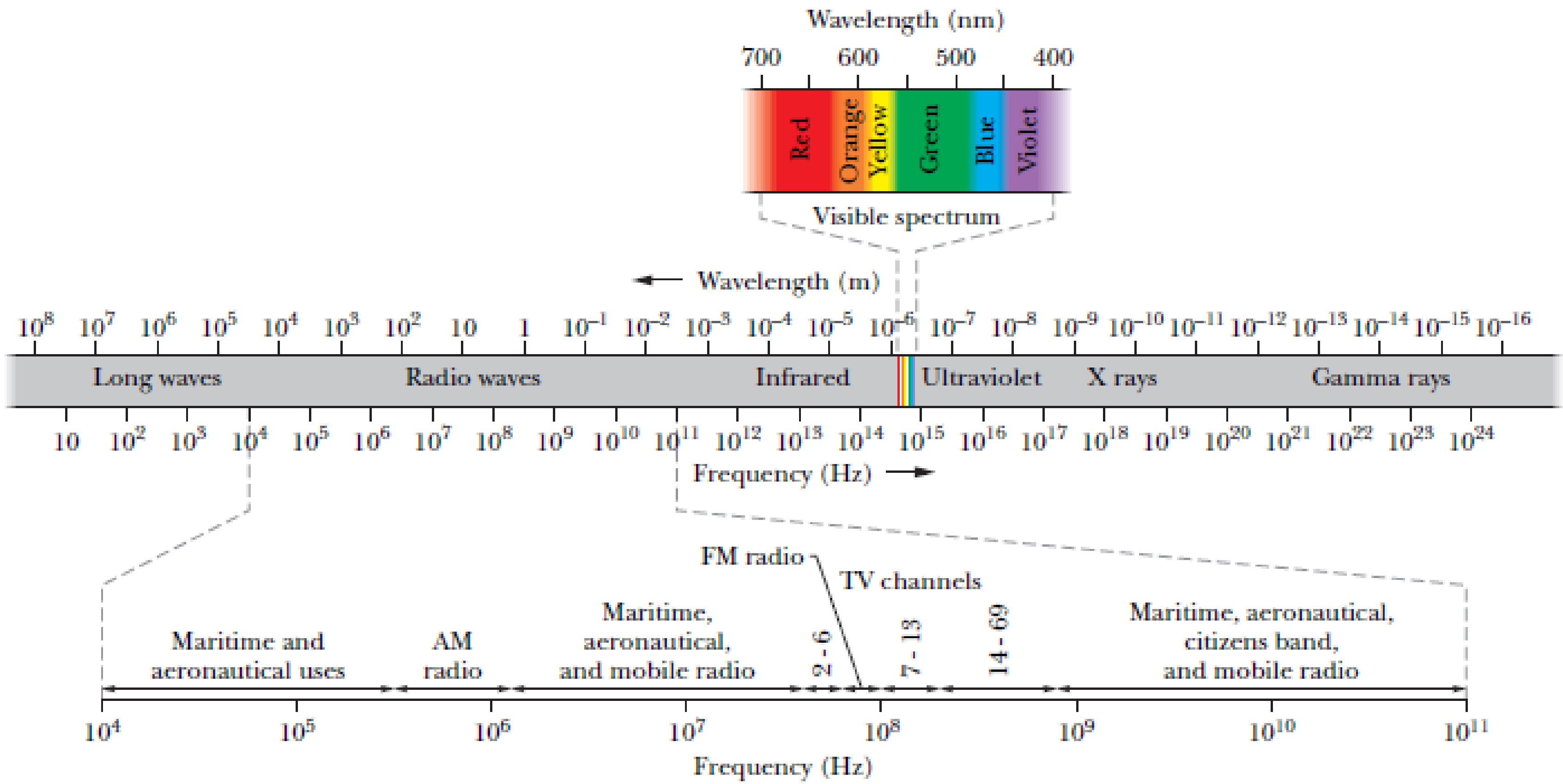
Magnetic field: From Eq. 33-5, we write

$$B_{rms} = \frac{E_{rms}}{c} = \frac{1.24 \times 10^{-3}\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = 4.1 \times 10^{-12}\ \text{T} = 4.1\ \text{pT}.$$

Cannot compare the fields: Note that E_{rms} (= 1.2 mV/m) is small as judged by ordinary laboratory standards, but B_{rms} (= 4.1 pT) is quite small. This difference helps to explain why most instruments used for the detection and measurement of electromagnetic waves are designed to respond to the electric component of the wave. It is wrong, however, to say that the electric component of an electromagnetic wave is "stronger" than the magnetic component. You cannot compare quantities that are measured in different units. However, these electric and magnetic components are on an equal basis because their average energies, which *can* be compared, are equal.


ultraviolet
visible range
infrared
Derived by Max Planck 1900
BLACKBODY RADIATION
$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$
Radiation is quantized
In modes E = h ν
h = 6,626 10^-34 Ws²
(Planck´s constant
T = 6000 K
WIENs´ law
λ_{max} T = 2898 µm K
λ_{max}
T = 5000 K
T = 4000 K
T = 3000 K
Intensity
0
500
1000
1500
2000
Wavelength

Electromagnetic spectrum



Electromagnetic radiation

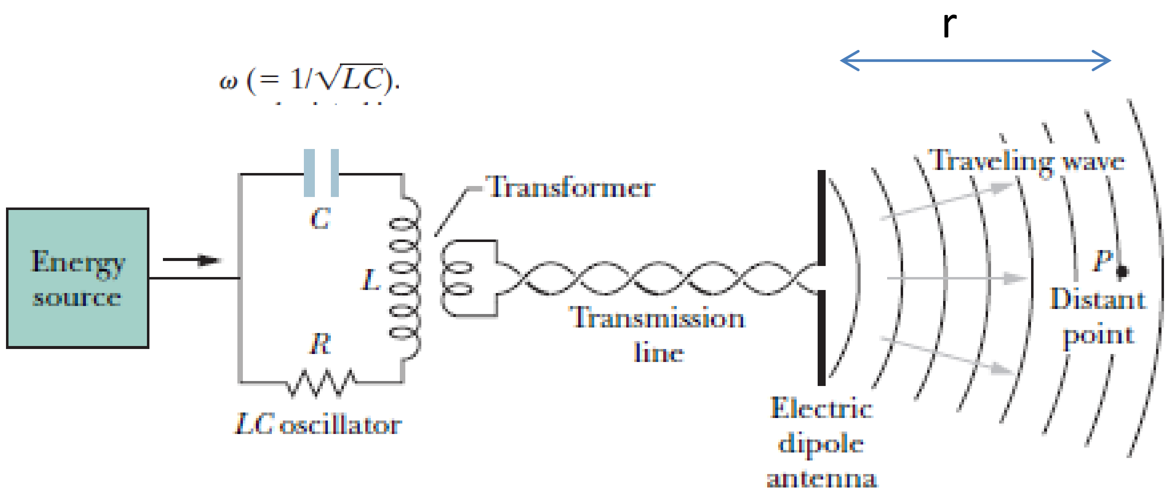
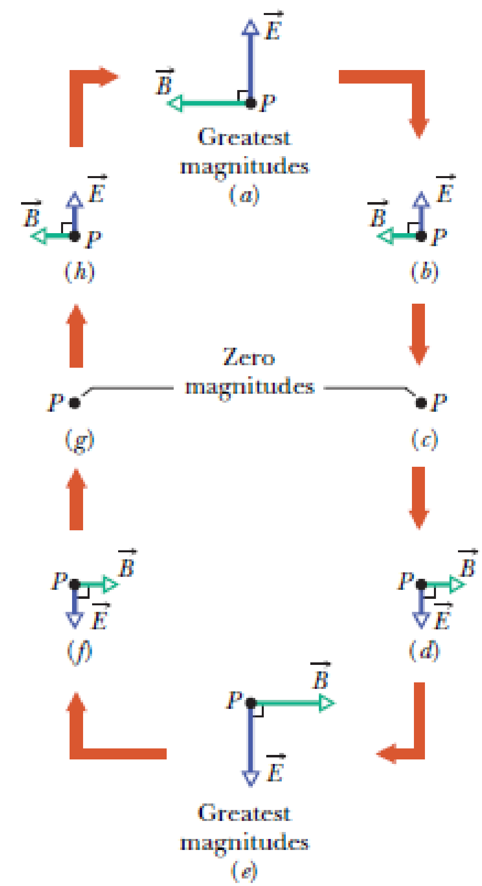


Fig. 33-3 An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an *LC* oscillator produces a sinusoidal current in the antenna, which generates the wave. *P* is a distant point at which a detector can monitor the wave traveling past it.

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2},$$

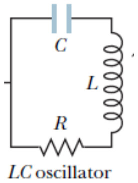
1. The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are always perpendicular to the direction in which the wave is traveling. Thus, the wave is a *transverse wave*, as discussed in Chapter 16.
2. The electric field is always perpendicular to the magnetic field.
3. The cross product $\vec{E} \times \vec{B}$ always gives the direction in which the wave travels.
4. The fields always vary sinusoidally, just like the transverse waves discussed in Chapter 16. Moreover, the fields vary with the same frequency and *in phase* (in step) with each other.



Variation of E and B at point P

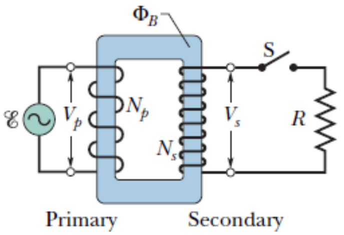
Elements of emission antenna

LC oscillator



$$\omega\ (= 1/\sqrt{LC}).$$

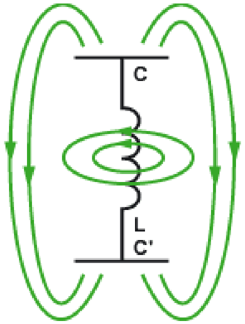
transformator



$$V_s = V_p \frac{N_s}{N_p} \quad \text{(transformation of voltage).}$$

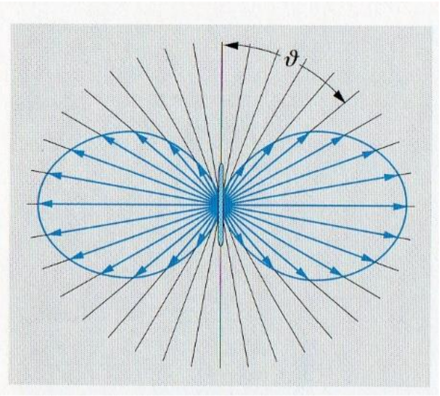
$$I_s = I_p \frac{N_p}{N_s} \quad \text{(transformation of currents).}$$

Linear oscillator = antenna

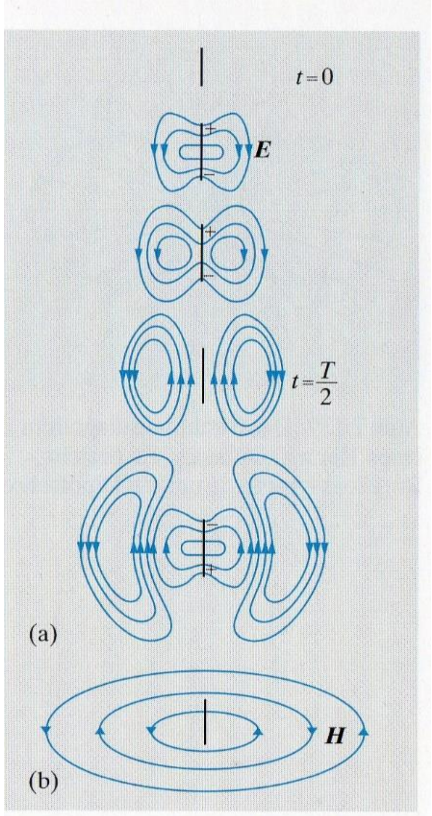


Elektronik-Kompendium.de

$$\upsilon = \frac{c}{\lambda} = \frac{c}{2l}$$



Linear oscillator



Hertz oscillator :
Dipol radiation

$$S = \frac{p^2\omega}{32\pi^2\varepsilon_0(\frac{\lambda}{2\pi})^5} sin^2\theta$$

Gerthsen: Physik, 22.Ed, pages 431ff

Wave emission always maximum perpendicular to direction of electronic oscillation with power P

$$P = \frac{p^2\omega^4}{6\pi\varepsilon_0 c^3}$$

Time dependence of emission

Wave propagation in media with conductivity σ

Per hand

Here phase velocity v = c/√ε

In conductive medium the current j=σE

$$rot\,\vec{B} = \mu_0\varepsilon\varepsilon_0\frac{d\vec{E}}{dt} + \mu_0\sigma\vec{E}$$

$$\Delta\vec{E} = \mu_0\varepsilon\varepsilon_0\frac{d^2\vec{E}}{dt^2} + \mu_0\sigma\frac{d\vec{E}}{dt}$$

Wave equation in media

$$\vec{E} = E_0 e^{i(\omega t - kx)} + e^{-\delta x}$$

$$(\delta + ik)^2 = \mu_0\sigma i\omega - \mu_0\varepsilon\varepsilon_0\omega^2$$

$$k^2 - \delta^2 = \mu_0\varepsilon\varepsilon_0\omega^2$$

$$2\delta k = \mu_0\sigma\omega$$

$\omega >> \mu_0\sigma c^2$ $\quad$ $\delta << k$ $\quad$ Penetration depth > wave length

$$I = |E^2| = I_0 e^{-2\delta x} = I_0 e^{-\alpha x}$$

$$\delta = \frac{1}{2}\mu_0\sigma c \qquad c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\varepsilon\varepsilon_0}}$$

$\omega << \mu_0\sigma c^2$ $\quad$ $k \approx \delta = \sqrt{\frac{1}{2}\mu_0\sigma\omega}$ $\quad$ Penetration depth < wave length examples

seewater : $\sigma \approx 12\ \Omega^{-1}m^{-1}$ $\varepsilon \approx 80$ → $c \approx 3\ 10^7$ m/s

$2\delta \approx 4\ \pi\ 10^{-7}$ Tm/A 12 A/V m $3\ 10^7$ m/s = 452 m^{-1}

Decay depth 1/e → $1/2\delta \approx 0.002$ m = 2 mm

= 4000 λ for visible light (0.5 µm) or
< 1 λ for radio waves

μ_0:
Tm/A
= Ws/mA²
=VAs/mA²

Copper: $\sigma \approx 6\ 10^7\ \Omega^{-1}m^{-1}$ $\varepsilon \approx 80$ → $c \approx 3\ 10^7$ m/s
$2\delta \approx 4\ \pi\ 10^{-7}$ Tm/A $6\ 10^7$ A/V m $3\ 10^7$ m/s = $2.25\ 10^9\ m^{-1}$
$1/2\delta = 0.4\ 10^{-9}$ m = 0.4 nm (2 atomic layers) → see later

Quartzglass: $\sigma \approx 10^{-15}\ \Omega^{-1}m^{-1}$ $\varepsilon \approx 2$ → $c \approx 2\ 10^9$ m/s
$2\delta \approx 4\ \pi\ 10^{-7}$ Tm/A 10^{-15} A/V m $2\ 10^9$ m/s = $25\ 10^{-13}\ m^{-1}$
$1/2\delta = 4\ 10^{11}$ m (limited by absorption)

Radiation due to accelerated charges

Dipole moment of antenna of lenght l

$$p = el$$

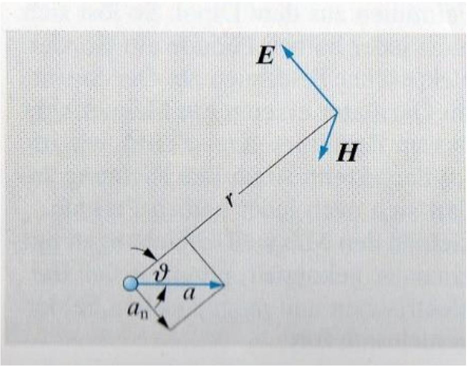
$$\frac{d}{dt}p = ev = e\omega l$$

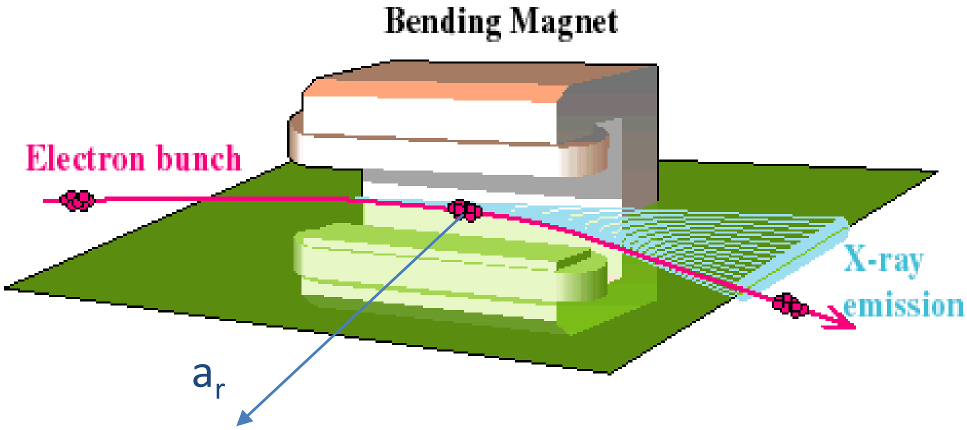
$$\frac{d^2}{dt^2}p = ea = e\omega^2 l$$

→Each charge accelerated by *a* radiates with energy density *S* and power *P*

$$S = \frac{1}{16\pi^2}\frac{e^2a^2}{\varepsilon_0 c^3}\frac{sin^2\theta}{r^2}$$

$$P = \frac{1}{6\pi}\frac{e^2a^2}{\varepsilon_0 c^3}$$





Name : Matr nr.

Physics 1 for Nanoscience & Nanotechnology

Level of knowledge 5 8.1.19

1. Give the equation of Ampere´s law, what is its physical meaning ?

2. Give the equation for Faraday´s law ? What is the physical meaning ?

3. Apply Gauss´law for the case of an enclosed electrical charge.

4. Show the analogy of Gauss´law for magnetic charges. Apply Gauss´law for the case of an enclosed magnetic dipole.