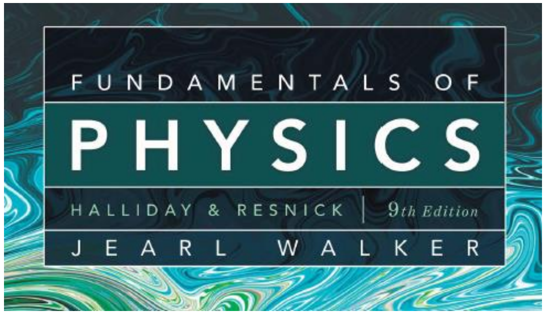
Physics 1

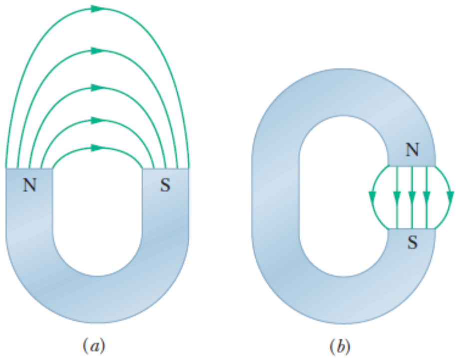
UNIVERSITÄT SIEGEN
FESTKÖRPERPHYSIK

Lecture 6: Current and magnetic field

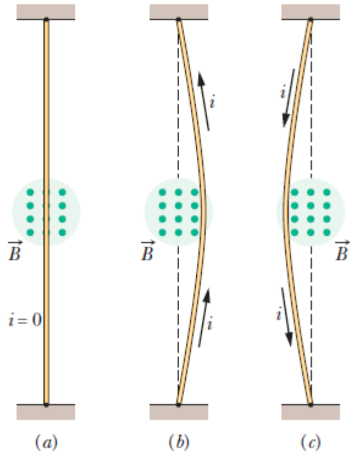
Prof. Dr. U. Pietsch



What creates a magnetic force ?



By electromagnets ,
By permanent magnets



Due to an
electric current

$$\vec{E} = \frac{\vec{F}_E}{q}.$$

Similar to electric field there
is a magnetic field, depends on
velocity of carrier movement

$$B = \frac{F_B}{|q|v},$$

Magnetic force

Lorentz Force

$$\vec{F}_B = q\vec{v} \times \vec{B};$$

$$1\ \text{T} = 1\,\frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1\,\frac{\text{N}}{\text{A}\cdot\text{m}}.$$

$$1\ \text{tesla} = 10^4\ \text{gauss}.$$

Cross $\vec{v}$ into $\vec{B}$ to get the new vector $\vec{v} \times \vec{B}$.

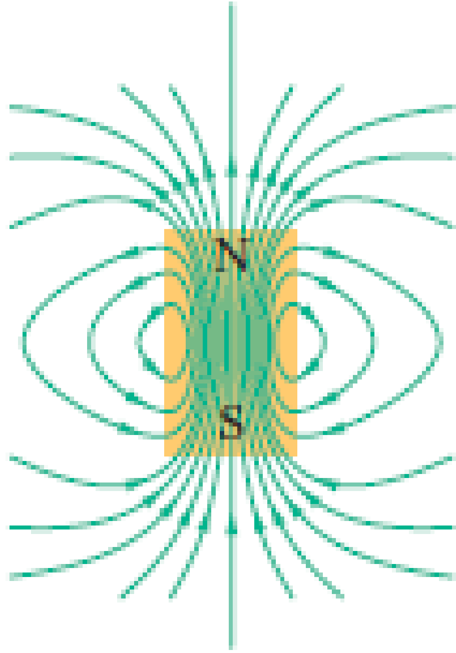
(a) (b) (c)

Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Magnetic field lines

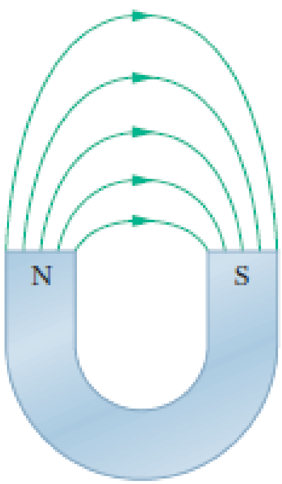


(a)

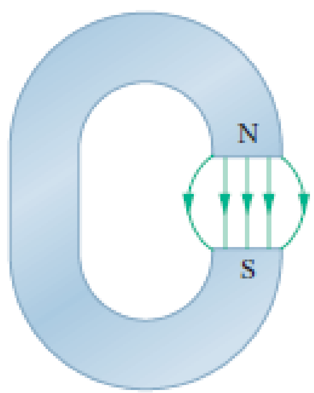
Always :

magnetic dipole

Opposite magnetic poles attract each other, and like magnetic poles repel each other.



(a)



(b)

The field lines run from the north pole to the south pole.

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

KEY IDEAS

Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}_B$ can act on it. Because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}_B$ is not simply zero.

Magnitude: To find the magnitude of $\vec{F}_B$, we can use Eq. 28-3 ($F_B = |q|vB \sin \phi$) provided we first find the proton's speed v. We can find v from the given kinetic energy because $K = \frac{1}{2}mv^2$. Solving for v, we obtain

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\text{ MeV})(1.60 \times 10^{-13}\text{ J/MeV})}{1.67 \times 10^{-27}\text{ kg}}}$$
$$= 3.2 \times 10^7\text{ m/s}.$$

Equation 28-3 then yields

$$F_B = |q|vB \sin \phi$$
$$= (1.60 \times 10^{-19}\text{ C})(3.2 \times 10^7\text{ m/s}) \times (1.2 \times 10^{-3}\text{ T})(\sin 90°)$$
$$= 6.1 \times 10^{-15}\text{ N}. \qquad \text{(Answer)}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration; namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15}\text{ N}}{1.67 \times 10^{-27}\text{ kg}} = 3.7 \times 10^{12}\text{ m/s}^2.$$

Direction: To find the direction of $\vec{F}_B$, we use the fact that $\vec{F}_B$ has the direction of the cross product $q\vec{v} \times \vec{B}$. Because the charge q is positive, $\vec{F}_B$ must have the same direction as $\vec{v} \times \vec{B}$, which can be determined with the right-hand rule for cross products (as in Fig. 28-2*d*). We know that $\vec{v}$ is directed horizontally from south to north and $\vec{B}$ is directed vertically up. The right-hand rule shows us that the deflecting force $\vec{F}_B$ must be directed horizontally from west to east, as Fig. 28-6 shows. (The array of dots in the figure represents a magnetic field directed out of the plane of the figure. An array of Xs would have represented a magnetic field directed into that plane.)

If the charge of the particle were negative, the magnetic deflecting force would be directed in the opposite direction—that is, horizontally from east to west. This is predicted automatically by Eq. 28-2 if we substitute a negative value for q.

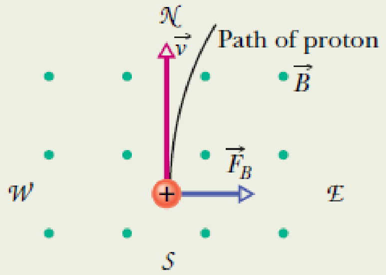
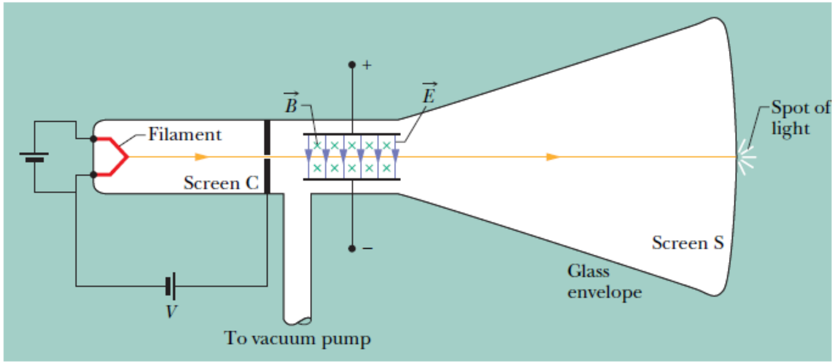
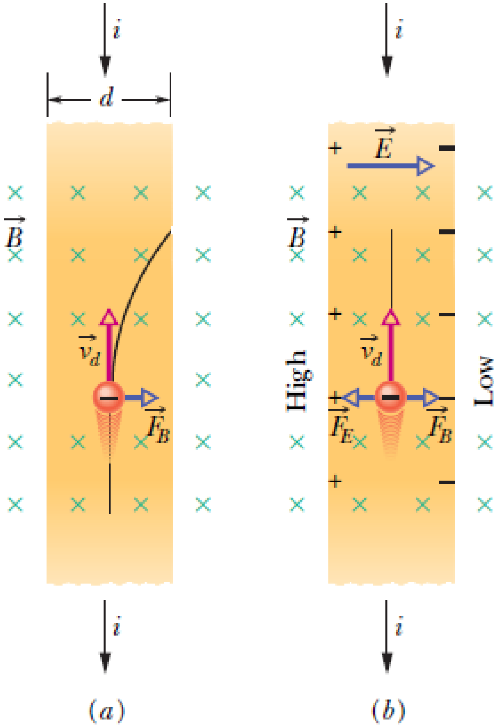


Fig. 28-6 An overhead view of a proton moving from south to north with velocity $\vec{v}$ in a chamber. A magnetic field is directed vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.

Crossed Fields: Discovery of the Electron

1. Set $E = 0$ and $B = 0$ and note the position of the spot on screen S due to the undeflected beam.
2. Turn on $\vec{E}$ and measure the resulting beam deflection.
3. Maintaining $\vec{E}$, now turn on $\vec{B}$ and adjust its value until the beam returns to the undeflected position. (With the forces in opposition, they can be made to cancel.)





$$|q|E = |q|vB\sin(90°) = |q|vB$$

$$v = \frac{E}{B}.$$ measure v

$$\frac{m}{|q|} = \frac{B^2L^2}{2yE},$$ measure m/|q|

Deflection on screen (TV tube): B || E , B moves e-beam along x, E moves it along y

Hall Effect

To measure charge carrier concentration of specimen

Charge carriers move perpendicular to magnetic field B, become deflected due to Lorentz force, this creates charge separation perpendiculat v and B, and subsequentely an electric field E

$$V = Ed.$$

Balance of E and B fields:

$$eE = ev_d B.$$

measure drift velocity v_d

$$v_d = \frac{J}{ne} = \frac{i}{neA},$$

measure charge carrier concentration n

$$n = \frac{Bi}{Vle},$$

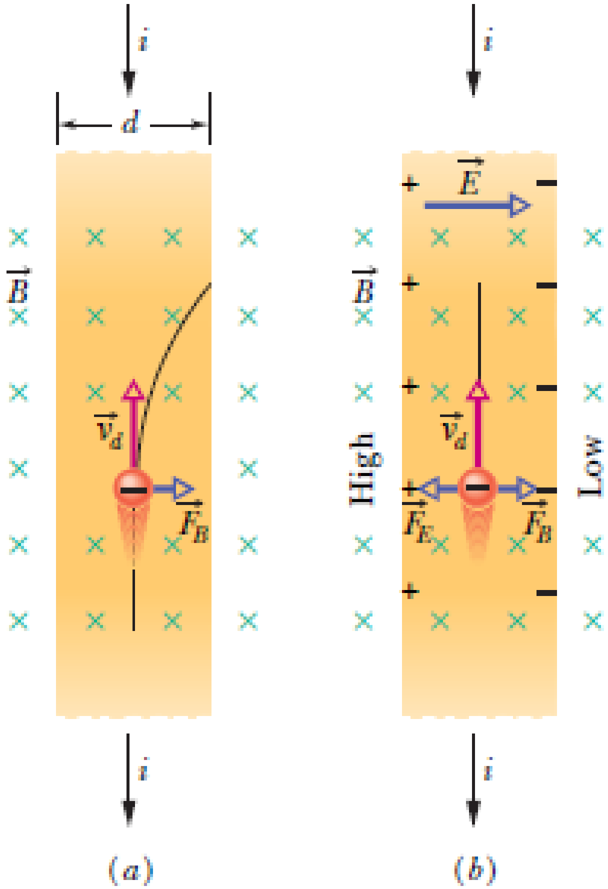
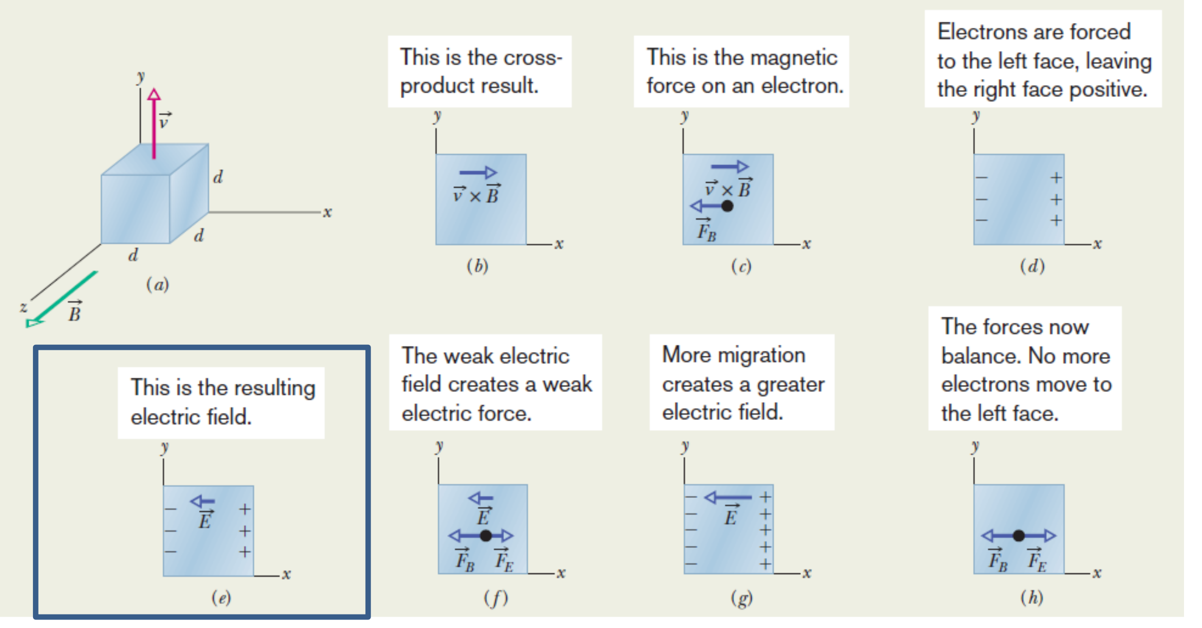


Figure 28-9*a* shows a solid metal cube, of edge length $d = 1.5$ cm, moving in the positive y direction at a constant velocity $\vec{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\vec{B}$ of magnitude 0.050 T in the positive z direction.

(a) Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?



(b) What is the potential difference between the faces of higher and lower electric potential?

$$|q|E = |q|vB \sin 90° = |q|vB.$$

This gives us $E = vB$; so $V = Ed$ becomes

$$V = vBd.$$

Substituting known values gives us

$$V = (4.0 \text{ m/s})(0.050 \text{ T})(0.015 \text{ m})$$
$$= 0.0030 \text{ V} = 3.0 \text{ mV}.$$

Circulating charged particle

$\vec{F}_B = q\vec{v} \times \vec{B};$ equals $F = m\frac{v^2}{r},$

$$|q|vB = \frac{mv^2}{r}.$$

$$r = \frac{mv}{|q|B} \quad \text{(radius)}.$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{(period)}.$$

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency)}.$$

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular frequency)}.$$

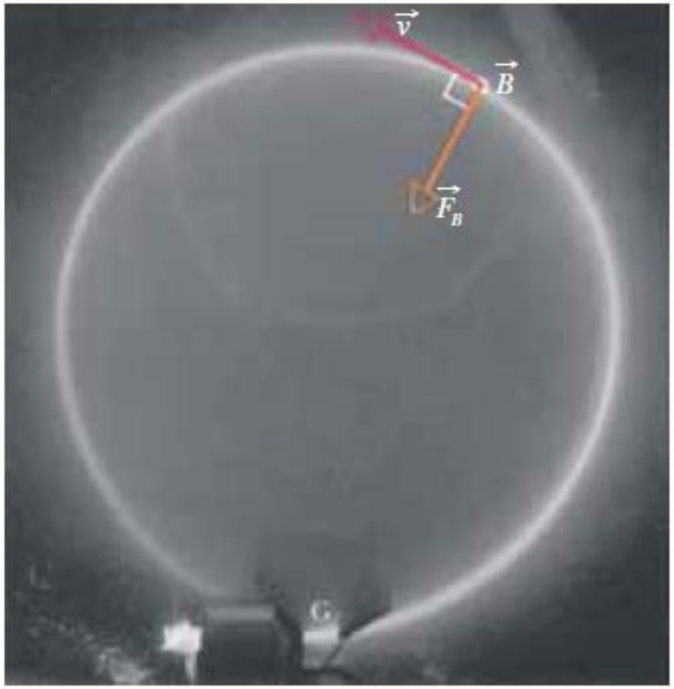


Figure 28-12 shows the essentials of a *mass spectrometer*, which can be used to measure the mass of an ion; an ion of mass m (to be measured) and charge q is produced in source S. The initially stationary ion is accelerated by the electric field due to a potential difference V. The ion leaves S and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the $\vec{B}$ causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000$ mT, $V = 1000.0$ V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the detector at a point that lies at $x = 1.6254$ m. What is the mass m of the individual ions, in atomic mass units (Eq. 1-7: 1 u = 1.6605×10^{-27} kg)?

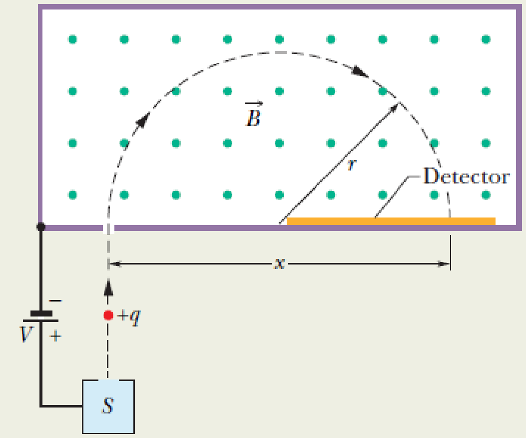


Fig. 28-12 Essentials of a mass spectrometer. A positive ion, after being accelerated from its source S by a potential difference V, enters a chamber of uniform magnetic field $\vec{B}$. There it travels through a semicircle of radius r and strikes a detector at a distance x from where it entered the chamber.

KEY IDEAS

(1) Because the (uniform) magnetic field causes the (charged) ion to follow a circular path, we can relate the ion's mass m to the path's radius r with Eq. 28-16 ($r = mv/|q|B$). From Fig. 28-12 we see that $r = x/2$ (the radius is half the diameter). From the problem statement, we know the magnitude B of the magnetic field. However, we lack the ion's speed v in the magnetic field after the ion has been accelerated due to the potential difference V. (2) To relate v and V, we use the fact that mechanical energy ($E_{mec} = K + U$) is conserved during the acceleration.

Finding speed: When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of $-V$. Thus, because the ion has positive charge q, its potential energy changes by $-qV$. If we now write the conservation of mechanical energy as

$$\Delta K + \Delta U = 0,$$

we get

$$\tfrac{1}{2}mv^2 - qV = 0$$

or

$$v = \sqrt{\frac{2qV}{m}}. \qquad (28\text{-}22)$$

Finding mass: Substituting this value for v into Eq. 28-16 gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Thus,

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

Solving this for m and substituting the given data yield

$$m = \frac{B^2qx^2}{8V}$$
$$= \frac{(0.080000\ \text{T})^2(1.6022 \times 10^{-19}\ \text{C})(1.6254\ \text{m})^2}{8(1000.0\ \text{V})}$$
$$= 3.3863 \times 10^{-25}\ \text{kg} = 203.93\ \text{u}. \qquad \text{(Answer)}$$

The Cyclotron

$$r = \frac{mv}{|q|B} \quad \text{(radius)}. \qquad f = \frac{|q|}{2\pi m} B$$

$f = f_{osc}$ (resonance condition).

$$|q|B = 2\pi m f_{osc}.$$

Changing B to maintain f_{osc}

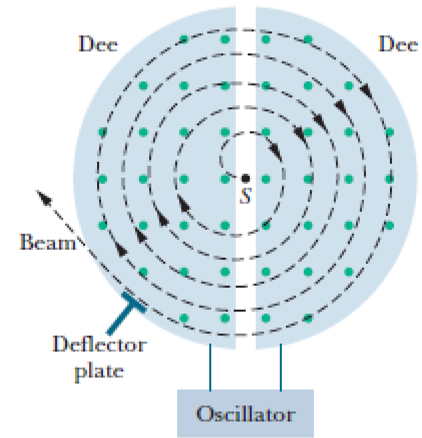
$$E = \frac{|q|^2}{2m}(RB)^2$$

Kinetic energy

For higher energies m → γm

$$f = \frac{|q|}{2\pi \gamma m} B \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The protons spiral outward in a cyclotron, picking up energy in the gap.



In Cyclotrons the final energy is limited

In a synchrotron R is constant. One increases B in order to increase kinetic energy of the accelerated particles. → K is increased synchronous to increase of B. The principle allows for accelation of e^- or p^+ to very large energies (GeV).

Suppose a cyclotron is operated at an oscillator frequency of 12 MHz and has a dee radius $R = 53$ cm.

(a) What is the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron? The deuteron mass is $m = 3.34 \times 10^{-27}$ kg (twice the proton mass).

KEY IDEA

For a given oscillator frequency f_{osc}, the magnetic field magnitude B required to accelerate any particle in a cyclotron depends on the ratio $m/|q|$ of mass to charge for the particle, according to Eq. 28-24 ($|q|B = 2\pi m f_{osc}$).

Calculation: For deuterons and the oscillator frequency $f_{osc} = 12$ MHz, we find

$$B = \frac{2\pi m f_{osc}}{|q|} = \frac{(2\pi)(3.34 \times 10^{-27}\,\text{kg})(12 \times 10^{6}\,\text{s}^{-1})}{1.60 \times 10^{-19}\,\text{C}}$$

$$= 1.57\,\text{T} \approx 1.6\,\text{T}. \qquad \text{(Answer)}$$

Note that, to accelerate protons, B would have to be reduced by a factor of 2, provided the oscillator frequency remained fixed at 12 MHz.

(b) What is the resulting kinetic energy of the deuterons?

KEY IDEAS

(1) The kinetic energy ($\frac{1}{2}mv^2$) of a deuteron exiting the cyclotron is equal to the kinetic energy it had just before exiting, when it was traveling in a circular path with a radius approximately equal to the radius R of the cyclotron dees. (2) We can find the speed v of the deuteron in that circular path with Eq. 28-16 ($r = mv/|q|B$).

Calculations: Solving that equation for v, substituting R for r, and then substituting known data, we find

$$v = \frac{R|q|B}{m} = \frac{(0.53\,\text{m})(1.60 \times 10^{-19}\,\text{C})(1.57\,\text{T})}{3.34 \times 10^{-27}\,\text{kg}}$$

$$= 3.99 \times 10^{7}\,\text{m/s}.$$

This speed corresponds to a kinetic energy of

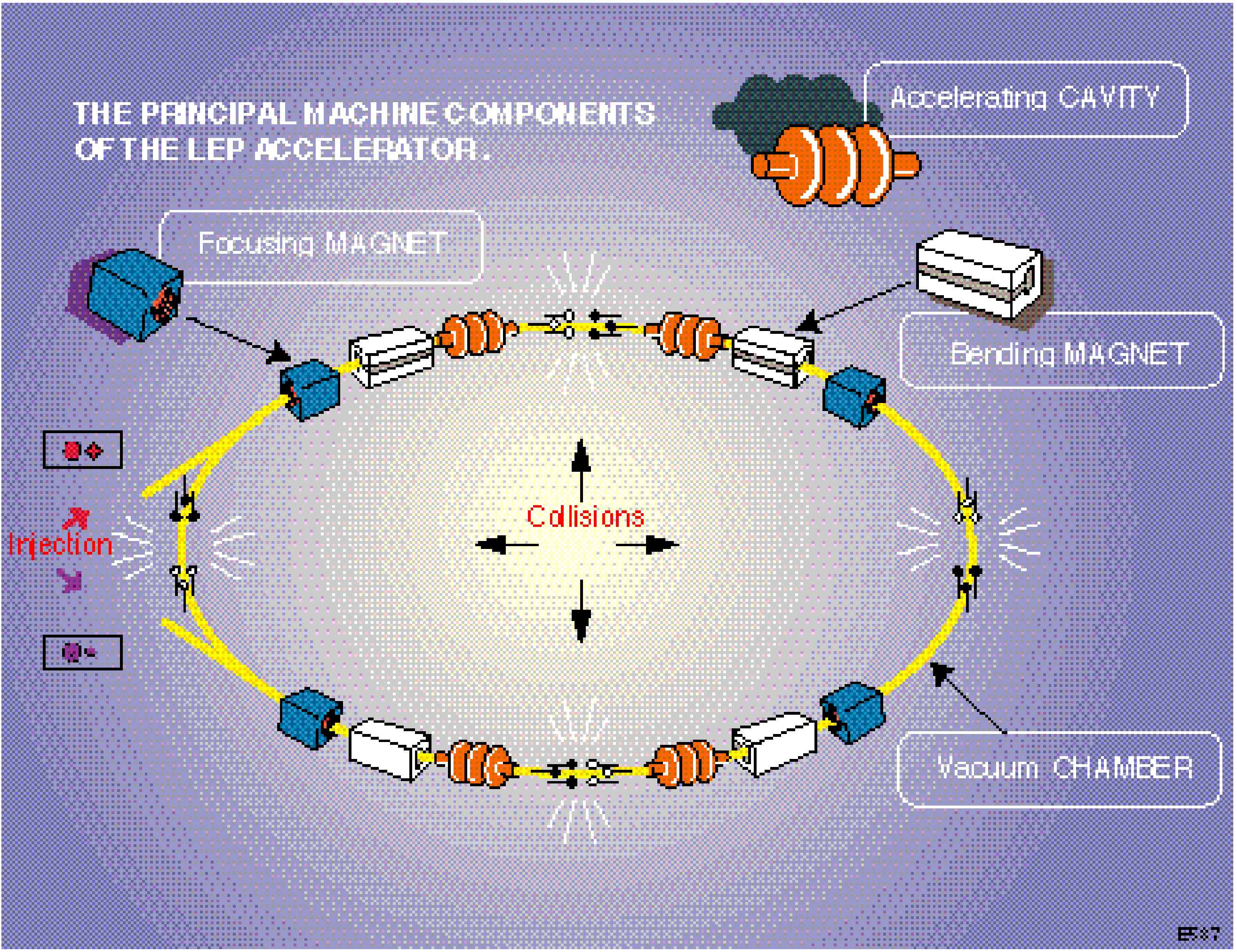
$$K = \tfrac{1}{2}mv^2$$

$$= \tfrac{1}{2}(3.34 \times 10^{-27}\,\text{kg})(3.99 \times 10^{7}\,\text{m/s})^2$$

$$= 2.7 \times 10^{-12}\,\text{J}, \qquad \text{(Answer)}$$

or about 17 MeV.

General scheme of a synchrotron



In a synchrotron R is constant. One increases B in order to increase kinetic energy of the accelerated particles.

→ K is increased synchronous to increase of B. The principle allows for accelation of e^- or p^+ to very large energies (GeV).

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Generation of Synchrotron radiation

At relativistic energies the orbiting electrons emit radiation = synchrotron radiation

Electron circulates with relatvististic energy in orbit of storage ring of radius R with orbit frequency $\omega = 10^6$ Hz → E_e = 5 GeV

$$E_e = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0 c^2}{\sqrt{1-ß^2}}$$

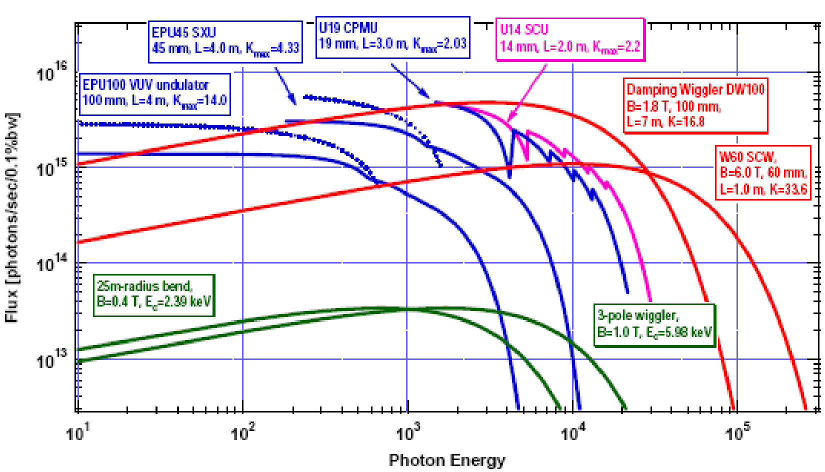
E_e is much larger compared to rest mass energy mc^2, γ = 5GeV/ 0.511 GeV≈ 10^4

value $1/\gamma$ is the vertical open angle

Using γ one can estimate the electron velocity

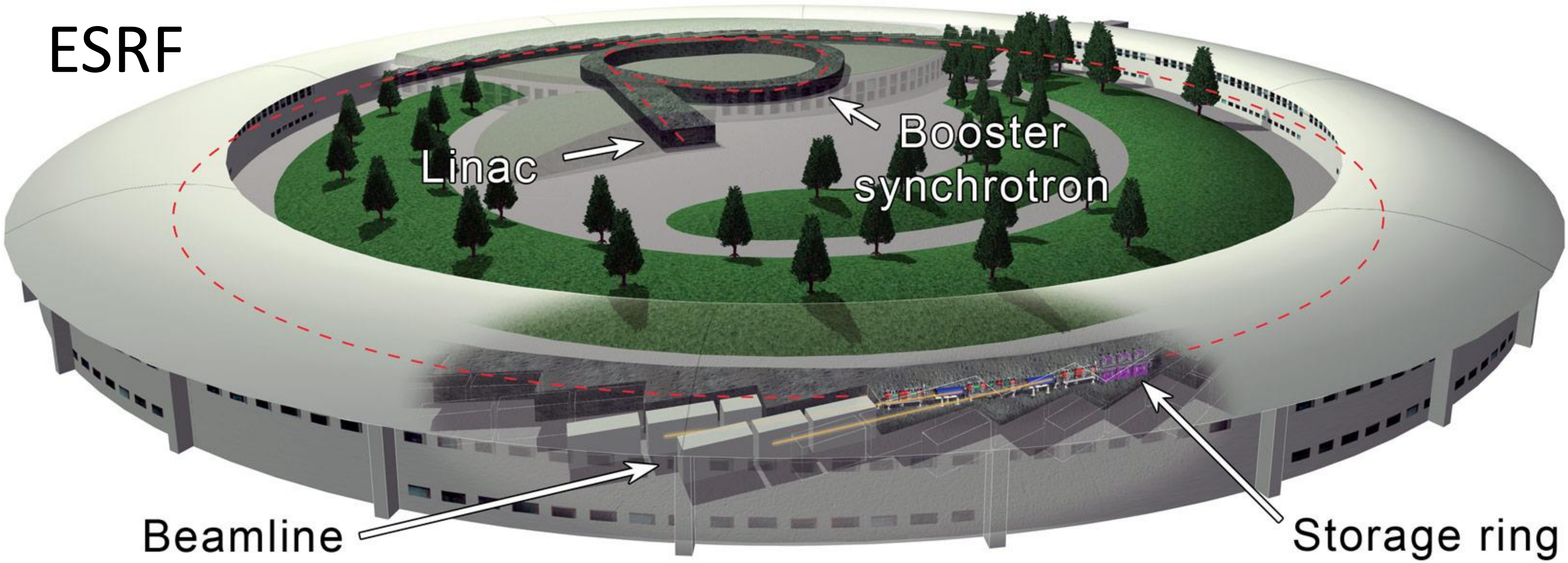
$$ß = [1-\frac{1}{\gamma^2}]^{1/2} \approx 1-\frac{1}{2\gamma^2} = 1-6*10^{-9}$$

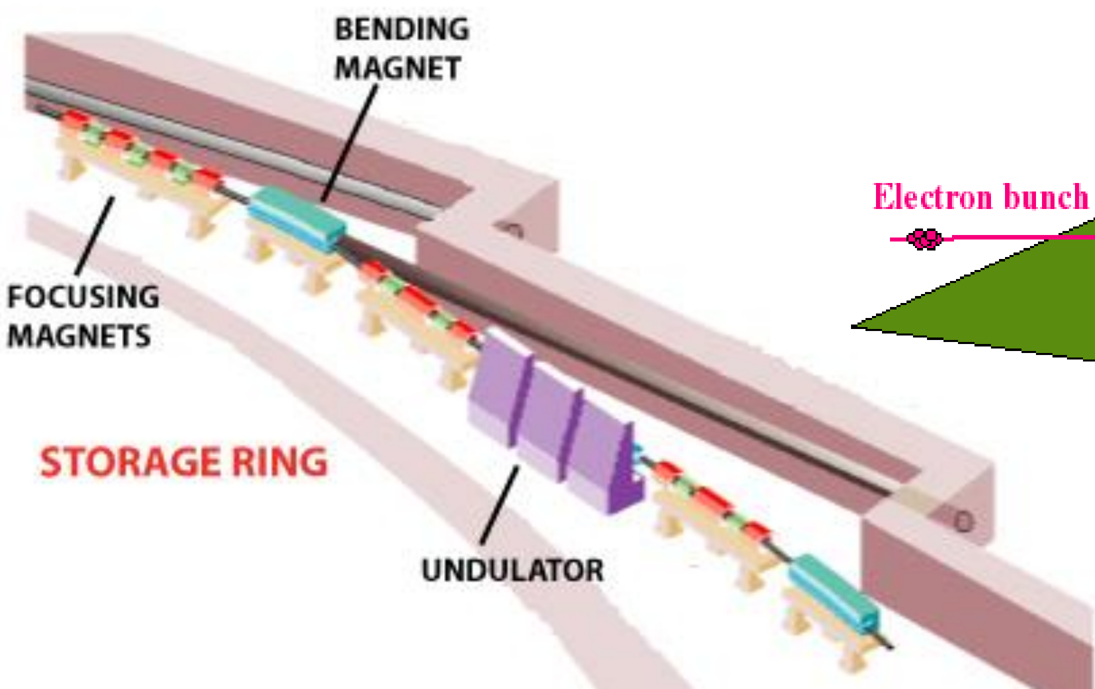
$$v \approx 1-6*10^{-9}c$$

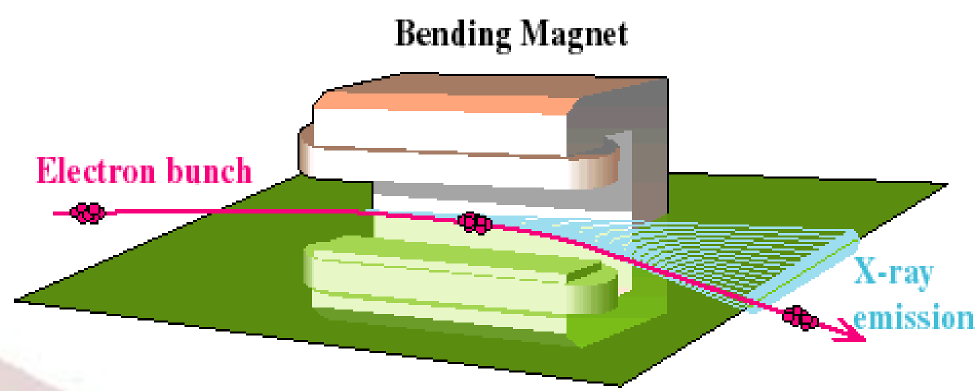


$$\hbar\omega_c = \frac{3}{2}\hbar\omega\gamma^3 \approx 0.655 * E_e^2(GeV)B(T)$$

ESRF


Linac
Booster
synchrotron
Beamline
Storage ring

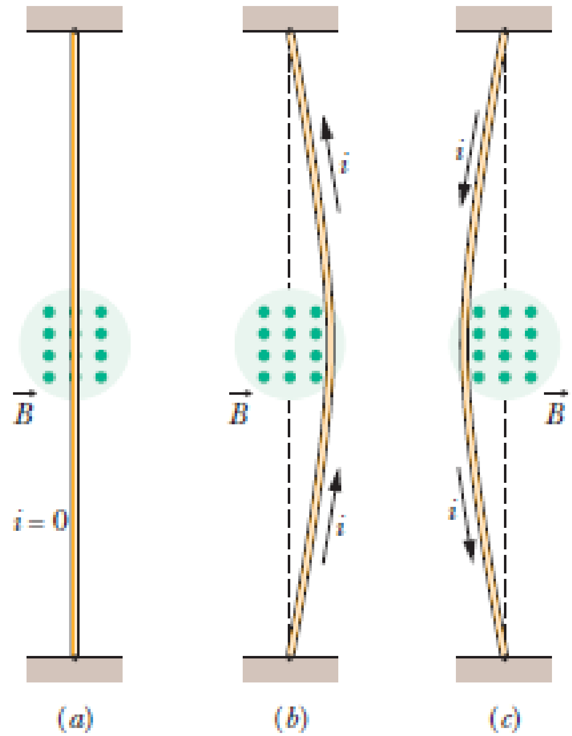

BENDING
MAGNET
FOCUSING
MAGNETS
STORAGE RING
UNDULATOR


Bending Magnet
Electron bunch
X-ray
emission

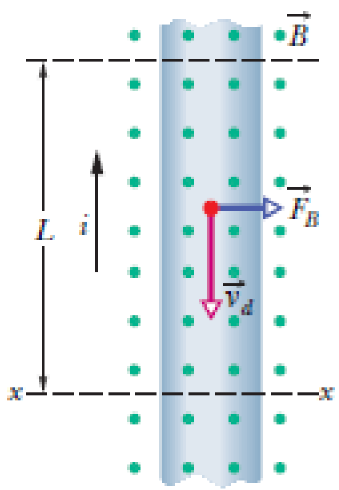
Magnetic force in a current-carrying wire

$$q = it = i\frac{L}{v_d}$$

$$F_B = qv_dB\sin\phi = \frac{iL}{v_d}v_dB\sin 90^\circ$$

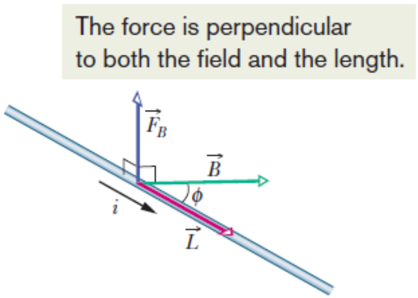


Deflection depends on current and direction



$$\vec{F}_B = i\vec{L} \times \vec{B} \qquad \text{(force on a current).}$$

$\vec{L}$ is a *length vector* that has magnitude L directed along the wire



A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

KEY IDEAS

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig. 28-17). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

Calculations: Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 28-17) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$ (Eq. 28-27). Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg, \tag{28-29}$$

where mg is the magnitude of $\vec{F}_g$ and m is the mass of the wire.

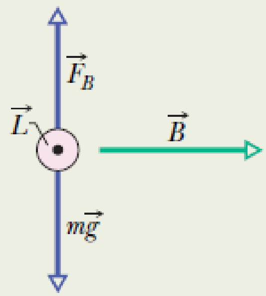


Fig. 28-17 A wire (shown in cross section) carrying current out of the page.

We also want the minimal field magnitude B for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin \phi$ in Eq. 28-29. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin \phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}. \tag{28-30}$$

We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3}\ \text{kg/m})(9.8\ \text{m/s}^2)}{28\ \text{A}}$$

$$= 1.6 \times 10^{-2}\ \text{T}. \qquad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field.

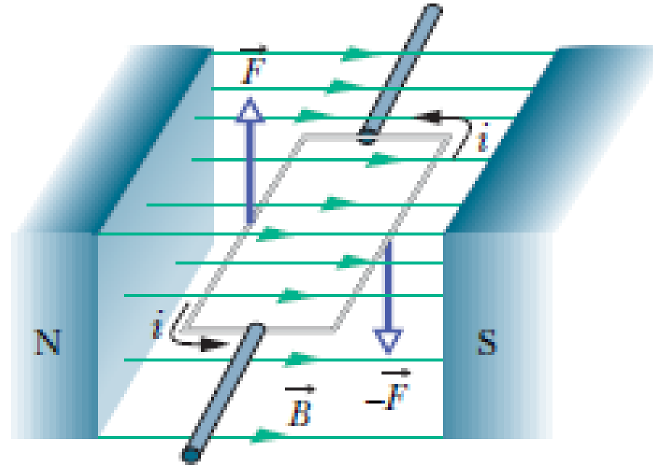
Torque on a current loop

$$F_2 = ibB \sin(90° - \theta) = ibB \cos\theta.$$

F2 = -F4

torque due to forces $\vec{F}_1$ and $\vec{F}_3$ i

$\vec{\tau} = \vec{r} \times \vec{F}$ r=b/2 F=*i*aBsinθ

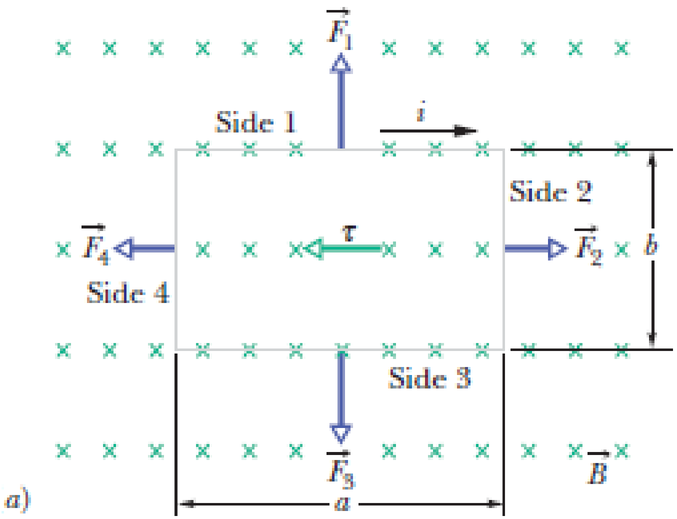


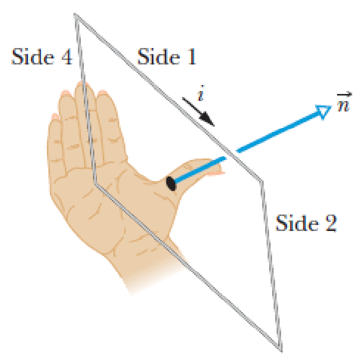
$$\tau' = \left(iaB\frac{b}{2}\sin\theta\right) + \left(iaB\frac{b}{2}\sin\theta\right) = iabB\sin\theta.$$

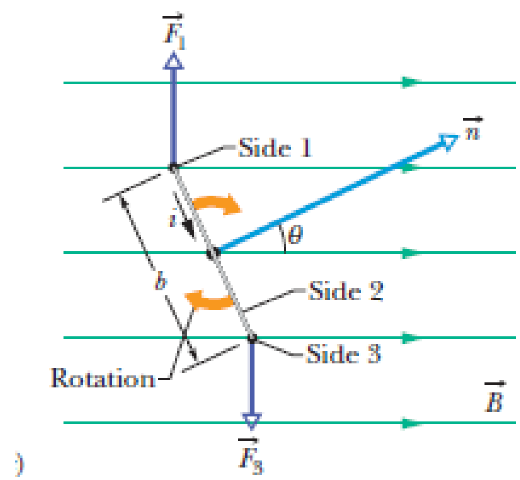
$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta,$$

Circular coil ,
N loops

$$\tau = (Ni\pi r^2)B\sin\theta.$$

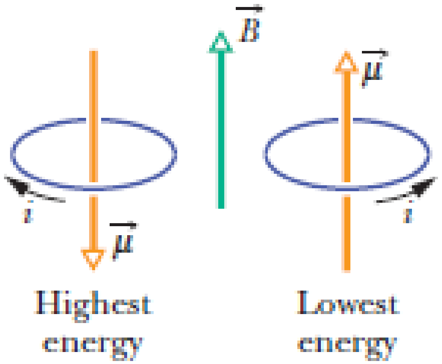






Magnetic dipole moment

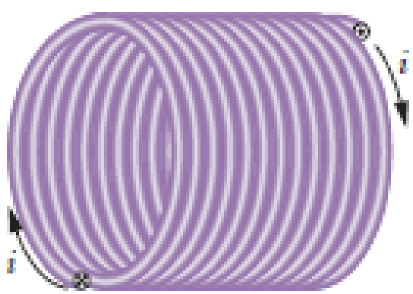
A current rotating on a circle produces an magnetic field, due to sign of B it defines a dipole moment



$\mu = NiA$ (magnetic moment),

$\tau = \mu B \sin\theta,$

$\vec{\tau} = \vec{\mu} \times \vec{B},$ compare $\vec{\tau} = \vec{p} \times \vec{E}.$



$U(\theta) = -\vec{\mu} \cdot \vec{B}.$ compare $U(\theta) = -\vec{p} \cdot \vec{E}.$

Figure 28-21 shows a circular coil with 250 turns, an area A of 2.52×10^{-4} m^2, and a current of 100 μA. The coil is at rest in a uniform magnetic field of magnitude $B = 0.85$ T, with its magnetic dipole moment $\vec{\mu}$ initially aligned with $\vec{B}$.

(a) In Fig. 28-21, what is the direction of the current in the coil?

Right-hand rule: Imagine cupping the coil with your right hand so that your right thumb is outstretched in the direction of $\vec{\mu}$. The direction in which your fingers curl around the coil is the direction of the current in the coil. Thus, in the wires on the near side of the coil—those we see in Fig. 28-21—the current is from top to bottom.

(b) How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its initial orientation, so that $\vec{\mu}$ is perpendicular to $\vec{B}$ and the coil is again at rest?

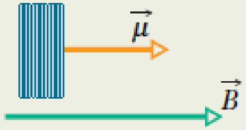


Fig. 28-21 A side view of a circular coil carrying a current and oriented so that its magnetic dipole moment is aligned with magnetic field $\vec{B}$.

KEY IDEA

The work W_a done by the applied torque would be equal to the change in the coil's orientation energy due to its change in orientation.

Calculations: From Eq. 28-39 ($W_a = U_f - U_i$), we find

$$W_a = U(90°) - U(0°)$$
$$= -\mu B \cos 90° - (-\mu B \cos 0°) = 0 + \mu B$$
$$= \mu B.$$

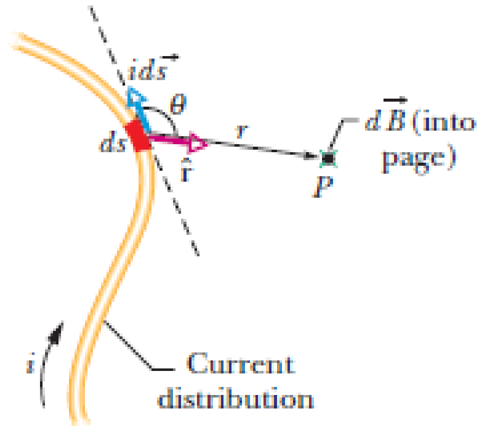
Substituting for μ from Eq. 28-35 ($\mu = NiA$), we find that

$$W_a = (NiA)B$$
$$= (250)(100 \times 10^{-6}\text{ A})(2.52 \times 10^{-4}\text{ m}^2)(0.85\text{ T})$$
$$= 5.355 \times 10^{-6}\text{ J} \approx 5.4\ \mu\text{J}. \qquad \text{(Answer)}$$

Magnetic field due to current

Magnitude of field *dB* produced at point *P* at distance *r* by a current length element *i ds* is:

This element of current creates a magnetic field at *P*, into the page.



$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

permeability constant,

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}.$$

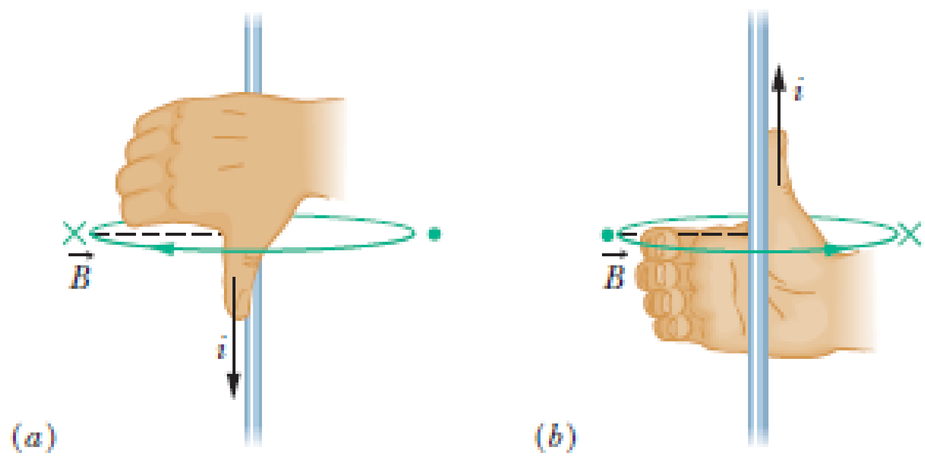
Compare: $E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$

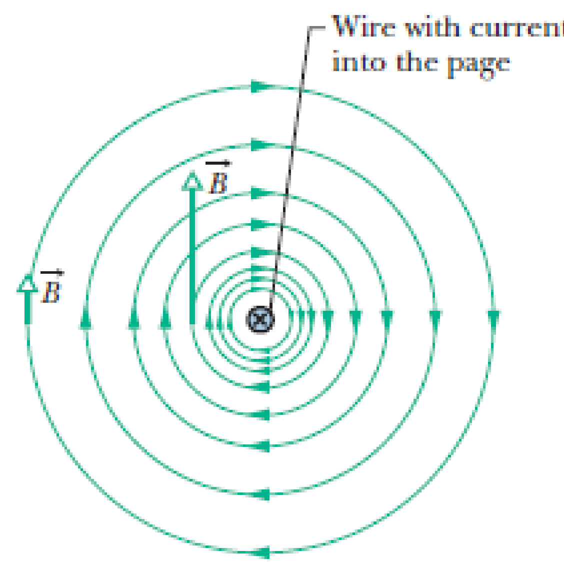
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{\mathrm{r}}}{r^2} \qquad \text{(Biot–Savart law).}$$

Long straight wire

$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire).}$$

Right-hand rule:





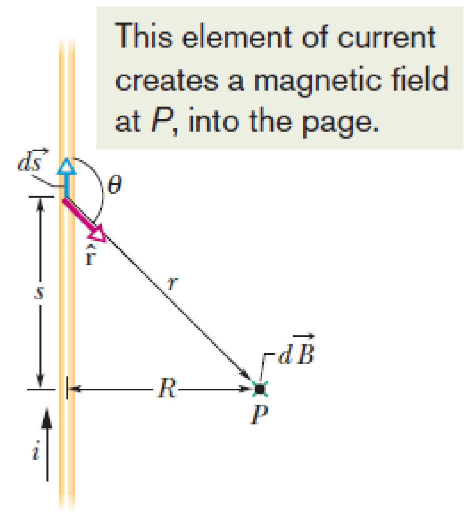
Proof of Biot-Savart equation:

$$B = 2\int_0^{\infty} dB = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{\sin\theta\, ds}{r^2}.$$

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}.$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{R\, ds}{(s^2 + R^2)^{3/2}} = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(s^2 + R^2)^{1/2}}\right]_0^{\infty} = \frac{\mu_0 i}{2\pi R},$$



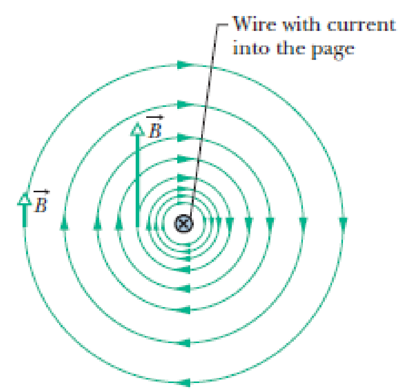
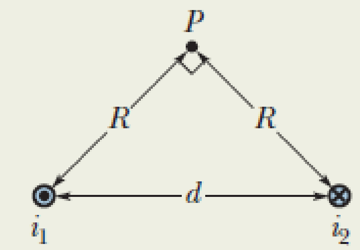


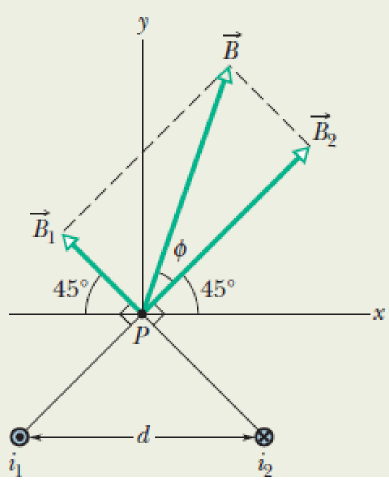
Figure 29-8*a* shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.



(*a*)

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$



Adding the vectors:

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)} \sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T·m/A})\sqrt{(15\ \text{A})^2 + (32\ \text{A})^2}}{(2\pi)(5.3 \times 10^{-2}\ \text{m})(\cos 45°)}$$

$$= 1.89 \times 10^{-4}\ \text{T} \approx 190\ \mu\text{T}. \qquad \text{(Answer)}$$

$$\phi = \tan^{-1}\frac{B_1}{B_2}, = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\ \text{A}}{32\ \text{A}} = 25°.$$

Force between two parallel currents

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

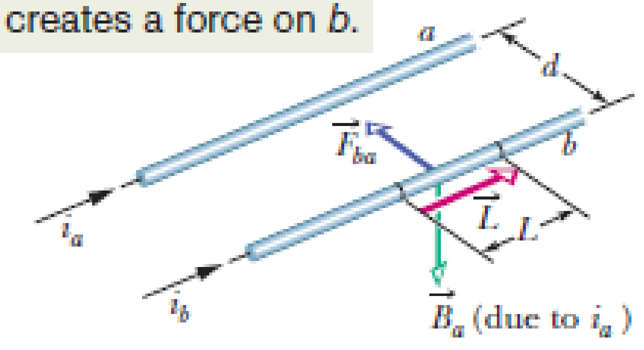
q dL/dt = dq/dt L= iL

$\vec{F}_B = q\vec{v} \times \vec{B};$

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

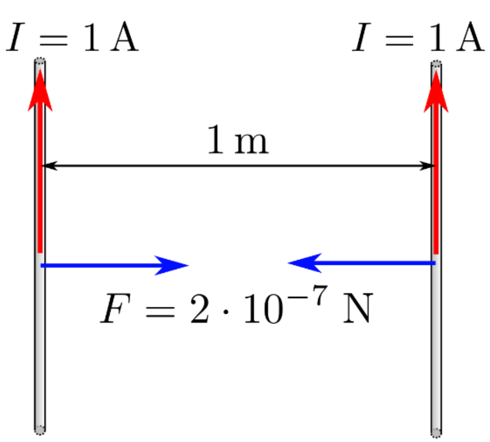
$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

The field due to *a* at the position of *b* creates a force on *b*.



Parallel currents attract each other, and antiparallel currents repel each other.

International definition of 1 A:
Current of 1 A is defined by the force of 2 x 10^{-7} N acting between two conductors in a distance of 1m



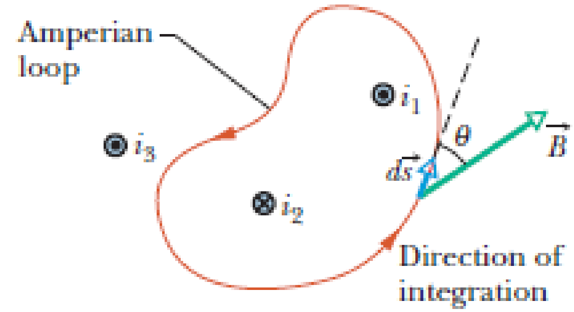
Ampere´s law

We have found the electric field of any charge distribution by summing up *dE* of each charge *q*

Similar approach to find net magnetic field by summing up dB of the currents i

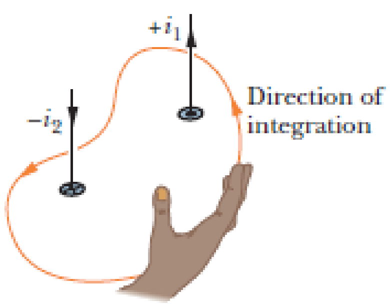
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{\mathrm{r}}}{r^2}$$

Only the currents encircled by the loop are used in Ampere's law.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \qquad \text{(Ampere's law).}$$

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.



$$\oint B \cos\theta\, ds = \mu_0(i_1 - i_2).$$

Magnetic field inside and outside a wire with current

$$\oint \vec{B}\cdot d\vec{s} = \oint B\cos\theta\, ds = B\oint ds = B(2\pi r).$$

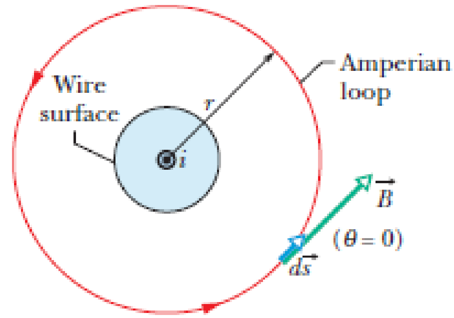
$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{(outside straight wire).}$$

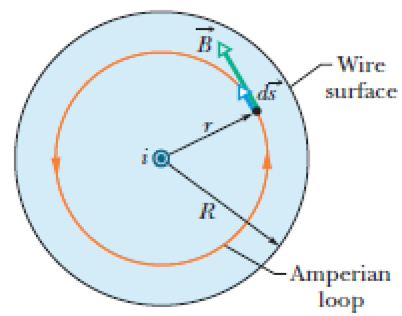
$$\oint \vec{B}\cdot d\vec{s} = B\oint ds = B(2\pi r).$$

$$i_{enc} = i\,\frac{\pi r^2}{\pi R^2}.$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r \qquad \text{(inside straight wire).}$$

Compare to E inside and outside a charged sphere





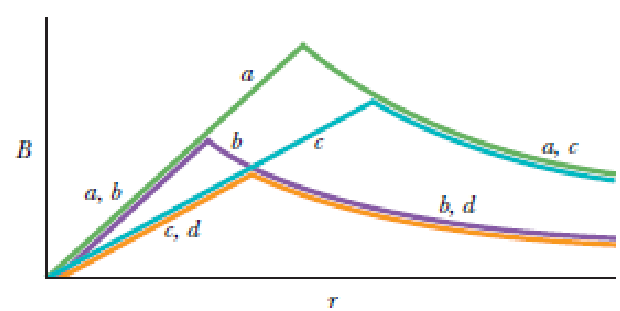


Figure 29-15*a* shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m^4 and r in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-15*a*, which is at radius $r = 3.0$ cm from the central axis of the cylinder?

KEY IDEAS

The point at which we want to evaluate $\vec{B}$ is inside the material of the conducting cylinder, between its inner and outer radii. We note that the current distribution has cylindrical symmetry (it is the same all around the cross section for any given radius). Thus, the symmetry allows us to use Ampere's law to find $\vec{B}$ at the point. We first draw the Amperian loop shown in Fig. 29-15*b*. The loop is concentric with the cylinder and has radius $r = 3.0$ cm because we want to evaluate $\vec{B}$ at that distance from the cylinder's central axis.

Next, we must compute the current i_{enc} that is encircled by the Amperian loop. However, we *cannot* set up a proportionality as in Eq. 29-19, because here the current is not uniformly distributed. Instead, we must integrate the current density magnitude from the cylinder's inner radius a to the loop radius r, using the steps shown in Figs. 29-15*c* through *h*.

Calculations: We write the integral as

$$i_{enc} = \int J\,dA = \int_a^r cr^2(2\pi r\,dr)$$

$$= 2\pi c \int_a^r r^3\,dr = 2\pi c\left[\frac{r^4}{4}\right]_a^r$$

$$= \frac{\pi c(r^4 - a^4)}{2}.$$

Note that in these steps we took the differential area dA to be the area of the thin ring in Figs. 29-15*d*–*f* and then replaced it with its equivalent, the product of the ring's circumference $2\pi r$ and its thickness dr.

For the Amperian loop, the direction of integration indicated in Fig. 29-15*b* is (arbitrarily) clockwise. Applying the right-hand rule for Ampere's law to that loop, we find that we should take i_{enc} as negative because the current is directed out of the page but our thumb is directed into the page.

We next evaluate the left side of Ampere's law exactly as we did in Fig. 29-14, and we again obtain Eq. 29-18. Then Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc},$$

gives us

$$B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4).$$

Solving for B and substituting known data yield

$$B = -\frac{\mu_0 c}{4r}(r^4 - a^4)$$

$$= -\frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(3.0 \times 10^6\ \mathrm{A/m^4})}{4(0.030\ \mathrm{m})}$$

$$\times [(0.030\ \mathrm{m})^4 - (0.020\ \mathrm{m})^4]$$

$$= -2.0 \times 10^{-5}\ \mathrm{T}.$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude

$$B = 2.0 \times 10^{-5}\ \mathrm{T} \qquad \text{(Answer)}$$

and forms magnetic field lines that are directed opposite our direction of integration, hence counterclockwise in Fig. 29-15*b*.

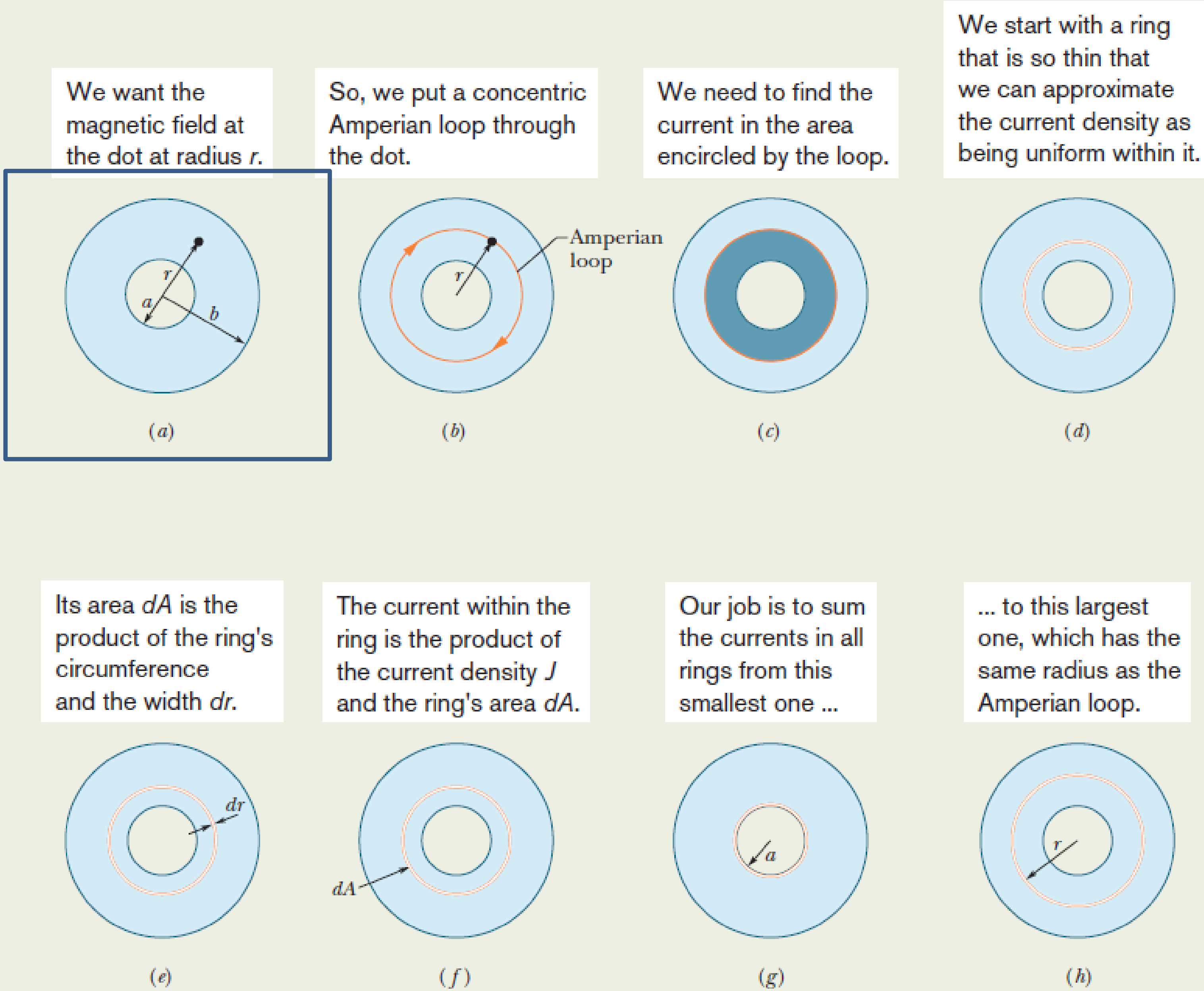
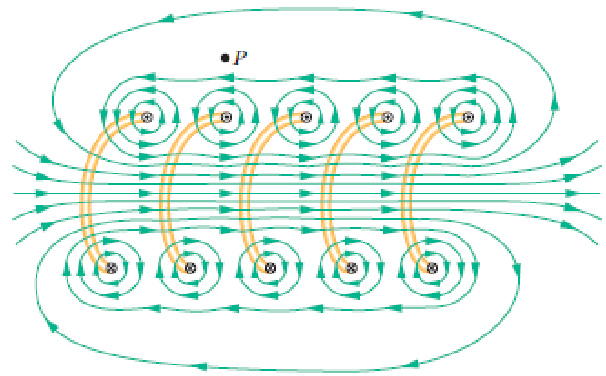
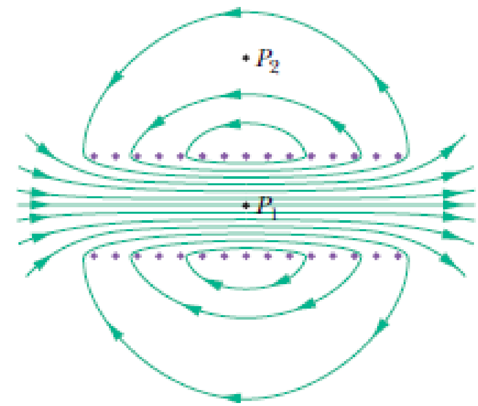


Fig. 29-15 (*a*) – (*b*) To find the magnetic field at a point within this conducting cylinder, we use a concentric Amperian loop through the point. We then need the current encircled by the loop. (*c*) – (*h*) Because the current density is nonuniform, we start with a thin ring and then sum (via integration) the currents in all such rings in the encircled area.

Magnetic field of a selenoid

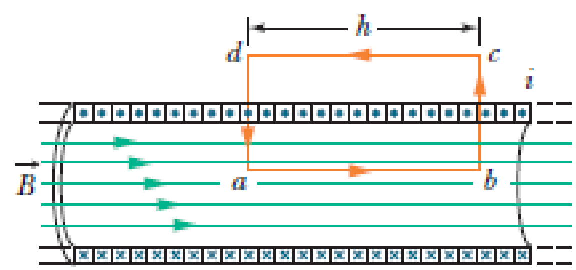


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc},$$



$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

$$B = \mu_0 in \qquad \text{(ideal solenoid).}$$



The field inside a solenoid (a long coil of current)

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

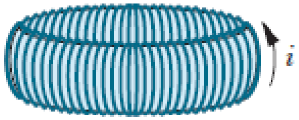
$$B = \mu_0 in = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.57\ \text{A})\frac{5 \times 850\ \text{turns}}{1.23\ \text{m}}$$

$$= 2.42 \times 10^{-2}\ \text{T} = 24.2\ \text{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.

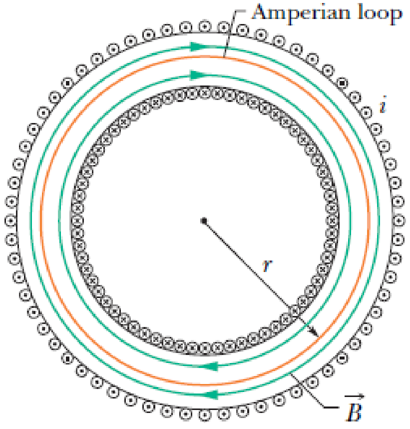
Additional examples, video, and practice available at *WileyPLUS*

Magnetic Field of a Toroid



(a)

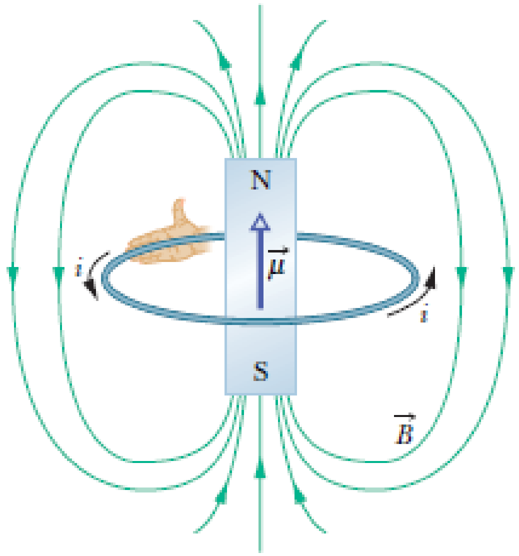
$$(B)(2\pi r) = \mu_0 iN,$$



(b)

$$B = \frac{\mu_0 iN}{2\pi}\frac{1}{r} \qquad \text{(toroid)}.$$

Magnet field of a coil (magnetic dipole)



$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}},$$

Z>>R

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3}.$$

A= πR^2

$$B(z) = \frac{\mu_0}{2\pi}\frac{NiA}{z^3}.$$

$\mu = NiA$:

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \qquad \text{(current-carrying coil).}$$

Compare to field of an electric dipole

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \text{(electric dipole).}$$