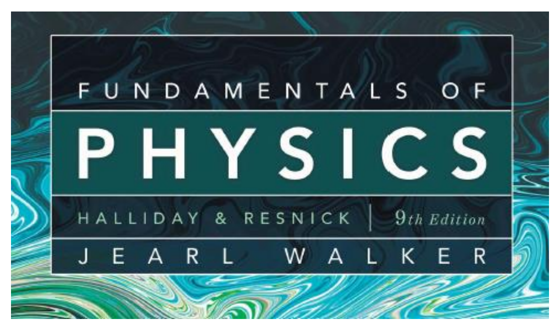
Physics 1

UNIVERSITÄT SIEGEN
FESTKÖRPERPHYSIK

Lecture 7: Inductance and LC oscillators

Prof. Dr. U. Pietsch



Faraday´s law of induction

An **EMF** is induced in a loop when the number of magnetic field lines that passing the loop is changing

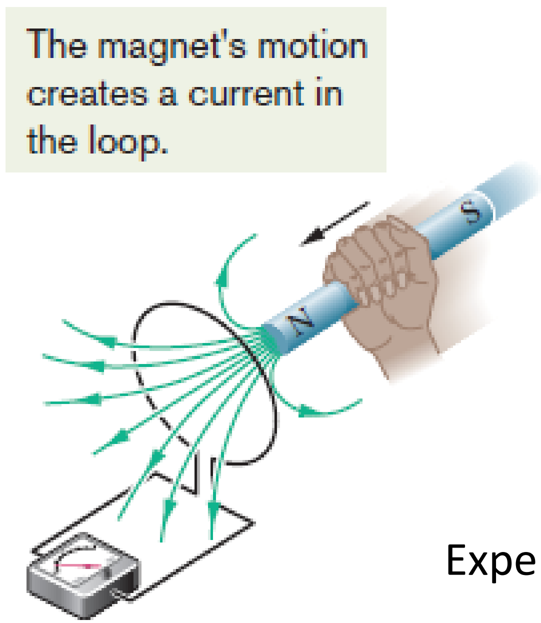
$$\Phi_B = \int \vec{B} \cdot d\vec{A} \qquad \text{(magnetic flux through area } A\text{).}$$

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

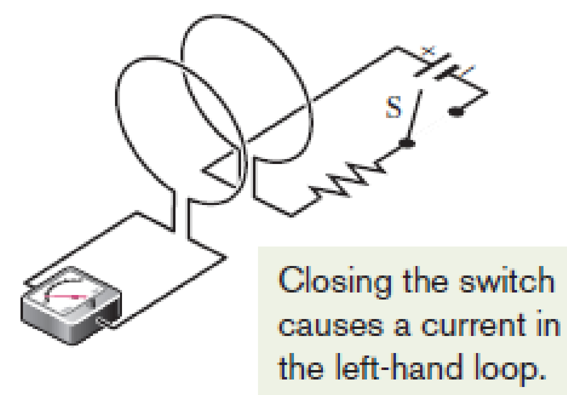
$$\mathscr{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law),}$$

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} \qquad \text{(coil of } N \text{ turns).}$$

The magitude of EMF induced in a conducting loop is equal to the rate at which the magnetic flux through the loop is changing



Experiment 1



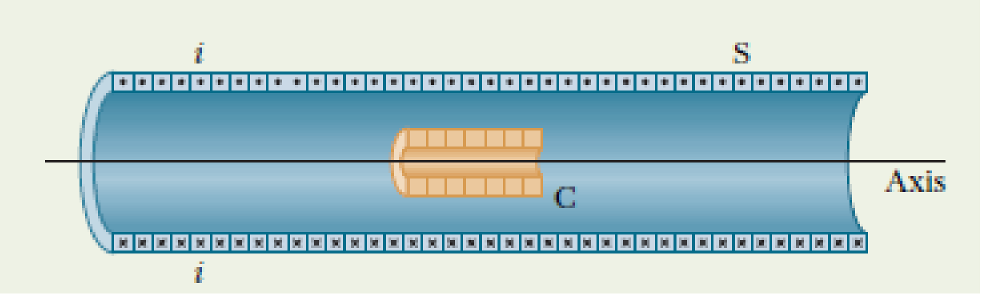
Experiment 2

Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.
2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).
3. Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil (for example, by rotating the coil so that field $\vec{B}$ is first perpendicular to the plane of the coil and then is along that plane).

example

The long solenoid S shown (in cross section) in Fig. 30-3 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?



1. Because it is located in the interior of the solenoid, coil C lies within the magnetic field produced by current i in the solenoid; thus, there is a magnetic flux Φ_B through coil C.
2. Because current i decreases, flux Φ_B also decreases.
3. As Φ_B decreases, emf $\mathscr{E}$ is induced in coil C.
4. The flux through each turn of coil C depends on the area A and orientation of that turn in the solenoid's magnetic field $\vec{B}$. Because $\vec{B}$ is uniform and directed perpendicular to area A, the flux is given by Eq. 30-2 ($\Phi_B = BA$).
5. The magnitude B of the magnetic field in the interior of a solenoid depends on the solenoid's current i and its number n of turns per unit length, according to Eq. 29-23 ($B = \mu_0 in$).

S: n =220 turns/cm

i= 1.5 A, reduced to i=0 within 25ms

C: N= 130 turns d=2.1 cm

What is the emf induced in coil C

example

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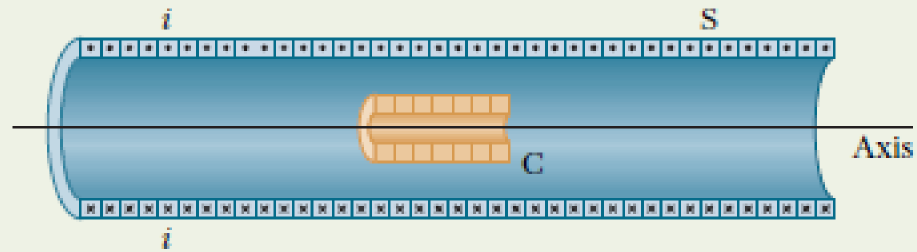

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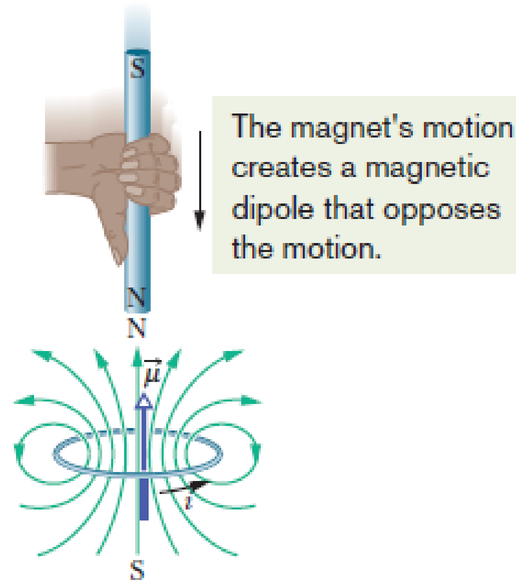
$$\begin{aligned}\Phi_{B,i} &= BA = (\mu_0 in)A \\ &= (4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(1.5\,\mathrm{A})(22\,000\ \mathrm{turns/m}) \\ &\quad \times (3.464 \times 10^{-4}\,\mathrm{m^2}) \\ &= 1.44 \times 10^{-5}\,\mathrm{Wb}.\end{aligned}$$

$$\begin{aligned}\frac{d\Phi_B}{dt} &= \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} \\ &= \frac{(0 - 1.44 \times 10^{-5}\,\mathrm{Wb})}{25 \times 10^{-3}\,\mathrm{s}} \\ &= -5.76 \times 10^{-4}\,\mathrm{Wb/s} = -5.76 \times 10^{-4}\,\mathrm{V}.\end{aligned}$$

$$\begin{aligned}\mathscr{E} &= N\frac{d\Phi_B}{dt} = (130\ \mathrm{turns})(5.76 \times 10^{-4}\,\mathrm{V}) \\ &= 7.5 \times 10^{-2}\,\mathrm{V} = 75\,\mathrm{mV}. \qquad \text{(Answer)}\end{aligned}$$

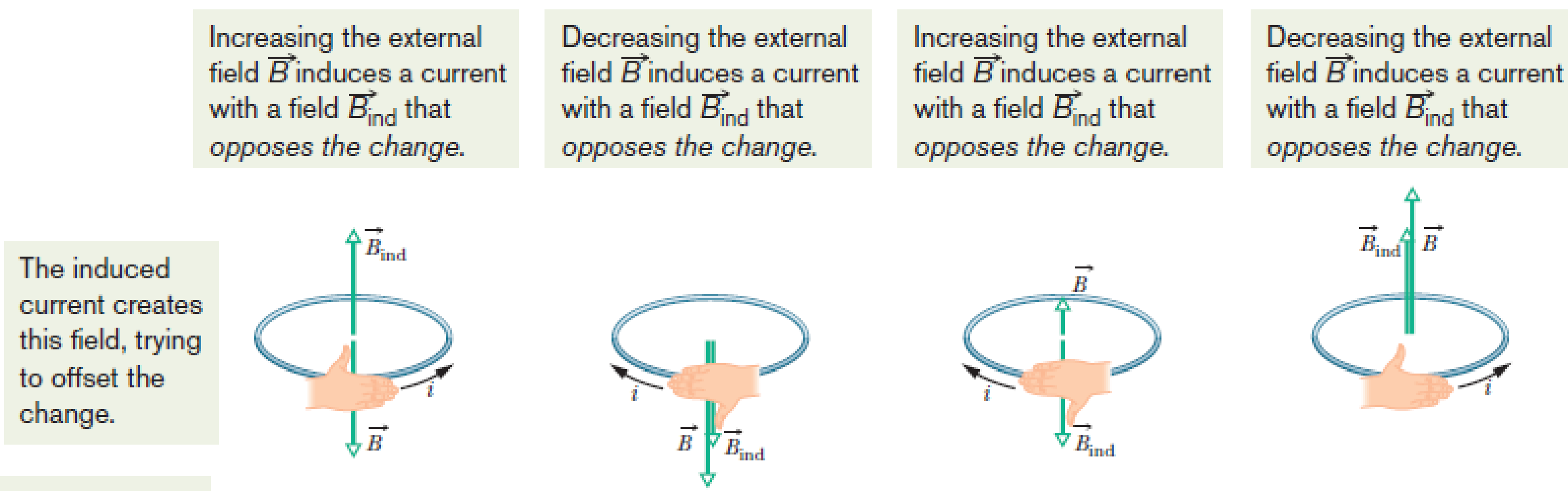
Lenz´s Law

1. ***Opposition to Pole Movement.***

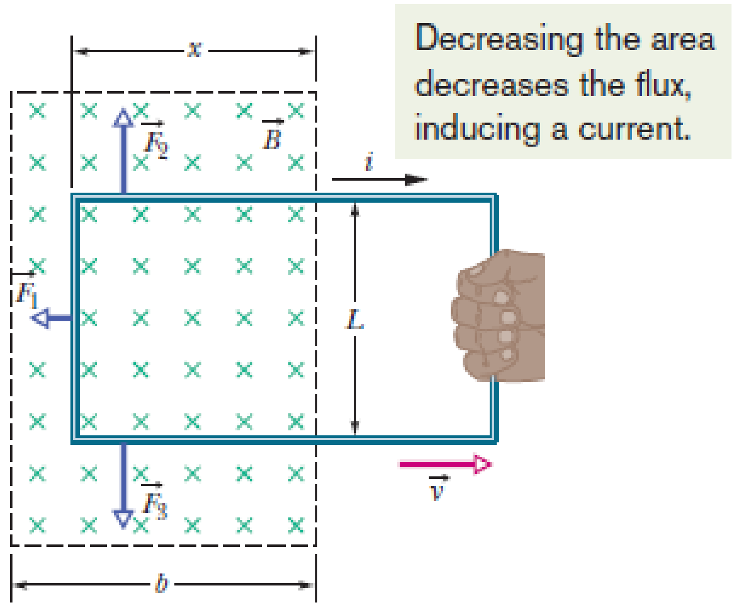


2. ***Opposition to Flux Change.***

An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.



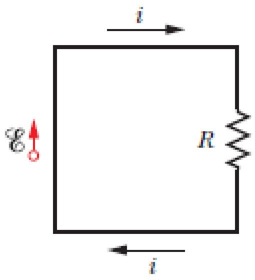
Induction and energy transfer



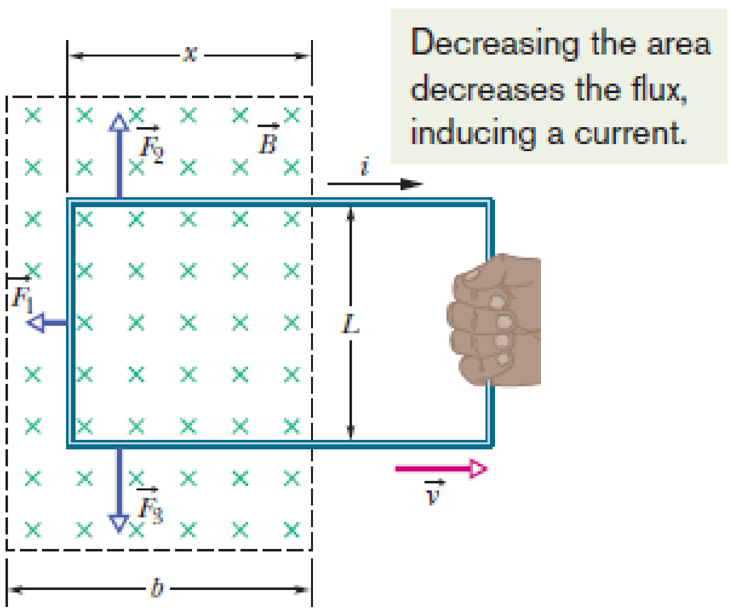
Electrical power

$$P = Fv,$$

i=ℰ/R



Induction and energy transfer



Electrical power

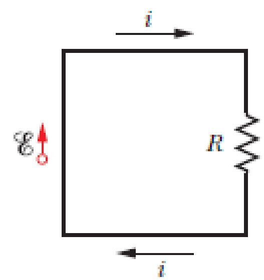
$$P = Fv,$$

$$\vec{F}_d = i\vec{L} \times \vec{B}.$$

$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL\frac{dx}{dt} = BLv,$$

i=ℰ/R

$$i = \frac{BLv}{R}.$$

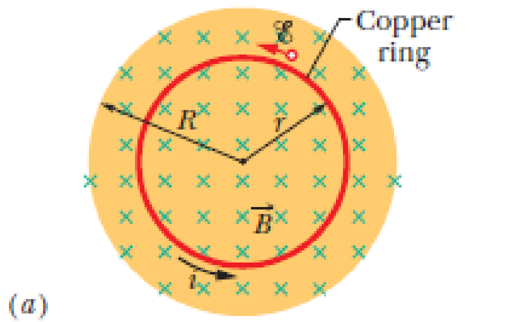


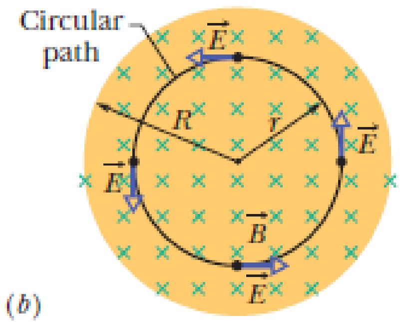
$$F = \frac{B^2L^2v}{R}.$$

$$P = Fv = \frac{B^2L^2v^2}{R} \quad \text{(rate of doing work).}$$

Induced electric field

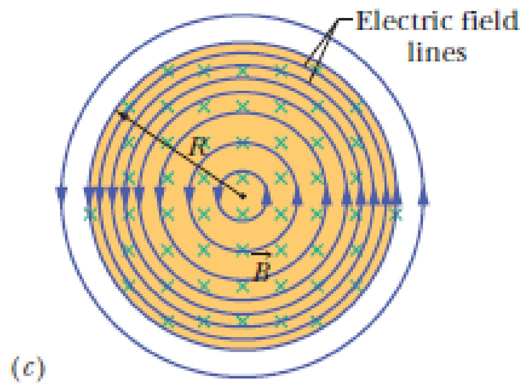
A changing magnetic field produces an electric field.

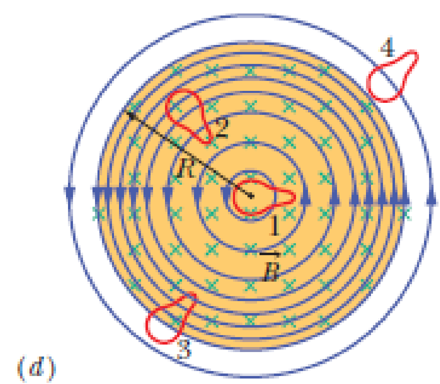




If the magnetic field increase, a constant current is induced

An electric field appears at Radius r





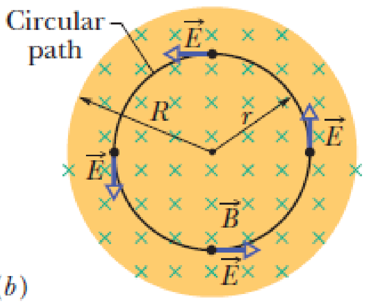
Electric field forms a ring system

Equal emf are induced in loops 1 and 2 but less in 3 and 4

Faraday´s Law improved

$W_{appl} = q\ \Delta V = q\ \mathscr{E}$

$$W = \int \vec{F} \cdot d\vec{s} = (q_0 E)(2\pi r),$$



(b)

It yields $$\mathscr{E} = 2\pi r E.$$

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}.$$

$(\mathscr{E} = -d\Phi_B/dt),$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law).}$$

field lines of induced electric fields form closed loops,

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

$$\oint \vec{E} \cdot d\vec{s} = 0.$$

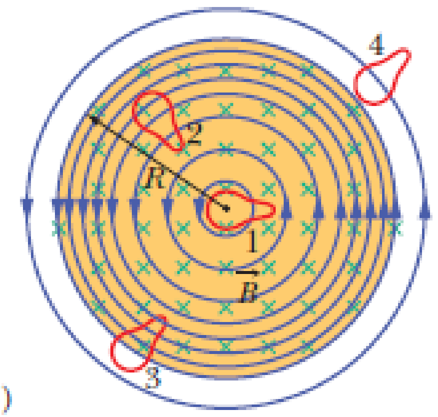
Static charges

Induced field

Example

R=8.5cm
dB/dt= 0.13 T/s

A)Find E at r=5.2 cm → inside the field

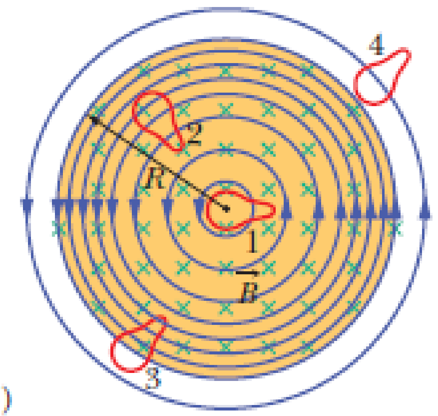


(d)

Find E at r=12.5.2 cm → outside the field

Example

R=8.5cm
dB/dt= 0.13 T/s



(d)

A)Find E at r=5.2 cm → inside the field

$$\oint \vec{E}\cdot d\vec{s} = \oint E\,ds = E\oint ds = E(2\pi r).$$

$$\Phi_B = BA = B(\pi r^2).$$

$$E(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

$$E = \frac{r}{2}\frac{dB}{dt}.$$

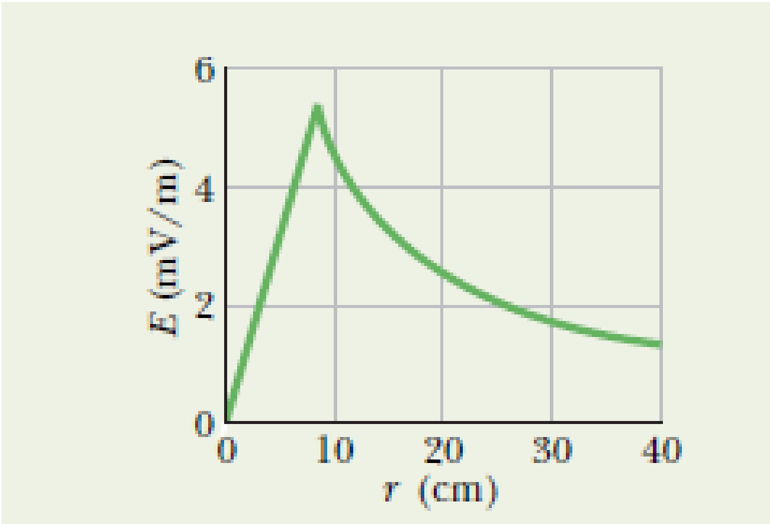
$$E = \frac{(5.2\times 10^{-2}\ \text{m})}{2}(0.13\ \text{T/s}) = 0.0034\ \text{V/m} = 3.4\ \text{mV/m}.$$

Find E at r=12.5.2 cm → outside the field

$$\Phi_B = BA = B(\pi R^2).$$

$$E = \frac{R^2}{2r}\frac{dB}{dt}.$$

$$E = \frac{(8.5\times 10^{-2}\ \text{m})^2}{(2)(12.5\times 10^{-2}\ \text{m})}(0.13\ \text{T/s}) = 3.8\times 10^{-3}\ \text{V/m} = 3.8\ \text{mV/m}.$$



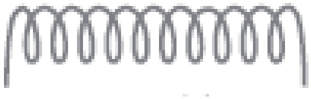
Inductors, Inductance

Changing magnetic flux induces an emf + Electric current produces magnetic field

SO

Changing current in one circuit ought to induce an emf and a cuurent in second nearby circuit and even induce an emf in itself.

Inductors can be used to produce a desired magnetic field.



If we establish a current i in the windings (turns) of the solenoid we are taking as our inductor, the current produces a magnetic flux Φ_B through the central region of the inductor. The **inductance** of the inductor is then

$$L = \frac{N\Phi_B}{i} \quad \text{(inductance defined),}$$

N is the number of turns.

$$L = \frac{N\Phi_B}{i} \quad \text{(inductance defined)},$$

1 henry = 1 H = 1 T·m²/A.

Inductance—like capacitance—depends only on the geometry of the device.

Inductance of a solenoid :

When a changing current passes through a solenoid, a changing magnetic flux is produced inside the coil, and this in turn induces an emf in that same coil. This induced emf opposes the change in the flux.The magnetic flux is proportional to the current by L.

long solenoid of cross-sectional area A.

$$N\Phi_B = (nl)(BA),$$

$N = n\,l$

$$\Phi_B = BA$$

$$B = \mu_0 i n,$$

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i} = \mu_0 n^2 l A.$$

$$\frac{L}{l} = \mu_0 n^2 A \quad \text{(solenoid)}.$$

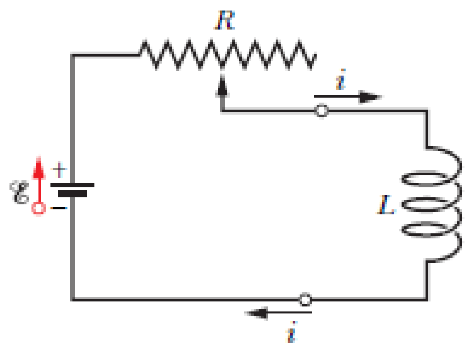
Inductance per unit length: ~n^2

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A} = 4\pi \times 10^{-7}\ \text{H/m}.$$

Self- inductance

If two coils—which we can now call inductors—are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced emf appears in the second coil according to Faraday's law. An induced emf appears in the first coil as well.

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.



If i is changed by varying the position of on a variable resistor, a self induced emf will appear in the coil.

$$L = \frac{N\Phi_B}{i} \qquad \text{(inductance defined)},$$

$$N\Phi_B = Li.$$

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

$$\mathscr{E}_L = -L\frac{di}{dt} \qquad \text{(self-induced emf)}.$$

Magnitude of i has no influence on magnitude of emf, only the rate of change in i.

$$N\Phi_B = Li.$$

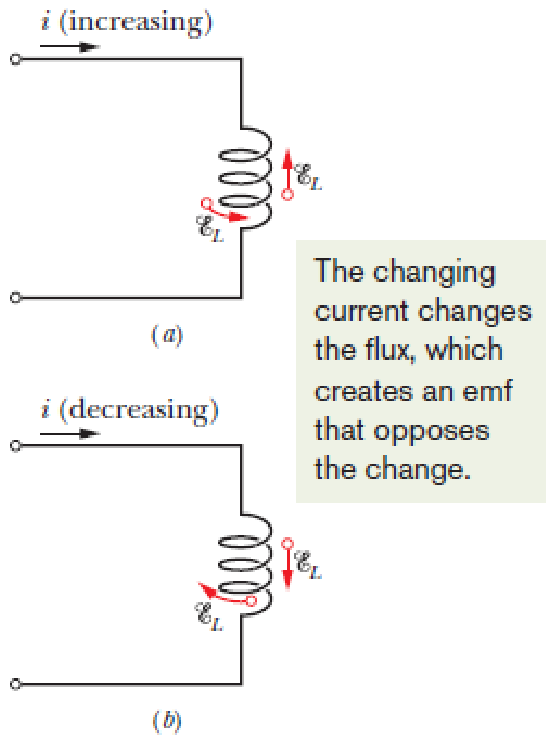
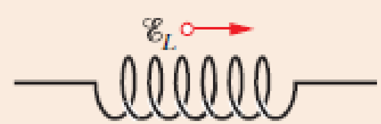


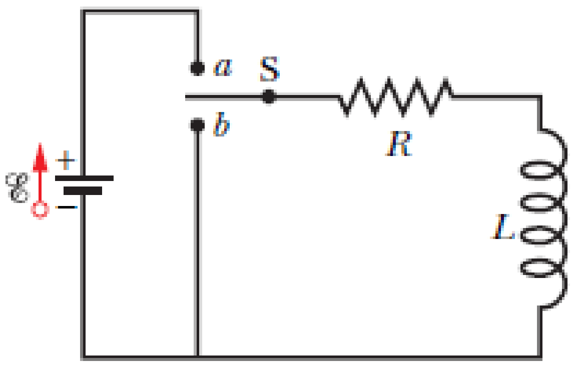
Fig. 30-14 (*a*) The current i is increasing, and the self-induced emf $\mathscr{E}_L$ appears along the coil in a direction such that it opposes the increase. The arrow representing $\mathscr{E}_L$ can be drawn along a turn of the coil or alongside the coil. Both are shown. (*b*) The current i is decreasing, and the self-induced emf appears in a direction such that it opposes the decrease.

CHECKPOINT 5

The figure shows an emf $\mathscr{E}_L$ induced in a coil. Which of the following can describe the current through the coil: (a) constant and rightward, (b) constant and leftward, (c) increasing and rightward, (d) decreasing and rightward, (e) increasing and leftward, (f) decreasing and leftward?



RL Circuits



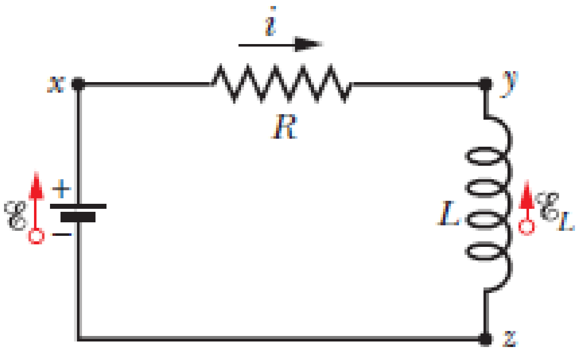
- *When S is closed on a, the current in the resistor starts to rise.*
- *If there is no L, it would rise rapidly to ε/R.*
- *As we have L, ε_L appears; from Lenz's Law, this emf opposes the rise of current.*
- *As long as ε_L is present, the current will be less than ε/R.*

- *As time goes on, the rate at which i increases becomes less rapid.*
- *The magnitude of ε_L becomes smaller.*
- *The current through the resistor approaches ε/R R asymptotically.*

Initially the inductor acts to oppose changes in the current through it. After a long time, it acts like ordinary connecting wire.

RL Circuits

current is increasing:



$$-iR - L\frac{di}{dt} + \mathscr{E} = 0$$

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) \quad \text{(rise of current).}$$

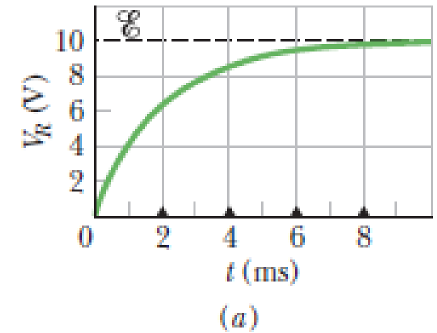
$$\tau_L = \frac{L}{R} \quad \text{(time constant).}$$

$$1\frac{\text{H}}{\Omega} = 1\frac{\text{H}}{\Omega}\left(\frac{1\ \text{V}\cdot\text{s}}{1\ \text{H}\cdot\text{A}}\right)\left(\frac{1\ \Omega\cdot\text{A}}{1\ \text{V}}\right) = 1\ \text{s}.$$

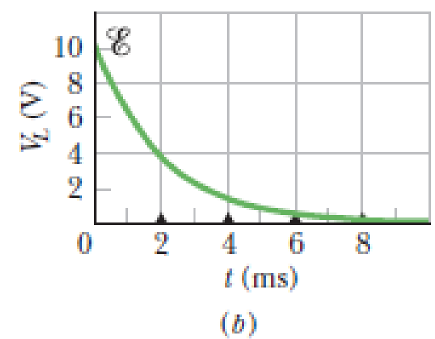
τ_L is the time it takes i to reach 63% of ε/R.

The resistor's potential difference turns on. The inductor's potential difference turns off.

$V_R = iR$



$V_L = L\ di/dt$



- *If the switch is thrown to b, the battery will be removed from the circuit.*
- *The current through th eresistor will decrease.*
- *It must decay to zero over time.*

$$L\frac{di}{dt} + iR = 0.$$

$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L} \qquad \text{(decay of current).}$$

Example

i=0 at t>=0

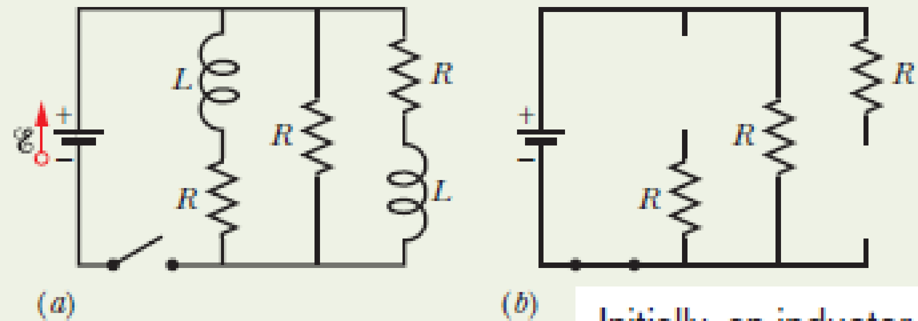
R= 9.0 Ω, L = 2.0mH, $\mathscr{E}$= 18V

(a) What is the current i through the battery just after the switch is closed?

Just after the switch is closed, the inductor acts to oppose a change in the current through it.

$$\mathscr{E} - iR = 0.$$

$$i = \frac{\mathscr{E}}{R} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}.$$

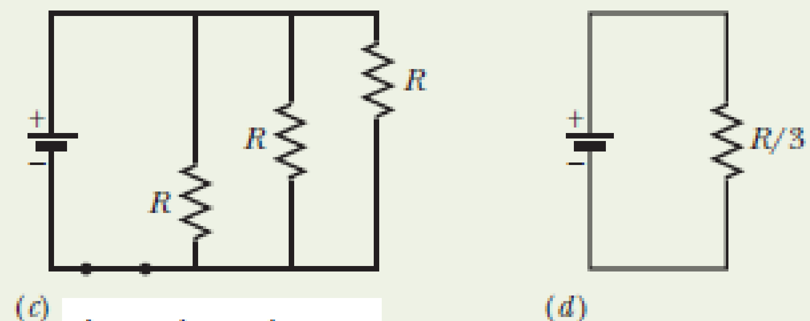


Initially, an inductor acts like broken wire.

(b) What is the current i through the battery long after the switch has been closed?

$$i = \frac{\mathscr{E}}{R_{eq}} = \frac{18\ \text{V}}{3.0\ \Omega} = 6.0\ \text{A}.$$

$$R_{eq} = R/3 = (9.0\ \Omega)/3 = 3.0\ \Omega.$$



Long later, it acts like ordinary wire.

1/R = 3/9.0Ω

RL circuit, current during the transition

A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω. If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value? (This is a *real solenoid* because we are considering its small, but nonzero, internal resistance.)

KEY IDEA

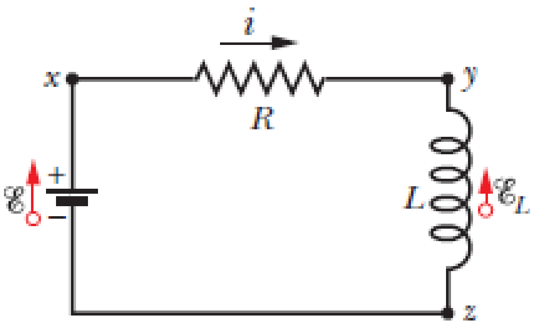
We can mentally separate the solenoid into a resistance and an inductance that are wired in series with a battery, as in Fig. 30-16. Then application of the loop rule leads to Eq. 30-39, which has the solution of Eq. 30-41 for the current i in the circuit.

Calculations: According to that solution, current i increases exponentially from zero to its final equilibrium value of $\mathscr{E}/R$. Let t_0 be the time that current i takes to reach half its equilibrium value. Then Eq. 30-41 gives us

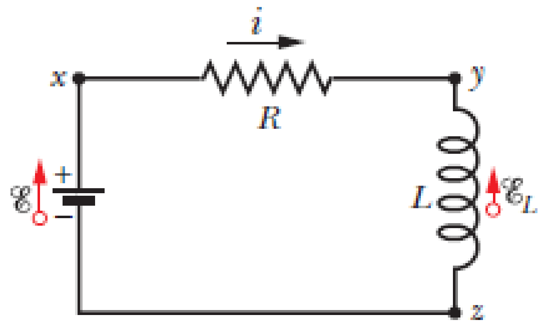
$$\frac{1}{2}\frac{\mathscr{E}}{R} = \frac{\mathscr{E}}{R}(1 - e^{-t_0/\tau_L}).$$

We solve for t_0 by canceling $\mathscr{E}/R$, isolating the exponential, and taking the natural logarithm of each side. We find

$$t_0 = \tau_L \ln 2 = \frac{L}{R}\ln 2 = \frac{53 \times 10^{-3}\ \text{H}}{0.37\ \Omega}\ln 2$$
$$= 0.10\ \text{s}. \qquad \text{(Answer)}$$



Energy stored in a magnetic field

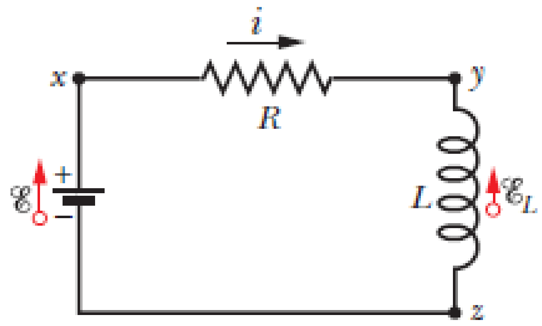


$$\mathscr{E} = L\frac{di}{dt} + iR,$$

$$\mathscr{E}i = Li\frac{di}{dt} + i^2R,$$

1. If a differential amount of charge dq passes through the battery of emf $\mathscr{E}$ in Fig. 30-16 in time dt, the battery does work on it in the amount $\mathscr{E}\,dq$. The rate at which the battery does work is $(\mathscr{E}\,dq)/dt$, or $\mathscr{E}i$. Thus, the left side of Eq. 30-47 represents the rate at which the emf device delivers energy to the rest of the circuit.
2. The rightmost term in Eq. 30-47 represents the rate at which energy appears as thermal energy in the resistor.
3. Energy that is delivered to the circuit but does not appear as thermal energy must, by the conservation-of-energy hypothesis, be stored in the magnetic field of the inductor. Because Eq. 30-47 represents the principle of conservation of energy for RL circuits, the middle term must represent the rate dU_B/dt at which magnetic potential energy U_B is stored in the magnetic field.

Energy stored in a magnetic field



$$\frac{dU_B}{dt} = Li\frac{di}{dt}.$$

$$U_B = \tfrac{1}{2}Li^2 \qquad \text{(magnetic energy),}$$

$$u_B = \frac{B^2}{2\mu_0} \qquad \text{(magnetic energy density).}$$

Remember energy stored by a capacitor;

$$U_E = \frac{q^2}{2C}.$$

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2,$$

Energy stored in a magnetic field

A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

KEY IDEA

The energy stored in the magnetic field of a coil at any time depends on the current through the coil at that time, according to Eq. 30-49 ($U_B = \frac{1}{2}Li^2$).

Calculations: Thus, to find the energy $U_{B\infty}$ stored at equilibrium, we must first find the equilibrium current. From Eq. 30-41, the equilibrium current is

$$i_\infty = \frac{\mathscr{E}}{R} = \frac{12\text{ V}}{0.35\ \Omega} = 34.3\text{ A}. \qquad (30\text{-}51)$$

Then substitution yields

$$U_{B\infty} = \tfrac{1}{2}Li_\infty^2 = (\tfrac{1}{2})(53 \times 10^{-3}\text{ H})(34.3\text{ A})^2$$
$$= 31\text{ J}. \qquad \text{(Answer)}$$

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

Calculations: Now we are being asked: At what time t will the relation

$$U_B = \tfrac{1}{2}U_{B\infty}$$

be satisfied? Using Eq. 30-49 twice allows us to rewrite this energy condition as

$$\tfrac{1}{2}Li^2 = (\tfrac{1}{2})\tfrac{1}{2}Li_\infty^2$$

or

$$i = \left(\frac{1}{\sqrt{2}}\right)i_\infty. \qquad (30\text{-}52)$$

This equation tells us that, as the current increases from its initial value of 0 to its final value of i_∞, the magnetic field will have half its final stored energy when the current has increased to this value. In general, we know that i is given by Eq. 30-41, and here i_∞ (see Eq. 30-51) is $\mathscr{E}/R$; so Eq. 30-52 becomes

$$\frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) = \frac{\mathscr{E}}{\sqrt{2}R}.$$

By canceling $\mathscr{E}/R$ and rearranging, we can write this as

$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293,$$

which yields

$$\frac{t}{\tau_L} = -\ln 0.293 = 1.23$$

or

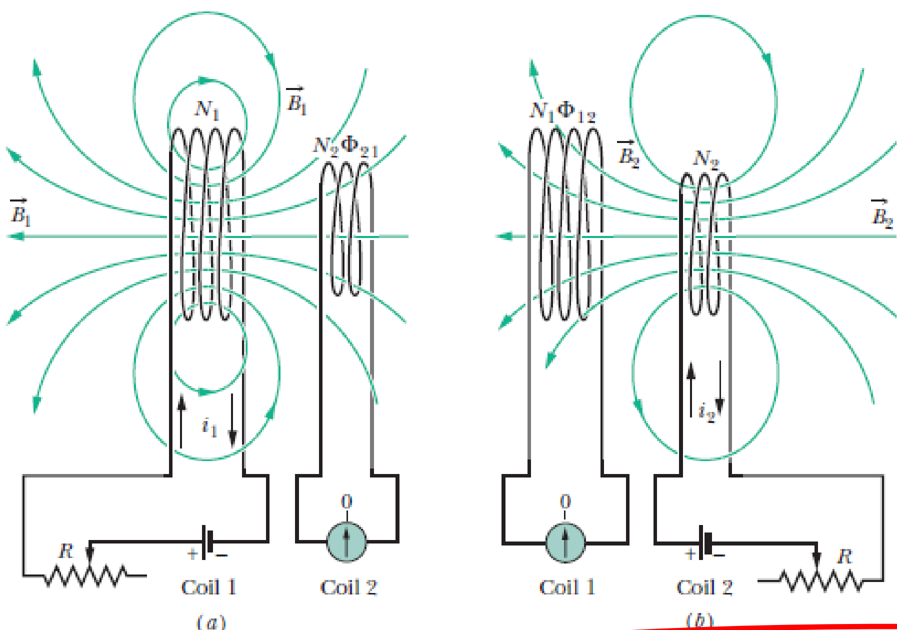
$$t \approx 1.2\tau_L. \qquad \text{(Answer)}$$

Thus, the energy stored in the magnetic field of the coil by the current will reach half its equilibrium value 1.2 time constants after the emf is applied.

Mutual induction

shows two circular close-packed coils near each other and sharing a common central axis. With the variable resistor set at a particular resistance R, the battery produces a steady current i_1 in coil 1. This current creates a magnetic field represented by the lines of $\vec{B}_1$ in the figure. Coil 2 is connected to a sensitive meter but contains no battery; a magnetic flux Φ_{21} (the flux through coil 2 associated with the current in coil 1) links the N_2 turns of coil 2.

We define the mutual inductance M_{21} of coil 2 with respect to coil 1 as



$$M_{21} = \frac{N_2\Phi_{21}}{i_1},$$ *By definition*

$$L = N\Phi/i,$$ *remember*

$$M_{21}i_1 = N_2\Phi_{21}.$$

$$M_{21}\frac{di_1}{dt} = N_2\frac{d\Phi_{21}}{dt}.$$

Emf appearing in coil 2 due to the changing i in coil 1

$$\mathcal{E}_2 = -M_{21}\frac{di_1}{dt}, \qquad \mathcal{E}_1 = -M_{12}\frac{di_2}{dt}. \qquad M_{21} = M_{12} = M,$$

Mutual inductance of two parallel coils

Figure 30-20 shows two circular close-packed coils, the smaller (radius R_2, with N_2 turns) being coaxial with the larger (radius R_1, with N_1 turns) and in the same plane.

(a) Derive an expression for the mutual inductance M for this arrangement of these two coils, assuming that $R_1 \gg R_2$.

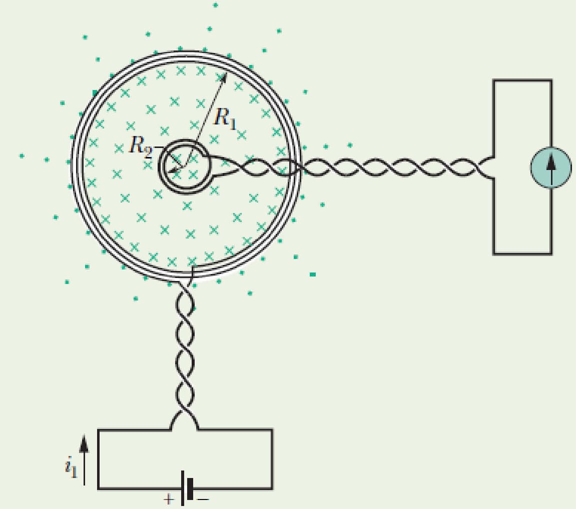


Fig. 30-20 A small coil is located at the center of a large coil. The mutual inductance of the coils can be determined by sending current i_1 through the large coil.

The magnetic flux through the small coil due to the current through the large coil is approximately uniform.

$$M = \frac{N_2\Phi_{21}}{i_1}.$$

$$\Phi_{21} = B_1 A_2,$$

Flux through each turn of the smaller co

$$N_2\Phi_{21} = N_2 B_1 A_2.$$

Flux linkage in the small coil with N_2 turns

$$M = \frac{N_2\Phi_{21}}{i_1} = \frac{\pi\mu_0 N_1 N_2 R_2^2}{2R_1}.$$

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}},$$

Set z=0, B1, the field the larger coil produces at points within the smaller coil

$$B_1 = N_1 \frac{\mu_0 i_1}{2R_1}$$

$$N_2\Phi_{21} = \frac{\pi\mu_0 N_1 N_2 R_2^2 i_1}{2R_1}.$$

Mutual inductance of two parallel coils

(b) What is the value of M for $N_1 = N_2 = 1200$ turns, $R_2 = 1.1$ cm, and $R_1 = 15$ cm?

$$M = \frac{(\pi)(4\pi \times 10^{-7}\ \text{H/m})(1200)(1200)(0.011\ \text{m})^2}{(2)(0.15\ \text{m})}$$

$$= 2.29 \times 10^{-3}\ \text{H} \approx 2.3\ \text{mH}. \qquad \text{(Answer)}$$

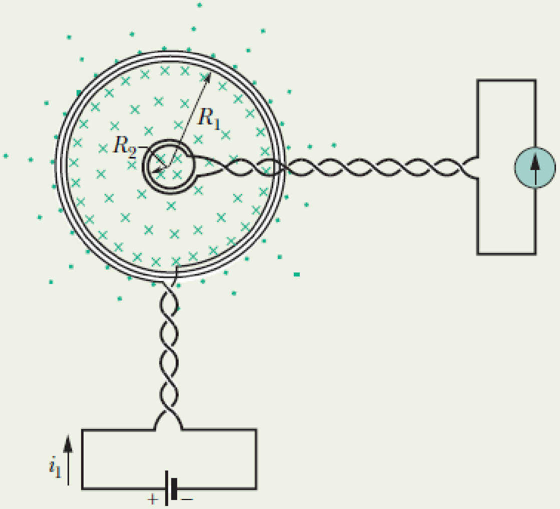


Fig. 30-20 A small coil is located at the center of a large coil. The mutual inductance of the coils can be determined by sending current i_1 through the large coil.

From here on !!!! At 17.12.