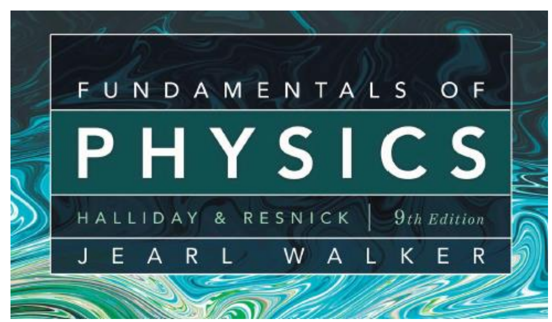
Physics 1

UNIVERSITÄT SIEGEN

FESTKÖRPERPHYSIK

Lecture 5b: Capacitors, dielectrics, electric circuits

Prof. Dr. U. Pietsch



A fixed potential difference V exists between a pair of close parallel plates carrying opposite charges $+Q$ and $-Q$. Which of the following would not increase the magnitude of charge that you could put on the plates?

(a) Increase the size of the plates.
(b) Move the plates farther apart.
(c) Fill the space between the plates with paper.
(d) Increase the fixed potential difference V.
(e) None of the above.

A **capacitor** is a device that can store electric charge, and normally consists of two conducting objects (usually plates or sheets) placed near each other but not touching. Capacitors are widely used in electronic circuits. They store charge for later use, such as in a camera flash, and as energy backup in computers if the power fails. Capacitors also block surges of charge and energy to protect circuits.

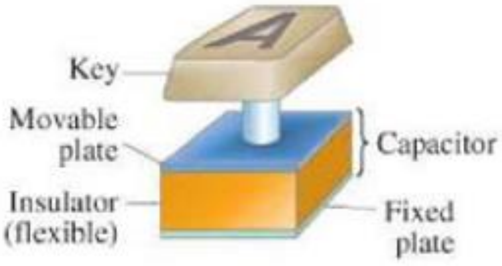
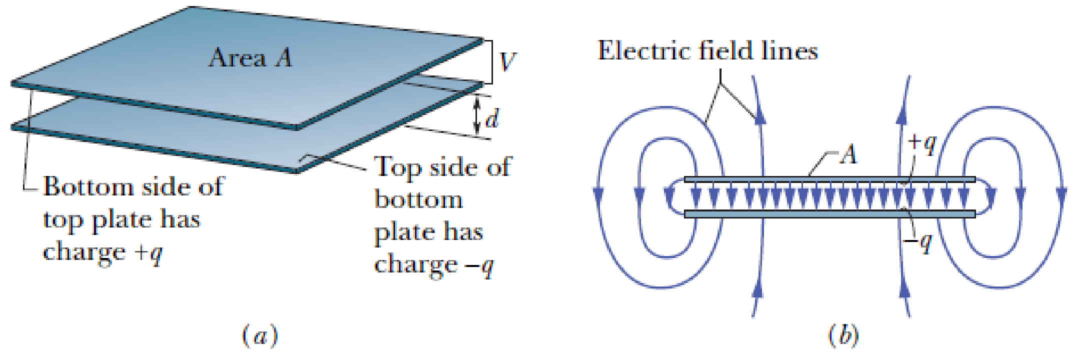


FIGURE 24–5 Key on a computer keyboard. Pressing the key reduces the capacitor spacing thus increasing the capacitance which can be detected electronically.

Capacity calculation

Two charged plates separated by distance d form a capacitor



Gaussian surface

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q.$$

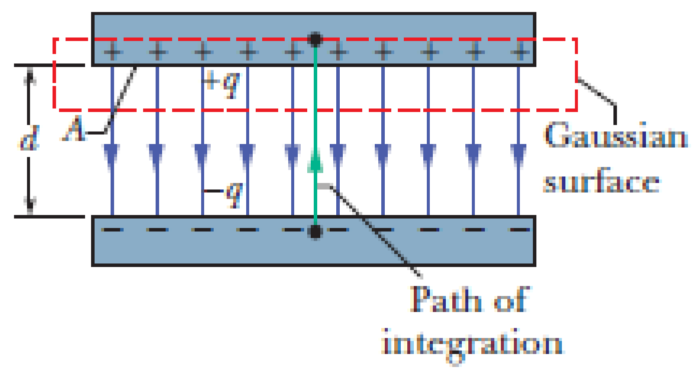
$$q = \varepsilon_0 E A,$$

The charge q and the potential difference V between the plates are proportional to each other

$$q = CV.$$

$$V = \int_{-}^{+} E\, ds = E \int_0^d ds = Ed.$$

C - capacity



$$\varepsilon_0 E A = C E d$$

$$C = \frac{\varepsilon_0 A}{d}$$

1 farad = 1 F = 1 coulomb per volt = 1 C/V.

Charging the plates in a parallel-plate capacitor

In Fig. 25-7*a*, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu F$ to the battery of potential difference $V = 12$ V. The lower capacitor plate has thickness $L = 0.50$ cm and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}$ electrons/m³. From what depth d within the plate (Fig. 25-7*b*) must electrons move to the plate face as the capacitor becomes charged?

KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25-1 ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6}\ F)(12\ V) = 3.0 \times 10^{-6}\ C.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ C}{1.602 \times 10^{-19}\ C} = 1.873 \times 10^{13}\ \text{electrons}.$$

These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$d = \frac{N}{An} = \frac{1.873 \times 10^{13}\ \text{electrons}}{(2.0 \times 10^{-4}\ m^2)(8.49 \times 10^{28}\ \text{electrons/m}^3)} = 1.1 \times 10^{-12}\ m = 1.1\ \text{pm}. \quad \text{(Answer)}$$

In common speech, we would say that the battery charges the capacitor by supplying the charged particles. But what the battery really does is set up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

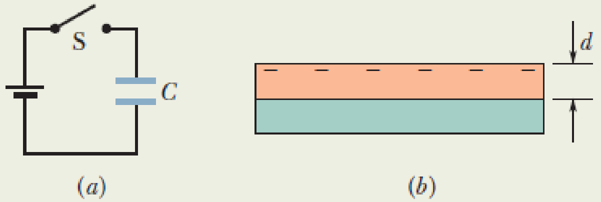


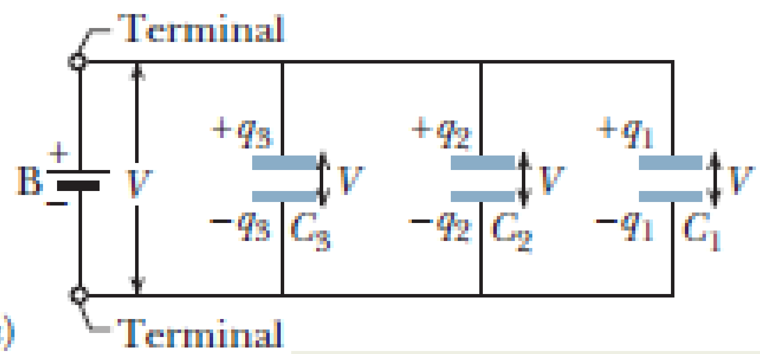
Fig. 25-7 (*a*) A battery and capacitor circuit. (*b*) The lower capacitor plate.

Parallel and Series connection of capacities

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

$$C_{eq} = \sum_{j=1}^{n} C_j$$



(a)

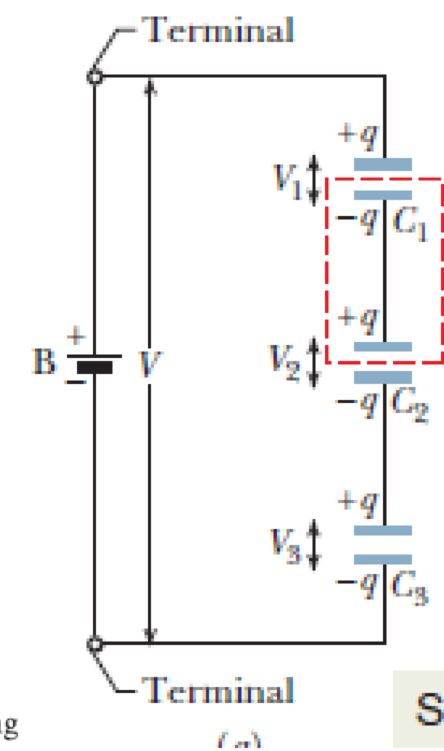
The net effect of connecting capacitors in parallel is thus to *increase* the capacitance. This makes sense because we are essentially increasing the area of the plates where charge can accumulate (see, for example, Eq. 24–2).

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$



Series capacitors

Notice that the equivalent capacitance C_{eq} is smaller than the smallest contributing capacitance.

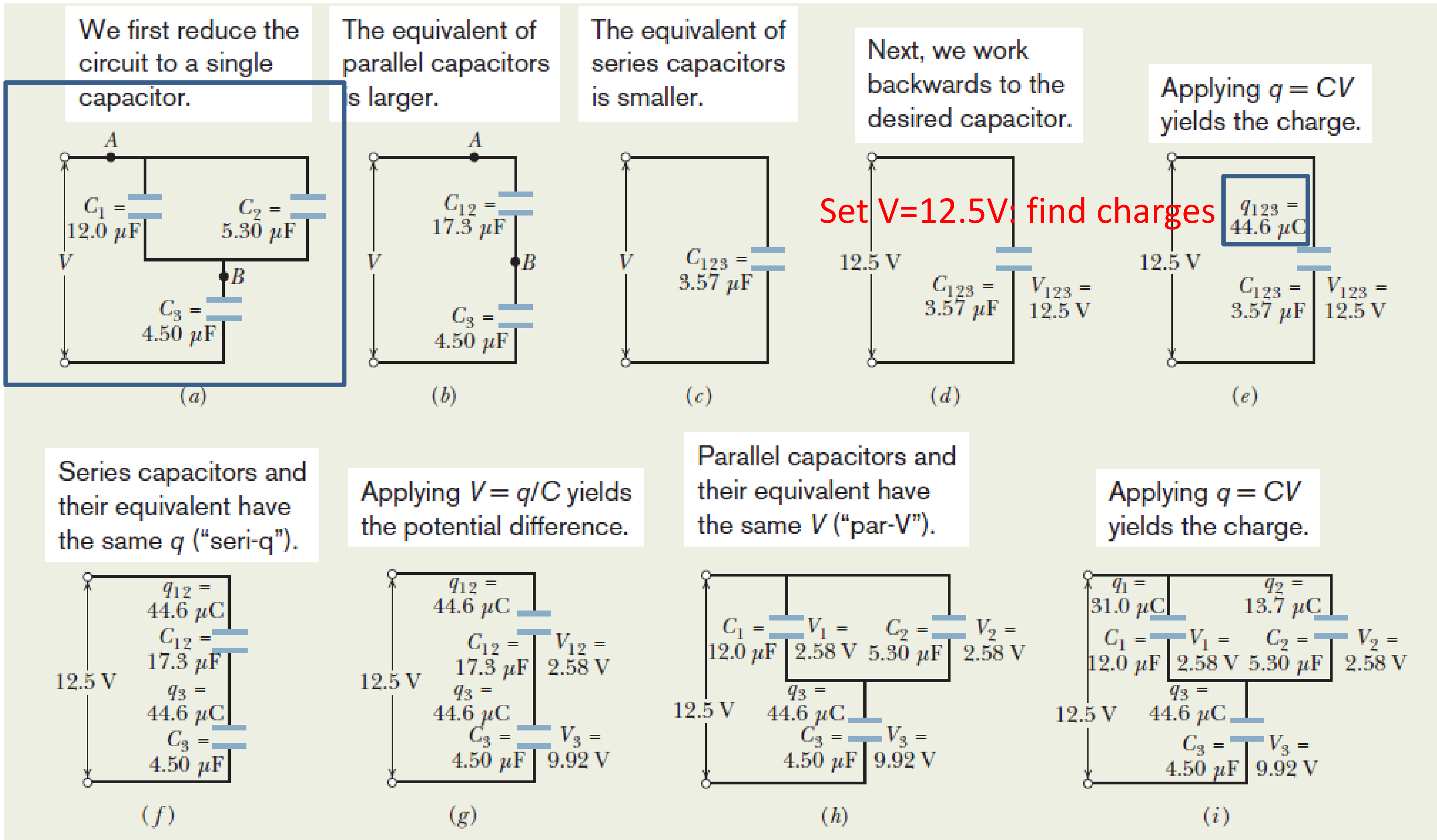
(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu\text{F}, \quad C_2 = 5.30\ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50\ \mu\text{F}.$$

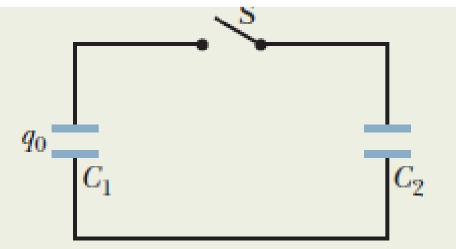
$$C_{12} = C_1 + C_2 = 12.0\ \mu\text{F} + 5.30\ \mu\text{F} = 17.3\ \mu\text{F}.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$= \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}} = 0.280\ \mu\text{F}^{-1},$$



Capacitor 1, with $C_1 = 3.55\ \mu F$, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu F$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.



$$q_0 = C_1 V_0 = (3.55 \times 10^{-6}\ \mathrm{F})\,(6.30\ \mathrm{V}) = 22.365 \times 10^{-6}\ \mathrm{C}.$$

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \qquad \text{(equilibrium).}$$

From Eq. 25-1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \text{(equilibrium).}$$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0 \quad \text{(charge conservation);}$$

thus $$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35\ \mu\mathrm{C}. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\ \mu C$) must be on capacitor 2:

$$q_2 = 16.0\ \mu\mathrm{C}. \qquad \text{(Answer)}$$

Energy stored in a capacitor

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

Work is stored as potential energy of the capacitor

$$U = \frac{q^2}{2C}$$

$$U = \tfrac{1}{2}CV^2$$

The potential energy of a charged capacitor is stored in the electric field between the plates

Energy Density

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

$$C = \varepsilon_0 A/d$$

$$u = \tfrac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2.$$

$$E = -\Delta V/\Delta s$$

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}$$

The electric energy stored per unit volume in any region of space is proportional to the square of the electric filed in that region.

CONCEPTUAL EXAMPLE 24–9 **Capacitor plate separation increased.** A parallel-plate capacitor carries charge Q and is then disconnected from a battery. The two plates are initially separated by a distance d. Suppose the plates are pulled apart until the separation is $2d$. How has the energy stored in this capacitor changed?

RESPONSE If we increase the plate separation d, we decrease the capacitance according to Eq. 24–2, $C = \epsilon_0 A/d$, by a factor of 2. The charge Q hasn't changed. So according to Eq. 24–5, where we choose the form $U = \frac{1}{2}Q^2/C$ because we know Q is the same and C has been halved, the reduced C means the potential energy stored increases by a factor of 2.

NOTE We can see why the energy stored increases from a physical point of view: the two plates are charged equal and opposite, so they attract each other. If we pull them apart, we must do work, so we raise their potential energy.

EXAMPLE 24–1 **Capacitor calculations.** (*a*) Calculate the capacitance of a parallel-plate capacitor whose plates are 20 cm × 3.0 cm and are separated by a 1.0-mm air gap. (*b*) What is the charge on each plate if a 12-V battery is connected across the two plates? (*c*) What is the electric field between the plates? (*d*) Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap *d*.

APPROACH The capacitance is found by using Eq. 24–2, $C = \epsilon_0 A/d$. The charge on each plate is obtained from the definition of capacitance, Eq. 24–1, $Q = CV$. The electric field is uniform, so we can use Eq. 23–4b for the magnitude $E = V/d$. In (*d*) we use Eq. 24–2 again.

SOLUTION (*a*) The area $A = (20 \times 10^{-2}\,\text{m})(3.0 \times 10^{-2}\,\text{m}) = 6.0 \times 10^{-3}\,\text{m}^2$. The capacitance C is then

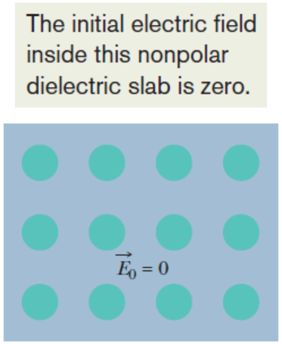
$$C = \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\frac{6.0 \times 10^{-3}\,\text{m}^2}{1.0 \times 10^{-3}\,\text{m}} = 53\,\text{pF}.$$

(*d*) We solve for A in Eq. 24–2 and substitute $C = 1.0\,\text{F}$ and $d = 1.0\,\text{mm}$ to find that we need plates with an area

$$A = \frac{Cd}{\epsilon_0} \approx \frac{(1\,\text{F})(1.0 \times 10^{-3}\,\text{m})}{(9 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} \approx 10^8\,\text{m}^2.$$

NOTE This is the area of a square 10^4 m or 10 km on a side. That is the size of a city like San Francisco or Boston! Large-capacitance capacitors will not be simple parallel plates.

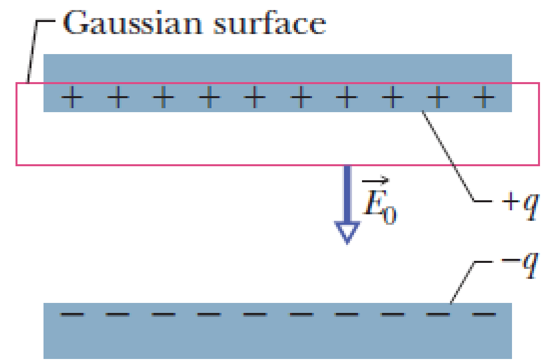
Dielectrics



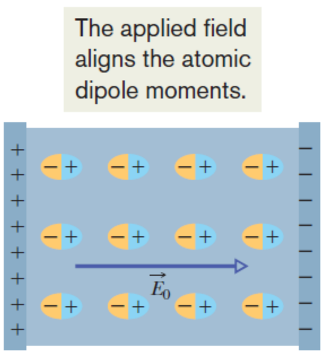
(a)

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q,$$

$$E_0 = \frac{q}{\varepsilon_0 A}.$$



(a)

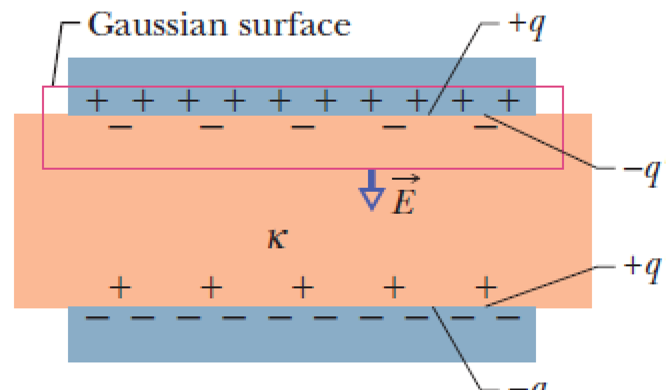


(b)

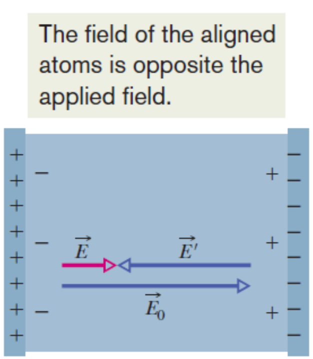
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q - q',$$

$$E = \frac{q - q'}{\varepsilon_0 A}.$$

$$q - q' = \frac{q}{\kappa}.$$



(b)



(c)

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \qquad \text{(Gauss' law with dielectric).}$$

$$C = KC_0,$$

$$C = K\epsilon_0 \frac{A}{d}$$

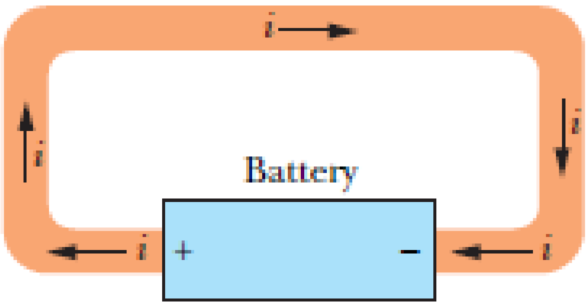
K: dielectric constant

Dielectric strength: the max. E before breakdown occurs

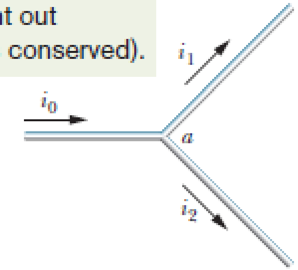
Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

Current and current density

i = dq/dt   1 Ampere = 1 A = 1 coulomb per seco

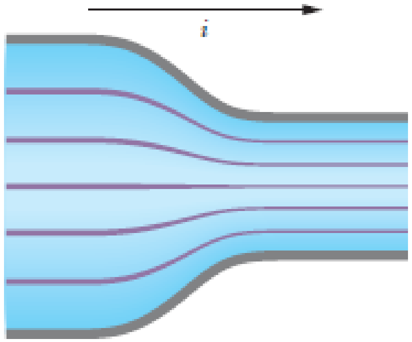


The current into the junction must equal the current out (charge is conserved).



$$i_0 = i_1 + i_2.$$

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.



current density $\vec{J}$, which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. For each element of the cross section, the magnitude J is equal to the current per unit area through that element. We can write the amount of current through the ele-

$$i = \int \vec{J} \cdot d\vec{A}.$$

$$J = \frac{i}{A},$$

Resistance and resistivity

Resisitance: R = V/I

1 Ohm = 1 Ω = 1 volt per ampere

Resisitivity: $\rho = E/J$ [Ωm]

Conductivity: $\sigma = 1/\rho$ [1/Ωm]

$J = \sigma E$ $R = \rho L/A$

Ohm's law is a statement that the current through a device is always directly proportional to the potential difference applied to the device.

Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

Series and Parallel Resistors and Capacitors

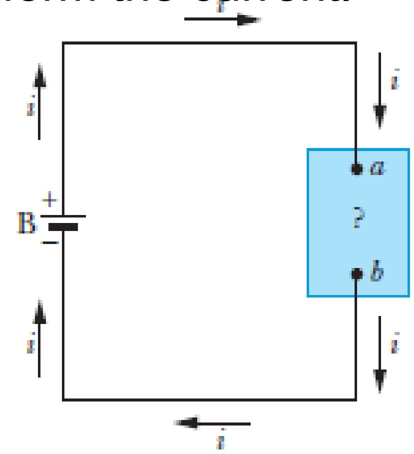
Series	Parallel
Resistors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24
Same current through all resistors	Same potential difference across all resistors

Series	Parallel
Capacitors	
$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same charge on all capacitors	Same potential difference across all capacitors

Power in Electric Circuits

The principle of conservation of energy tells us that the decrease in electric potential energy from *a* to *b* is accompanied by a transfer of energy to some other

The battery at the left supplies energy to the conduction electrons that form the current.



$$P = iV \qquad \text{(rate of electrical energy transfer).}$$

$$1\,\mathrm{V\cdot A} = \left(1\,\frac{\mathrm{J}}{\mathrm{C}}\right)\left(1\,\frac{\mathrm{C}}{\mathrm{s}}\right) = 1\,\frac{\mathrm{J}}{\mathrm{s}} = 1\,\mathrm{W}.$$

Using Ohm´s law

$$P = i^2R \qquad \text{(resistive dissipation)}$$

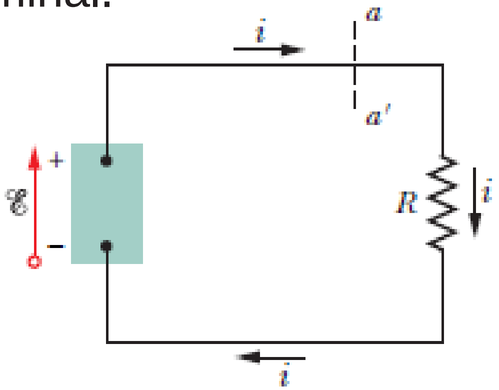
$$P = \frac{V^2}{R} \qquad \text{(resistive dissipation).}$$

In a resistor, electric potential energy is converted to internal thermal energy via collisions between charge carriers and atoms.

Electrical circuits

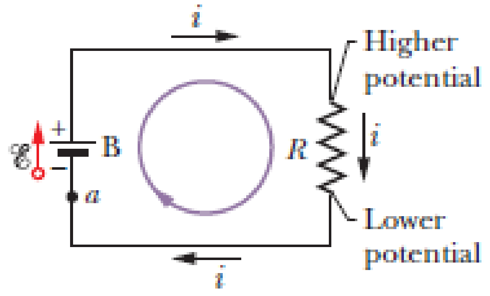
emf of an emf device is the work per unit charge that the device does in moving charge from its low-potential terminal to its high-potential terminal.

$$\mathscr{E} = \frac{dW}{dq} \qquad \text{(definition of } \mathscr{E}\text{).}$$



Single loop circuit

The battery drives current through the resistor, from high potential to low potential.

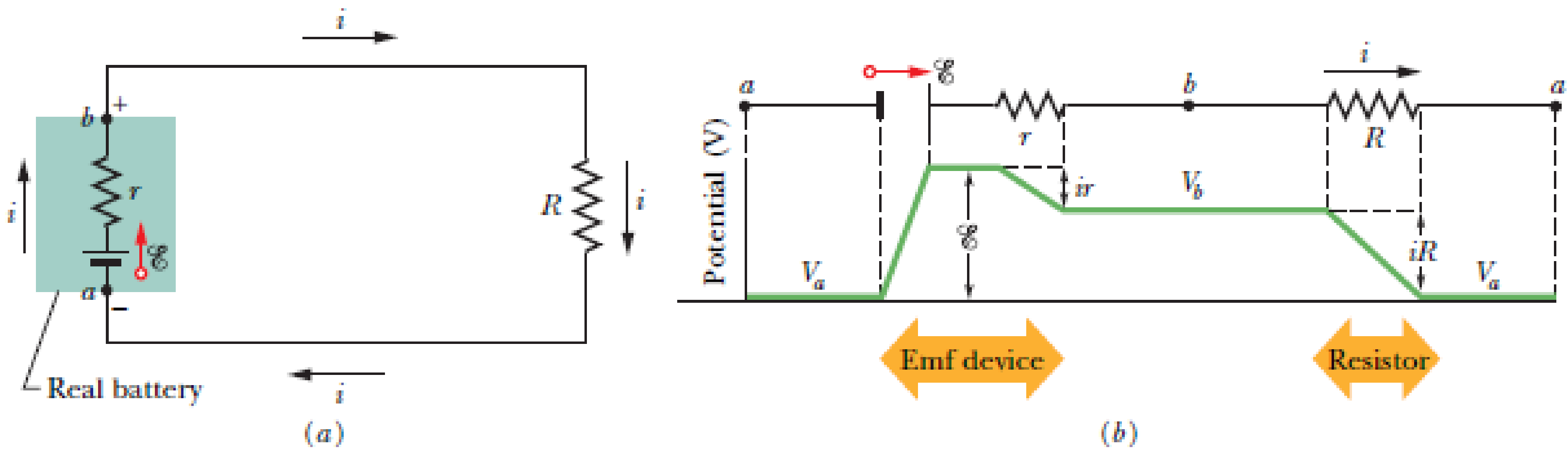


LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

$$V_a + \mathscr{E} - iR = V_a.$$

$$\mathscr{E} - iR = 0.$$

Potential loop of a circuits



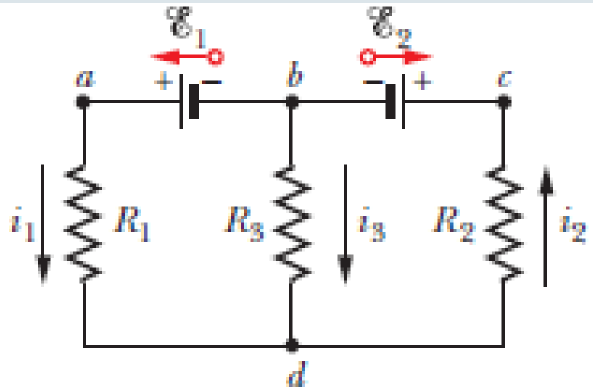
$$\mathscr{E} - ir - iR = 0.$$

$$i = \frac{\mathscr{E}}{R + r}.$$

Multiloop circuits

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

$i_1 + i_3 = i_2$



If we traverse the left-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$$

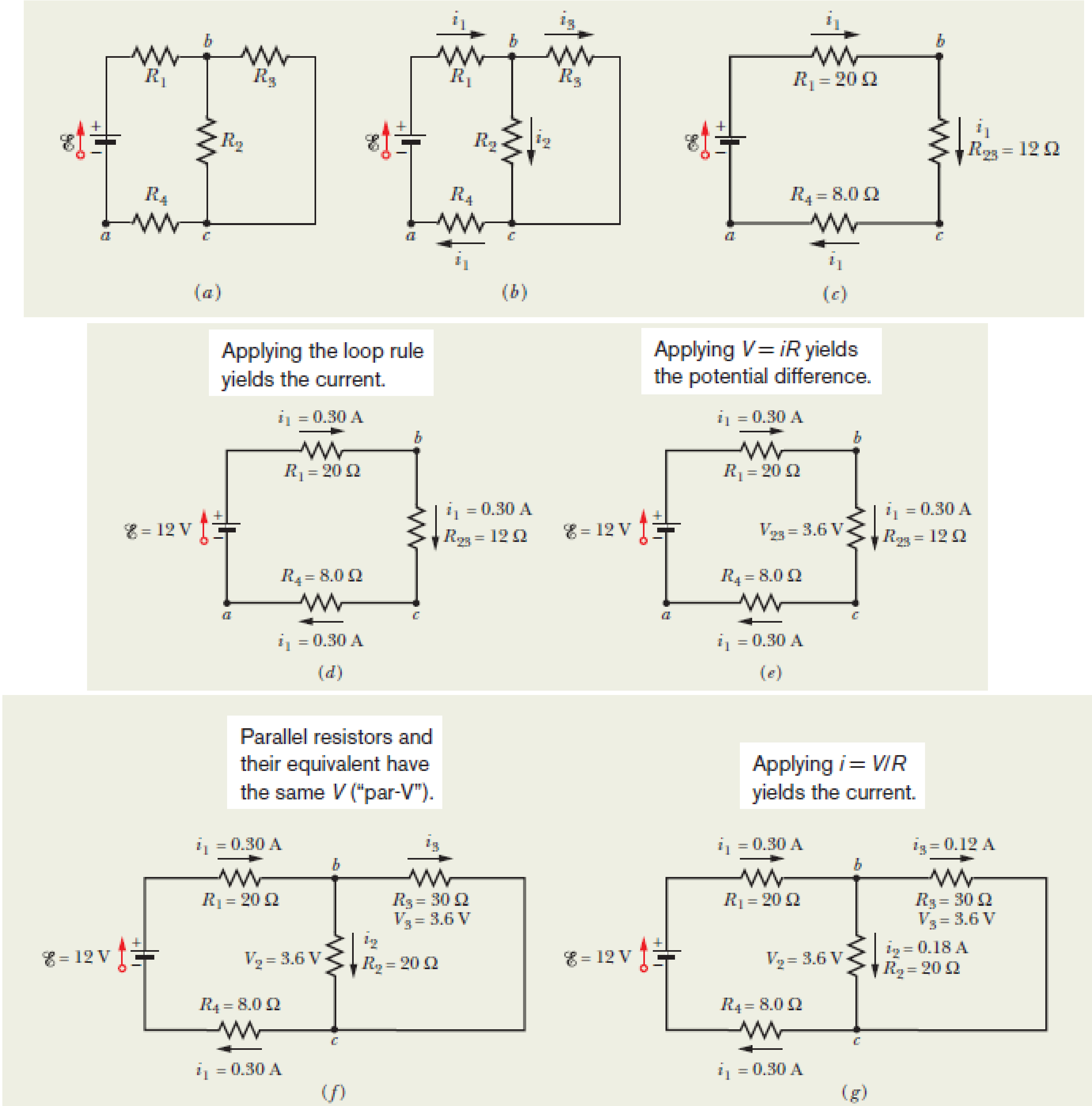
If we traverse the right-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$$

Substitute : i_3R_3

$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

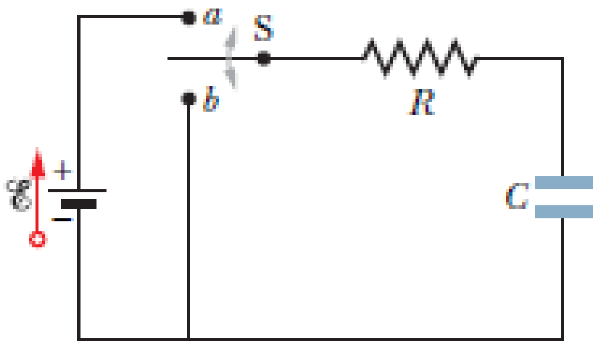
example



R1
R2
R3
R4
b
a
c
(a)
i1
i3
i2
(b)
R1 = 20 Ω
R23 = 12 Ω
R4 = 8.0 Ω
(c)
Applying the loop rule yields the current.
i1 = 0.30 A
ℰ = 12 V
(d)
Applying V = iR yields the potential difference.
V23 = 3.6 V
(e)
Parallel resistors and their equivalent have the same V ("par-V").
R3 = 30 Ω
V3 = 3.6 V
V2 = 3.6 V
R2 = 20 Ω
(f)
Applying i = V/R yields the current.
i3 = 0.12 A
i2 = 0.18 A
(g)

RC - circuits

When switch S is closed on *a*, the capacitor is *charged* through the resistor. When the switch is afterward closed on *b*, the capacitor *discharges* through the resistor..



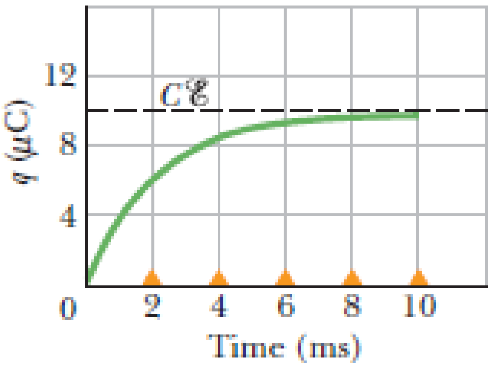
$$\mathscr{E} - iR - \frac{q}{C} = 0.$$

$$R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E} \qquad \text{(charging equation)}.$$

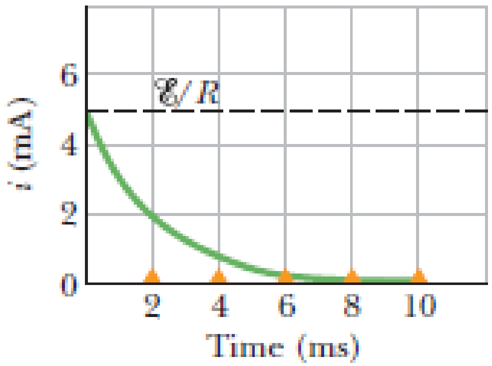
$$q = C\mathscr{E}(1 - e^{-t/RC}) \qquad \text{(charging a capacitor)}.$$

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC} \qquad \text{(charging a capacitor)}.$$

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \qquad \text{(charging a capacitor)}.$$



(*a*)



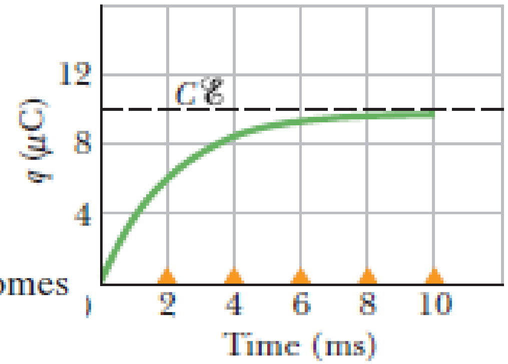
(*b*)

Time constant: τ= RC

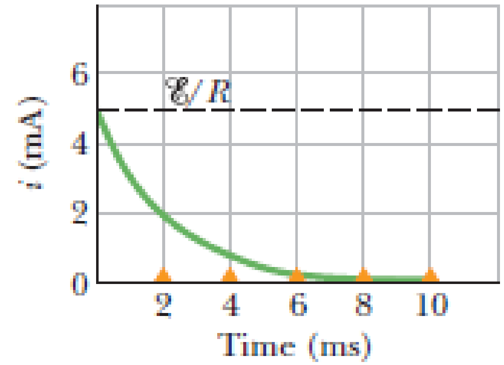
A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \qquad \text{(charging a capacitor).}$$

This tells us that $V_C = 0$ at $t = 0$ and that $V_C = \mathscr{E}$ when the capacitor becomes fully charged as $t \to \infty$.



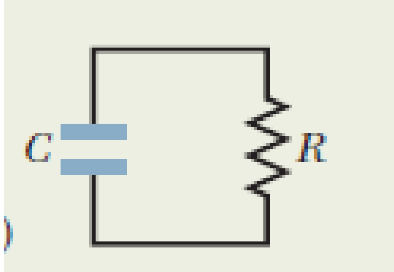
(a)



(b)

Discharging a Capacitor

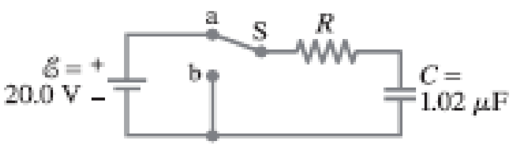
$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad \text{(discharging equation).}$$



$$q = q_0 e^{-t/RC} \qquad \text{(discharging a capacitor),}$$

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \qquad \text{(discharging a capacitor).}$$

FIGURE 26–19 Example 26–12.



EXAMPLE 26–12 Discharging *RC* circuit. In the RC circuit shown in Fig. 26–19, the battery has fully charged the capacitor, so $Q_0 = C\mathscr{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\ \mu\text{F}$. The current I is observed to decrease to 0.50 of its initial value in 40 μs. (*a*) What is the value of Q, the charge on the capacitor, at $t = 0$? (*b*) What is the value of R? (*c*) What is Q at $t = 60\ \mu\text{s}$?

APPROACH At $t = 0$, the capacitor has charge $Q_0 = C\mathscr{E}$, and then the battery is removed from the circuit and the capacitor begins discharging through the resistor, as in Fig. 26–18. At any time t later (Eq. 26–9a) we have

$$Q = Q_0 e^{-t/RC} = C\mathscr{E}e^{-t/RC}.$$

SOLUTION (*a*) At $t = 0$,

$$Q = Q_0 = C\mathscr{E} = (1.02 \times 10^{-6}\ \text{F})(20.0\ \text{V}) = 2.04 \times 10^{-5}\ \text{C} = 20.4\ \mu\text{C}.$$

(*b*) To find R, we are given that at $t = 40\ \mu\text{s}$, $I = 0.50 I_0$. Hence

$$0.50 I_0 = I_0 e^{-t/RC}.$$

Taking natural logs on both sides $(\ln 0.50 = -0.693)$:

$$0.693 = \frac{t}{RC}$$

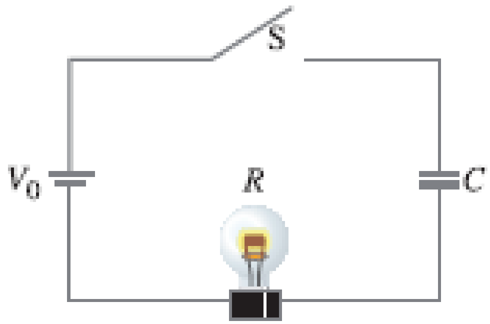
so

$$R = \frac{t}{(0.693)C} = \frac{(40 \times 10^{-6}\ \text{s})}{(0.693)(1.02 \times 10^{-6}\ \text{F})} = 57\ \Omega.$$

(*c*) At $t = 60\ \mu\text{s}$,

$$Q = Q_0 e^{-t/RC} = (20.4 \times 10^{-6}\ \text{C})e^{-\frac{60 \times 10^{-6}\ \text{s}}{(57\ \Omega)(1.02 \times 10^{-6}\ \text{F})}} = 7.3\ \mu\text{C}.$$

CONCEPTUAL EXAMPLE 26–13 **Bulb in *RC* circuit.** In the circuit of Fig. 26–20, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.



RESPONSE When the switch is first closed, the current in the circuit is high and the lightbulb burns brightly. As the capacitor charges, the voltage across the capacitor increases causing the current to be reduced, and the lightbulb dims. As the potential difference across the capacitor approaches the same voltage as the battery, the current decreases toward zero and the lightbulb goes out.

CONCEPTUAL EXAMPLE 25–3 **Current and potential.** Current I enters a resistor R as shown in Fig. 25–10. (*a*) Is the potential higher at point A or at point B? (*b*) Is the current greater at point A or at point B?

RESPONSE (*a*) Positive charge always flows from + to −, from high potential to low potential. Think again of the gravitational analogy: a mass will fall down from high gravitational potential to low. So for positive current I, point A is at a higher potential than point B.

(*b*) Conservation of charge requires that whatever charge flows into the resistor at point A, an equal amount of charge emerges at point B. Charge or current does not get "used up" by a resistor, just as an object that falls through a gravitational potential difference does not gain or lose mass. So the current is the same at A and B.

EXAMPLE 25–10 ESTIMATE Lightning bolt. Lightning is a spectacular example of electric current in a natural phenomenon (Fig. 25–18). There is much variability to lightning bolts, but a typical event can transfer 10^9 J of energy across a potential difference of perhaps 5×10^7 V during a time interval of about 0.2 s. Use this information to estimate (*a*) the total amount of charge transferred between cloud and ground, (*b*) the current in the lightning bolt, and (*c*) the average power delivered over the 0.2 s.

APPROACH We estimate the charge Q, recalling that potential energy change equals the potential difference ΔV times the charge Q, Eq. 23–3. We equate ΔU with the energy transferred, $\Delta U \approx 10^9$ J. Next, the current I is Q/t (Eq. 25–1a) and the power P is energy/time.

SOLUTION (*a*) From Eq. 23–3, the energy transformed is $\Delta U = Q\,\Delta V$. We solve for Q:

$$Q = \frac{\Delta U}{\Delta V} \approx \frac{10^9\ \text{J}}{5 \times 10^7\ \text{V}} = 20\ \text{coulombs}.$$

(*b*) The current during the 0.2 s is about

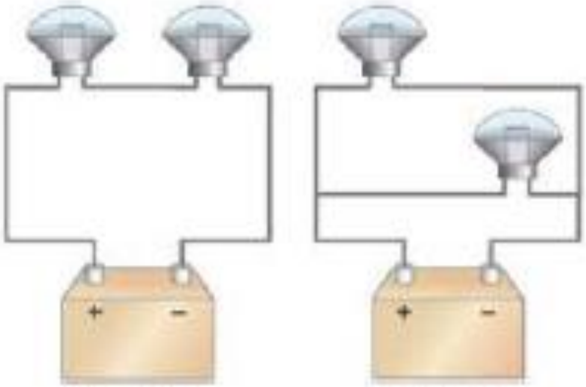
$$I = \frac{Q}{t} \approx \frac{20\ \text{C}}{0.2\ \text{s}} = 100\ \text{A}.$$

(*c*) The average power delivered is

$$P = \frac{\text{energy}}{\text{time}} = \frac{10^9\ \text{J}}{0.2\ \text{s}} = 5 \times 10^9\ \text{W} = 5\ \text{GW}.$$

We can also use Eq. 25–6:

$$P = IV = (100\ \text{A})(5 \times 10^7\ \text{V}) = 5\ \text{GW}.$$



(1) Series (2) Parallel

CONCEPTUAL EXAMPLE 26–2 **Series or parallel?** (*a*) The lightbulbs in Fig. 26–6 are identical. Which configuration produces more light? (*b*) Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.

RESPONSE (*a*) The equivalent resistance of the parallel circuit is found from Eq. 26–4, $1/R_{eq} = 1/R + 1/R = 2/R$. Thus $R_{eq} = R/2$. The parallel combination then has lower resistance ($= R/2$) than the series combination $(R_{eq} = R + R = 2R)$. There will be more total current in the parallel configuration (2), since $I = V/R_{eq}$ and V is the same for both circuits. The total power transformed, which is related to the light produced, is $P = IV$, so the greater current in (2) means more light produced. (*b*) Headlights are wired in parallel (2), because if one bulb goes out, the other bulb can stay lit. If they were in series (1), when one bulb burned out (the filament broke), the circuit would be open and no current would flow, so neither bulb would light.

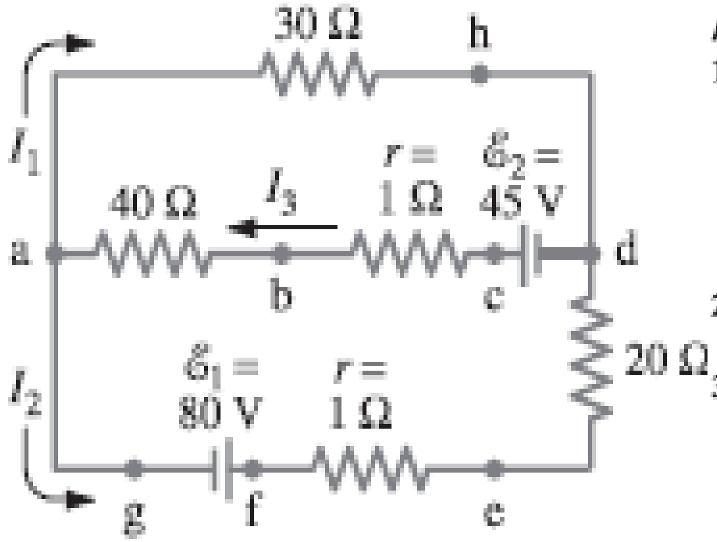


FIGURE 26–13 Currents can be calculated using Kirchhoff's rules. See Example 26–9.

APPROACH AND SOLUTION

1. **Label the currents** and their directions. Figure 26–13 uses the labels I_1, I_2, and I_3 for the current in the three separate branches. Since (positive) current tends to move away from the positive terminal of a battery, we choose I_2 and I_3 to have the directions shown in Fig. 26–13. The direction of I_1 is not obvious in advance, so we arbitrarily chose the direction indicated. If the current actually flows in the opposite direction, our answer will have a negative sign.
2. **Identify the unknowns.** We have three unknowns and therefore we need three equations, which we get by applying Kirchhoff's junction and loop rules.
3. **Junction rule:** We apply Kirchhoff's junction rule to the currents at point a, where I_3 enters and I_2 and I_1 leave:

$$I_3 = I_1 + I_2. \qquad (a)$$

This same equation holds at point d, so we get no new information by writing an equation for point d.

4. **Loop rule:** We apply Kirchhoff's loop rule to two different closed loops. First we apply it to the upper loop ahdcba. We start (and end) at point a. From a to h we have a potential decrease $V_{ha} = -(I_1)(30\,\Omega)$. From h to d there is no change, but from d to c the potential increases by 45 V: that is, $V_{cd} = +45\text{ V}$. From c to a the potential decreases through the two resistances by an amount $V_{ac} = -(I_3)(40\,\Omega + 1\,\Omega) = -(41\,\Omega)I_3$. Thus we have $V_{ha} + V_{cd} + V_{ac} = 0$, or

$$-30I_1 + 45 - 41I_3 = 0, \qquad (b)$$

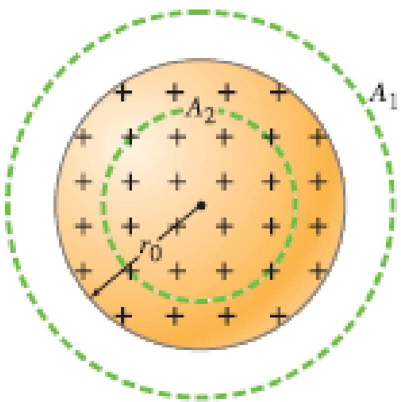
where we have omitted the units (volts and amps) so we can more easily do the algebra. For our second loop, we take the outer loop ahdefga. (We could have chosen the lower loop abcdefga instead.) Again we start at point a and have $V_{ha} = -(I_1)(30\,\Omega)$, and $V_{dh} = 0$. But when we take our positive test charge from d to e, it actually is going uphill, against the current—or at least against the *assumed* direction of the current, which is what counts in this calculation. Thus $V_{ed} = I_2(20\,\Omega)$ has a *positive* sign. Similarly, $V_{fe} = I_2(1\,\Omega)$. From f to g there is a decrease in potential of 80 V since we go from the high potential terminal of the battery to the low. Thus $V_{gf} = -80\text{ V}$. Finally, $V_{ag} = 0$, and the sum of the potential changes around this loop is

$$-30I_1 + (20 + 1)I_2 - 80 = 0. \qquad (c)$$

Our major work is done. The rest is algebra.

EXAMPLE 22–4 **Solid sphere of charge.** An electric charge Q is distributed uniformly throughout a nonconducting sphere of radius r_0, Fig. 22–12. Determine the electric field (a) outside the sphere $(r > r_0)$ and (b) inside the sphere $(r < r_0)$.

APPROACH Since the charge is distributed symmetrically in the sphere, the electric field at all points must again be symmetric. $\vec{\mathbf{E}}$ depends only on r and is directed radially outward (or inward if $Q < 0$).



SOLUTION (a) For our gaussian surface we choose a sphere of radius r $(r > r_0)$, labeled A_1 in Fig. 22–12. Since E depends only on r, Gauss's law gives, with $Q_{encl} = Q$,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

or

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}.$$

Again, the field outside a spherically symmetric distribution of charge is the same as that for a point charge of the same magnitude located at the center of the sphere.

(b) Inside the sphere, we choose for our gaussian surface a concentric sphere of radius r $(r < r_0)$, labeled A_2 in Fig. 22–12. From symmetry, the magnitude of $\vec{\mathbf{E}}$ is the same at all points on A_2, and $\vec{\mathbf{E}}$ is perpendicular to the surface, so

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(4\pi r^2).$$

We must equate this to Q_{encl}/ϵ_0 where Q_{encl} is the charge enclosed by A_2. Q_{encl} is not the total charge Q but only a portion of it. We define the **charge density**, ρ_E, as the charge per unit volume $(\rho_E = dQ/dV)$, and here we are given that ρ_E = constant. So the charge enclosed by the gaussian surface A_2, a sphere of radius r, is

$$Q_{encl} = \left(\frac{\frac{4}{3}\pi r^3 \rho_E}{\frac{4}{3}\pi r_0^3 \rho_E}\right) Q = \frac{r^3}{r_0^3} Q.$$

Hence, from Gauss's law,

$$E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0} = \frac{r^3}{r_0^3}\frac{Q}{\epsilon_0}$$

or

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^3}r. \qquad [r < r_0]$$