Physics 1

UNIVERSITÄT SIEGEN
FESTKÖRPERPHYSIK

Lecture 4a: oscillations and waves

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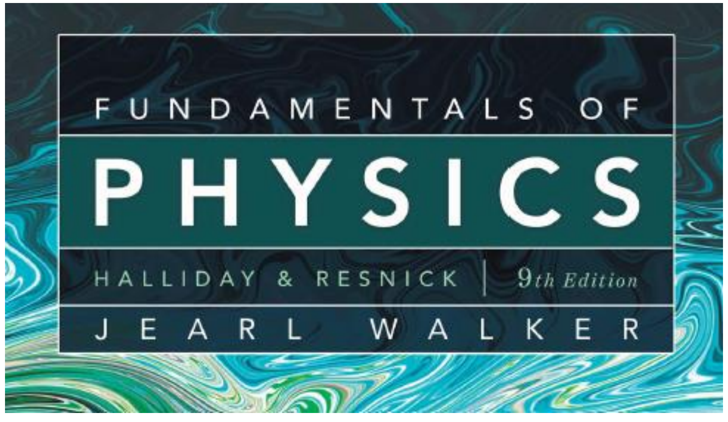
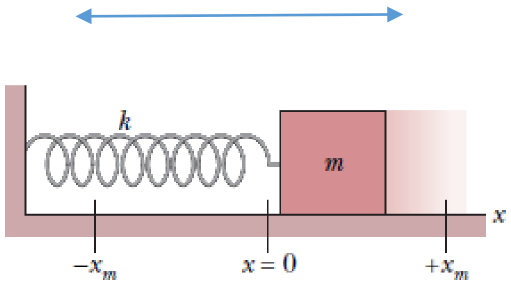
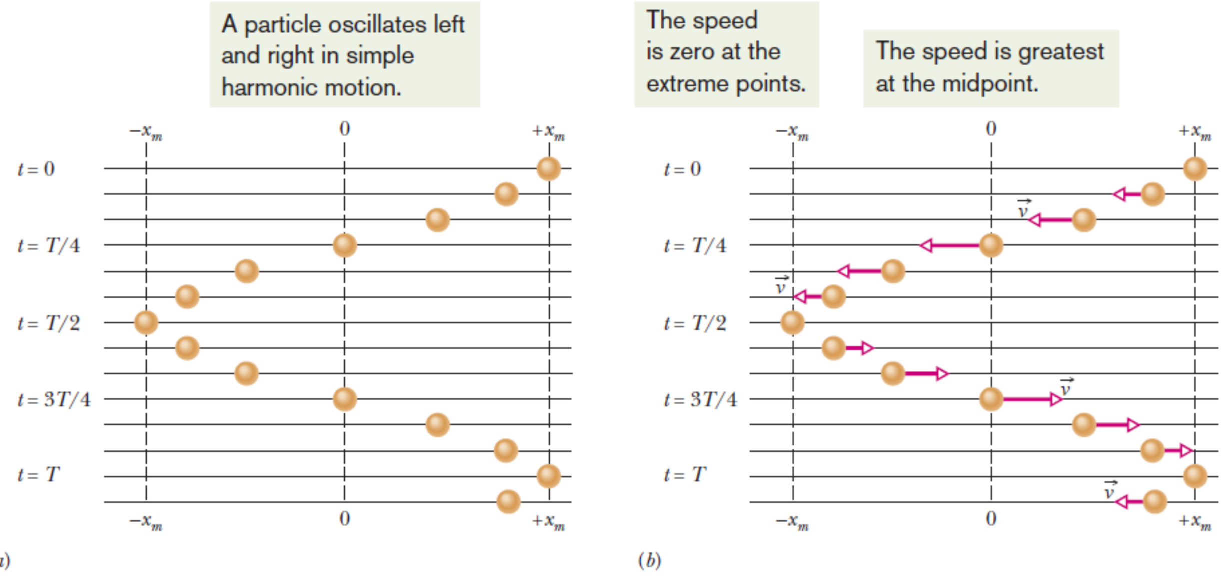
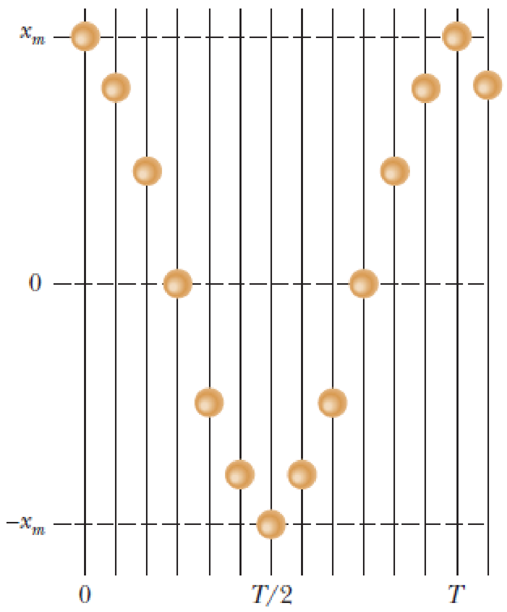
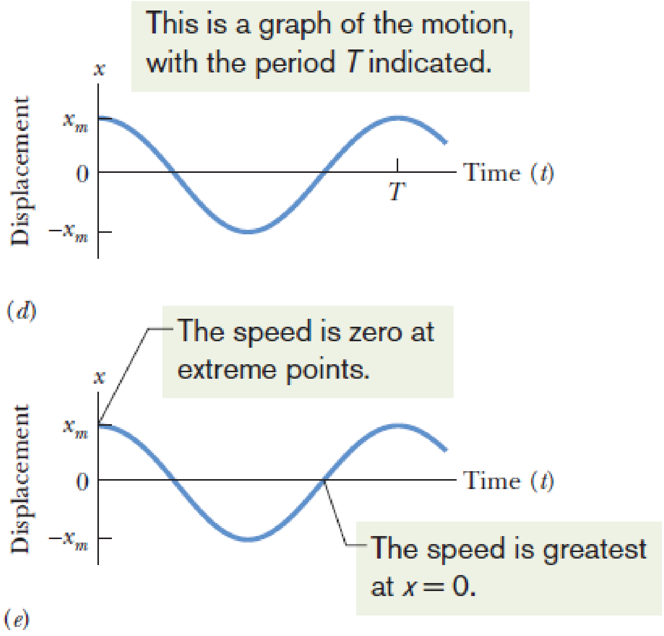


Simple Harmonic Motion









$$1 \text{ hertz} = 1 \text{ Hz} = 1 \text{ oscillation per second} = 1 \text{ s}^{-1}.$$

$$T = \frac{1}{f}.$$

Period T

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

$$x(t) = x_m \cos(\omega t + \phi) \qquad \text{(displacement)},$$

$$x(t) = x_m \cos(\omega t + \phi)$$

Displacemnt at time t — $x(t)$; Amplitude — x_m; Phase — $\omega t + \phi$; Angular frequency — ω; Time — t; Phase constant or phase angle — ϕ

$$x_m \cos \omega t = x_m \cos \omega(t + T).$$

$$\omega(t + T) = \omega t + 2\pi$$

$$\omega T = 2\pi.$$

$$T = 2\pi/\omega$$

amplitude

$$x(t) = x_m \cos(\omega t + \phi) \qquad \text{(displacement)},$$

velocity

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$$

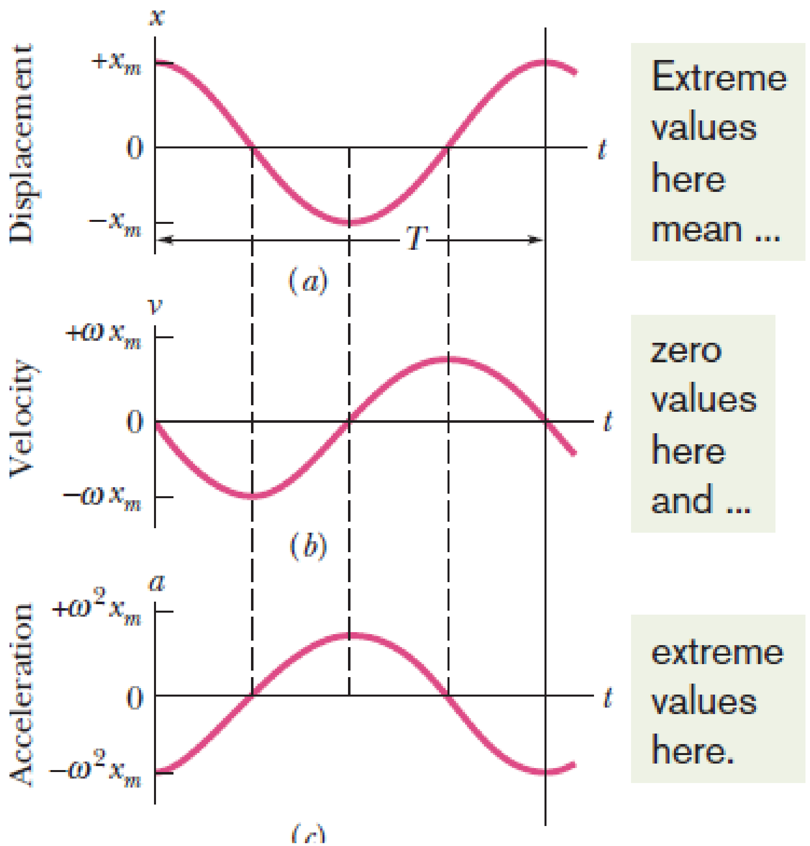
$$v(t) = -\omega x_m \sin(\omega t + \phi) \qquad \text{(velocity)}.$$

acceleration

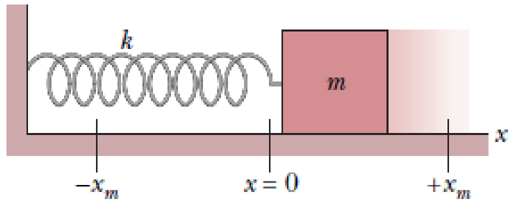
$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}[-\omega x_m \sin(\omega t + \phi)]$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi) \qquad \text{(acceleration)}.$$

$$a(t) = -\omega^2 x(t),$$



Harmonic oscillation of a spring



$$F = -kx,$$

$$F = ma = -(m\omega^2)x.$$

$$k = m\omega^2. \qquad a(t) = -\omega^2 x(t),$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{(angular frequency).}$$

$T=2\pi/\omega$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{(period).}$$

Solution via differential equation

$F = - kx$

$ma + kx = 0$ $\qquad a = d^2x/dt^2$

$m\, d^2x/dt^2 + k\, x = 0$

$d^2x/dt^2 + k/m\, x = 0$

Ansatz: $x(t) = x_0 \cos(\omega t)$

$d^2x/dt^2 = -\omega^2 x$

$-\omega^2 x + k/m\, x = 0$ $\qquad \rightarrow \omega^2 = k/m$

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

KEY IDEA

The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by Eq. 15-12:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65 \text{ N/m}}{0.68 \text{ kg}}} = 9.78 \text{ rad/s}$$
$$\approx 9.8 \text{ rad/s.} \qquad \text{(Answer)}$$

The frequency follows from Eq. 15-5, which yields

$$f = \frac{\omega}{2\pi} = \frac{9.78 \text{ rad/s}}{2\pi \text{ rad}} = 1.56 \text{ Hz} \approx 1.6 \text{ Hz.} \qquad \text{(Answer)}$$

The period follows from Eq. 15-2, which yields

$$T = \frac{1}{f} = \frac{1}{1.56 \text{ Hz}} = 0.64 \text{ s} = 640 \text{ ms.} \qquad \text{(Answer)}$$

(b) What is the amplitude of the oscillation?

KEY IDEA

With no friction involved, the mechanical energy of the spring–block system is conserved.

Reasoning: The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system at a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm:

$$x_m = 11 \text{ cm.} \qquad \text{(Answer)}$$

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

KEY IDEA

The maximum speed v_m is the velocity amplitude ωx_m in Eq. 15-6.

Calculation: Thus, we have

$$v_m = \omega x_m = (9.78 \text{ rad/s})(0.11 \text{ m})$$
$$= 1.1 \text{ m/s.} \qquad \text{(Answer)}$$

This maximum speed occurs when the oscillating block is rushing through the origin; compare Figs. 15-4*a* and 15-4*b*, where you can see that the speed is a maximum whenever $x = 0$.

(d) What is the magnitude a_m of the maximum acceleration of the block?

KEY IDEA

The magnitude a_m of the maximum acceleration is the acceleration amplitude $\omega^2 x_m$ in Eq. 15-7.

Calculation: So, we have

$$a_m = \omega^2 x_m = (9.78 \text{ rad/s})^2(0.11 \text{ m})$$
$$= 11 \text{ m/s}^2. \qquad \text{(Answer)}$$

This maximum acceleration occurs when the block is at the ends of its path. At those points, the force acting on the block has its maximum magnitude; compare Figs. 15-4*a* and 15-4*c*, where you can see that the magnitudes of the displacement and acceleration are maximum at the same times.

(e) What is the phase constant ϕ for the motion?

Calculations: Equation 15-3 gives the displacement of the block as a function of time. We know that at time $t = 0$, the block is located at $x = x_m$. Substituting these *initial conditions,* as they are called, into Eq. 15-3 and canceling x_m give us

$$1 = \cos \phi. \qquad (15\text{-}14)$$

Taking the inverse cosine then yields

$$\phi = 0 \text{ rad.} \qquad \text{(Answer)}$$

(Any angle that is an integer multiple of 2π rad also satisfies Eq. 15-14; we chose the smallest angle.)

(f) What is the displacement function $x(t)$ for the spring–block system?

Calculation: The function $x(t)$ is given in general form by Eq. 15-3. Substituting known quantities into that equation gives us

$$x(t) = x_m \cos(\omega t + \phi)$$
$$= (0.11 \text{ m}) \cos[(9.8 \text{ rad/s})t + 0]$$
$$= 0.11 \cos(9.8t), \qquad \text{(Answer)}$$

where x is in meters and t is in seconds.

At $t = 0$, the displacement $x(0)$ of the block in a linear oscillator like that of Fig. 15-5 is -8.50 cm. (Read $x(0)$ as "x at time zero.") The block's velocity $v(0)$ then is -0.920 m/s, and its acceleration $a(0)$ is $+47.0\ \text{m/s}^2$.

(a) What is the angular frequency ω of this system?

KEY IDEA

With the block in SHM, Eqs. 15-3, 15-6, and 15-7 give its displacement, velocity, and acceleration, respectively, and each contains ω.

Calculations: Let's substitute $t = 0$ into each to see whether we can solve any one of them for ω. We find

$$x(0) = x_m \cos \phi, \qquad (15\text{-}15)$$

$$v(0) = -\omega x_m \sin \phi, \qquad (15\text{-}16)$$

and

$$a(0) = -\omega^2 x_m \cos \phi. \qquad (15\text{-}17)$$

In Eq. 15-15, ω has disappeared. In Eqs. 15-16 and 15-17, we know values for the left sides, but we do not know x_m and ϕ. However, if we divide Eq. 15-17 by Eq. 15-15, we neatly eliminate both x_m and ϕ and can then solve for ω as

$$\omega = \sqrt{-\frac{a(0)}{x(0)}} = \sqrt{-\frac{47.0\ \text{m/s}^2}{-0.0850\ \text{m}}}$$

$$= 23.5\ \text{rad/s}. \qquad \text{(Answer)}$$

(b) What are the phase constant ϕ and amplitude x_m?

Calculations: We know ω and want ϕ and x_m. If we divide Eq. 15-16 by Eq. 15-15, we eliminate one of those unknowns and reduce the other to a single trig function:

$$\frac{v(0)}{x(0)} = \frac{-\omega x_m \sin \phi}{x_m \cos \phi} = -\omega \tan \phi.$$

Solving for $\tan \phi$, we find

$$\tan \phi = -\frac{v(0)}{\omega x(0)} = -\frac{-0.920\ \text{m/s}}{(23.5\ \text{rad/s})(-0.0850\ \text{m})}$$

$$= -0.461.$$

This equation has two solutions:

$$\phi = -25^\circ \quad \text{and} \quad \phi = 180^\circ + (-25^\circ) = 155^\circ.$$

Normally only the first solution here is displayed by a calculator, but it may not be the physically possible solution. To choose the proper solution, we test them both by using them to compute values for the amplitude x_m. From Eq. 15-15, we find that if $\phi = -25^\circ$, then

$$x_m = \frac{x(0)}{\cos \phi} = \frac{-0.0850\ \text{m}}{\cos(-25^\circ)} = -0.094\ \text{m}.$$

We find similarly that if $\phi = 155^\circ$, then $x_m = 0.094$ m. Because the amplitude of SHM must be a positive constant, the correct phase constant and amplitude here are

$$\phi = 155^\circ \quad \text{and} \quad x_m = 0.094\ \text{m} = 9.4\ \text{cm}. \qquad \text{(Answer)}$$

Energy in Simple Harmonic Motion

Potential energy of a spring, → x(t)

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

Kinetic energy of a spring → v(t) = ω x(t)

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t + \phi).$$

$\omega^2 = k/m$

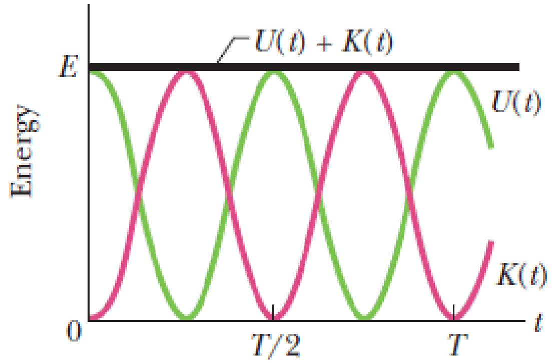
$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi).$$

$$\begin{aligned} E &= U + K \\ &= \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi) + \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi) \\ &= \tfrac{1}{2}kx_m^2 [\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)]. \end{aligned}$$

$\sin^2 a + \cos^2 a = 1$

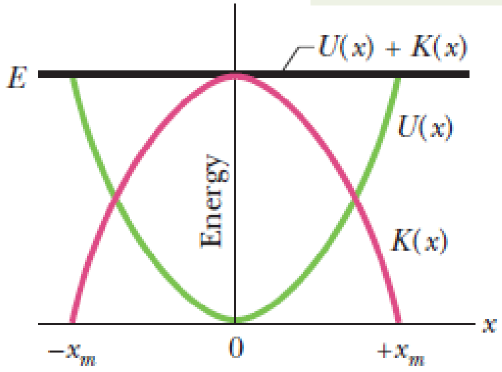
Total energy of a spring

$$E = U + K = \tfrac{1}{2}kx_m^2.$$



(a)

As *time* changes, the energy shifts between the two types, but the total is constant.



(b)

As *position* changes, the energy shifts between the two types, but the total is constant.

SHM potential energy, kinetic energy, mass dampers

Many tall buildings have *mass dampers*, which are anti-sway devices to prevent them from oscillating in a wind. The device might be a block oscillating at the end of a spring and on a lubricated track. If the building sways, say, eastward, the block also moves eastward but delayed enough so that when it finally moves, the building is then moving back westward. Thus, the motion of the oscillator is out of step with the motion of the building.

Suppose the block has mass $m = 2.72 \times 10^5$ kg and is designed to oscillate at frequency $f = 10.0$ Hz and with amplitude $x_m = 20.0$ cm.

(a) What is the total mechanical energy E of the spring–block system?

KEY IDEA

The mechanical energy E (the sum of the kinetic energy $K = \frac{1}{2}mv^2$ of the block and the potential energy $U = \frac{1}{2}kx^2$ of the spring) is constant throughout the motion of the oscillator. Thus, we can evaluate E at any point during the motion.

Calculations: Because we are given amplitude x_m of the oscillations, let's evaluate E when the block is at position $x = x_m$, where it has velocity $v = 0$. However, to evaluate U at that point, we first need to find the spring constant k. From Eq. 15-12 ($\omega = \sqrt{k/m}$) and Eq. 15-5 ($\omega = 2\pi f$), we find

$$\begin{aligned} k &= m\omega^2 = m(2\pi f)^2 \\ &= (2.72 \times 10^5 \text{ kg})(2\pi)^2(10.0 \text{ Hz})^2 \\ &= 1.073 \times 10^9 \text{ N/m}. \end{aligned}$$

We can now evaluate E as

$$\begin{aligned} E &= K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \\ &= 0 + \tfrac{1}{2}(1.073 \times 10^9 \text{ N/m})(0.20 \text{ m})^2 \\ &= 2.147 \times 10^7 \text{ J} \approx 2.1 \times 10^7 \text{ J}. \qquad \text{(Answer)} \end{aligned}$$

(b) What is the block's speed as it passes through the equilibrium point?

Calculations: We want the speed at $x = 0$, where the potential energy is $U = \frac{1}{2}kx^2 = 0$ and the mechanical energy is entirely kinetic energy. So, we can write

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

$$2.147 \times 10^7 \text{ J} = \tfrac{1}{2}(2.72 \times 10^5 \text{ kg})v^2 + 0,$$

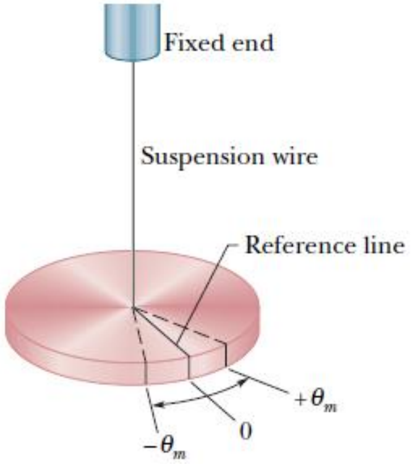
or

$$v = 12.6 \text{ m/s}. \qquad \text{(Answer)}$$

Because E is entirely kinetic energy, this is the maximum speed v_m.

Other oscillators

torsion pendulum,

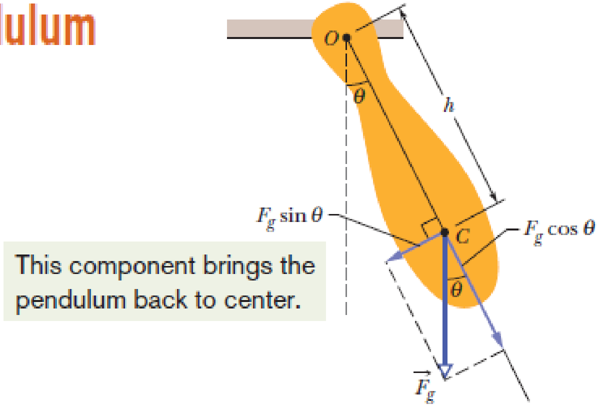


torque

$$\tau = -\kappa\theta.$$

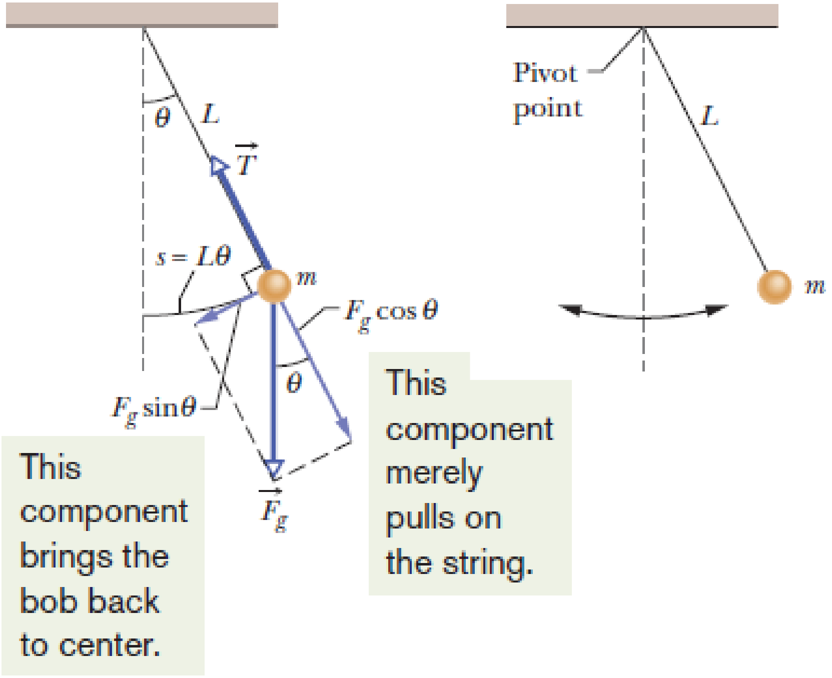
$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad \text{(torsion pendulum).}$$

The Physical Pendulum



$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

The Simple Pendulum



torque

$$(\tau = r_\perp F),$$

$$\tau = -L(F_g \sin\theta), \qquad (\tau = I\alpha)$$

$$-L(mg\sin\theta) = I\alpha,$$

$$\alpha = -\frac{mgL}{I}\theta. \qquad \alpha=\omega^2\,\theta$$

$$a(t) = -\omega^2 x(t),$$

$$T = 2\pi\sqrt{\frac{I}{mgL}}. \qquad T=2\pi/\omega \qquad I = mL^2$$

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{(simple pendulum,}$$

In Fig. 15-11*a*, a meter stick swings about a pivot point at one end, at distance h from the stick's center of mass.

(a) What is the period of oscillation T?

KEY IDEA

The stick is not a simple pendulum because its mass is not concentrated in a bob at the end opposite the pivot point—so the stick is a physical pendulum.

Calculations: The period for a physical pendulum is given by Eq. 15-29, for which we need the rotational inertia I of the stick about the pivot point. We can treat the stick as a uniform rod of length L and mass m. Then Eq. 15-30 tells us that $I = \frac{1}{3}mL^2$, and the distance h in Eq. 15-29 is $\frac{1}{2}L$. Substituting these quantities into Eq. 15-29, we find

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg(\frac{1}{2}L)}} \tag{15-32}$$

$$= 2\pi\sqrt{\frac{2L}{3g}} \tag{15-33}$$

$$= 2\pi\sqrt{\frac{(2)(1.00\text{ m})}{(3)(9.8\text{ m/s}^2)}} = 1.64\text{ s.} \quad \text{(Answer)}$$

Note the result is independent of the pendulum's mass m.

(b) What is the distance L_0 between the pivot point O of the stick and the center of oscillation of the stick?

Calculations: We want the length L_0 of the simple pendulum (drawn in Fig. 15-11*b*) that has the same period as the physical pendulum (the stick) of Fig. 15-11*a*. Setting Eqs. 15-28 and 15-33 equal yields

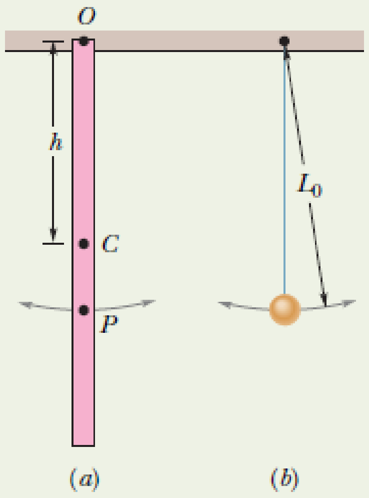


Fig. 15-11 (*a*) A meter stick suspended from one end as a physical pendulum. (*b*) A simple pendulum whose length L_0 is chosen so that the periods of the two pendulums are equal. Point P on the pendulum of (*a*) marks the center of oscillation.

$$T = 2\pi\sqrt{\frac{L_0}{g}} = 2\pi\sqrt{\frac{2L}{3g}}. \tag{15-34}$$

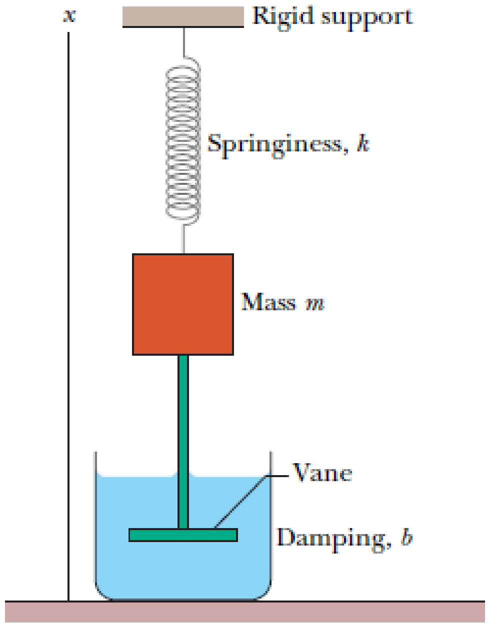
You can see by inspection that

$$L_0 = \tfrac{2}{3}L \tag{15-35}$$

$$= (\tfrac{2}{3})(100\text{ cm}) = 66.7\text{ cm.} \quad \text{(Answer)}$$

In Fig. 15-11*a*, point P marks this distance from suspension point O. Thus, point P is the stick's center of oscillation for the given suspension point. Point P would be different for a different suspension choice.

Damped Simple Harmonic Motion



Damping force

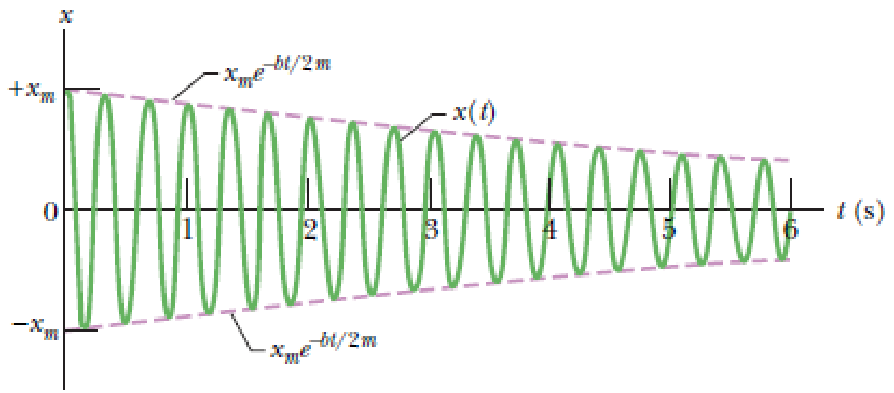
$$F_d = -bv,$$

$$-bv - kx = ma.$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi),$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$



Ansatz: $x(t) = x_m e^{i(\omega t+\varphi)}$

$$\frac{d^2x}{dt^2} = -\omega^2 x_m e^{i(\omega t+\varphi)}$$

$$-m\omega^2 x - i\omega b x + kx = 0$$

$$\omega^2 - i\omega\frac{b}{m} + \frac{k}{m} = 0 \qquad \omega = \frac{ib}{2m} \pm \sqrt{\frac{ib}{2m}^2 + \frac{k}{m}}$$

$$x(t) = x_m e^{i\left(\frac{ibt}{2m} \pm \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)}t + \varphi\right)} = x_m e^{\frac{-b^2 t}{2m}} e^{i(\omega t+\varphi)}$$

$$x(t) = x_m e^{\frac{-bt}{2m}} \cos(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

For the damped oscillator of Fig. 15-14, $m = 250$ g, $k = 85$ N/m, and $b = 70$ g/s.

(a) What is the period of the motion?

KEY IDEA

Because $b \ll \sqrt{km} = 4.6$ kg/s, the period is approximately that of the undamped oscillator.

Calculation: From Eq. 15-13, we then have

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25\text{ kg}}{85\text{ N/m}}} = 0.34\text{ s.} \qquad \text{(Answer)}$$

(b) How long does it take for the amplitude of the damped oscillations to drop to half its initial value?

KEY IDEA

The amplitude at time t is displayed in Eq. 15-42 as $x_m e^{-bt/2m}$.

Calculations: The amplitude has the value x_m at $t = 0$. Thus, we must find the value of t for which

$$x_m e^{-bt/2m} = \tfrac{1}{2}x_m.$$

Canceling x_m and taking the natural logarithm of the equation that remains, we have $\ln\frac{1}{2}$ on the right side and

$$\ln(e^{-bt/2m}) = -bt/2m$$

on the left side. Thus,

$$t = \frac{-2m\ln\frac{1}{2}}{b} = \frac{-(2)(0.25\text{ kg})(\ln\frac{1}{2})}{0.070\text{ kg/s}}$$
$$= 5.0\text{ s.} \qquad \text{(Answer)}$$

Because $T = 0.34$ s, this is about 15 periods of oscillation.

(c) How long does it take for the mechanical energy to drop to one-half its initial value?

KEY IDEA

From Eq. 15-44, the mechanical energy at time t is $\frac{1}{2}kx_m^2 e^{-bt/m}$.

Calculations: The mechanical energy has the value $\frac{1}{2}kx_m^2$ at $t = 0$. Thus, we must find the value of t for which

$$\tfrac{1}{2}kx_m^2 e^{-bt/m} = \tfrac{1}{2}(\tfrac{1}{2}kx_m^2).$$

If we divide both sides of this equation by $\frac{1}{2}kx_m^2$ and solve for t as we did above, we find

$$t = \frac{-m\ln\frac{1}{2}}{b} = \frac{-(0.25\text{ kg})(\ln\frac{1}{2})}{0.070\text{ kg/s}} = 2.5\text{ s.} \qquad \text{(Answer)}$$

This is exactly half the time we calculated in (b), or about 7.5 periods of oscillation. Figure 15-15 was drawn to illustrate this sample problem.

Forced Oscillations and Resonance

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega^2 x = f cos(\omega_c t) \qquad x(t) = x(t)_{hom} + x(t)_{inhom}$$

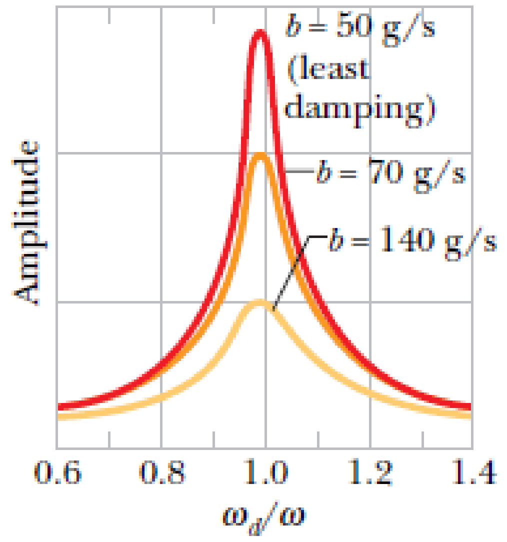
$$x(t)_{inhom} = X_m cos(\omega_e t + \varphi] = X_m[cos(\varphi)cos(\omega_e t) - sin(\varphi)sin(\omega_e t)]$$

$$\frac{d^2 x_{inhom}}{dt^2} = -X_m\omega^2[cos(\varphi)cos(\omega_e t) - sin(\varphi)sin(\omega_e t)]$$

$$cos(\omega_e t)A + sin(\omega_e t)B = 0$$

$$A = X_m[(\omega^2 - \omega_e^2)cos(\varphi) - 2\delta_e sin(\varphi)] - f$$

$$B = X_m[(\omega_e^2 - \omega^2)sin(\varphi) - 2\delta_e cos(\varphi)]$$



A and B have to be zero, using A=0 or B=0 we get

$$X_m = \frac{f}{(\omega^2 - \omega_e^2)cos(\varphi) - 2\delta\omega_e sin(\varphi)} \rightarrow X_m = \frac{f}{\sqrt{(\omega^2 - \omega_e^2)^2 + 4\delta^2\omega_e}}$$

Complete solution

$$x(t) = x(t)_{hom} + x(t)_{inhom} = x_m e^{-\delta t}cos(\omega t + \varphi) + X_m cos(\omega_e t + \varphi)$$

For t >> $1/\delta$ $\qquad x(t) = x(t)_{inhom} = X_m cos(\omega_e t + \varphi)$

For δ <>0 → X = X_{max} for $\omega = \omega_e$ Resonance !!!

For δ =0 → X=∞ divergence !!!

Physics 1

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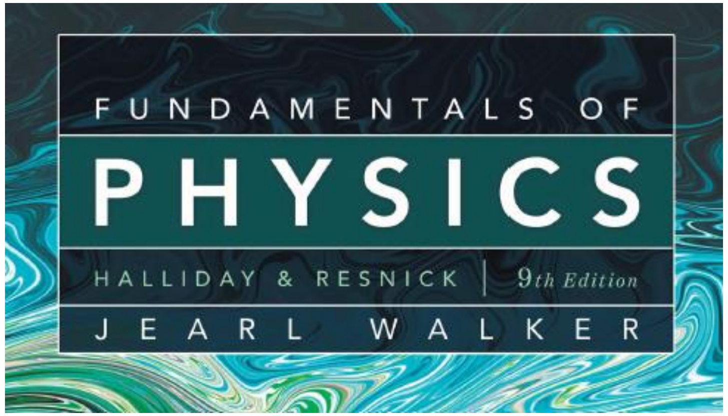
Lecture 4b: waves

Prof. Dr. U. Pietsch



Name : Matr nr.

Physics 1 for Nanoscience & Nanotechnology

Level of knowledge 2 06.11.19

1. Give the relation between angular velocity and trangential velocity ? Give a relation for kinetik energy for translation and for rotation ?

2. Give expression for angular force (torque) and angular momentum. For which quantity yields the law of conservation ?.

3. Give the relation between Force and potential energy

4. What characterized a „conservative force“ ?

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1. Give the relation between angular velocity and trangential velocity ? Gvie a relation for kinetik energy of translation and for rotation ?

$V = \omega r$ $K = \frac{1}{2} m v^2$ $K = \frac{1}{2} \Theta \omega^2$

2. Give expression for angular force (torque) and angular momentum. For which quantity yields the law of conservation ?.

$\vec{\tau} = \vec{r} \times \vec{F}$ $\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ conserved

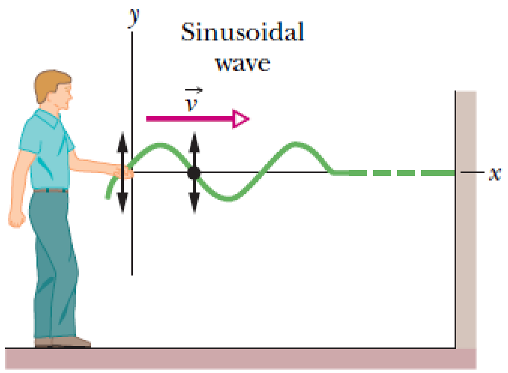
3. Give the relation between Force and potential energy

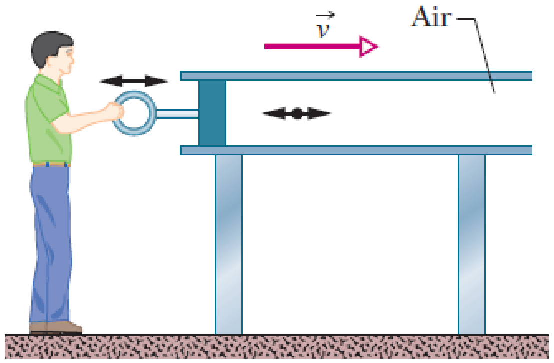
$$F(x) = -\frac{dU(x)}{dx}$$

4. What characterized a „conservative force“ ?

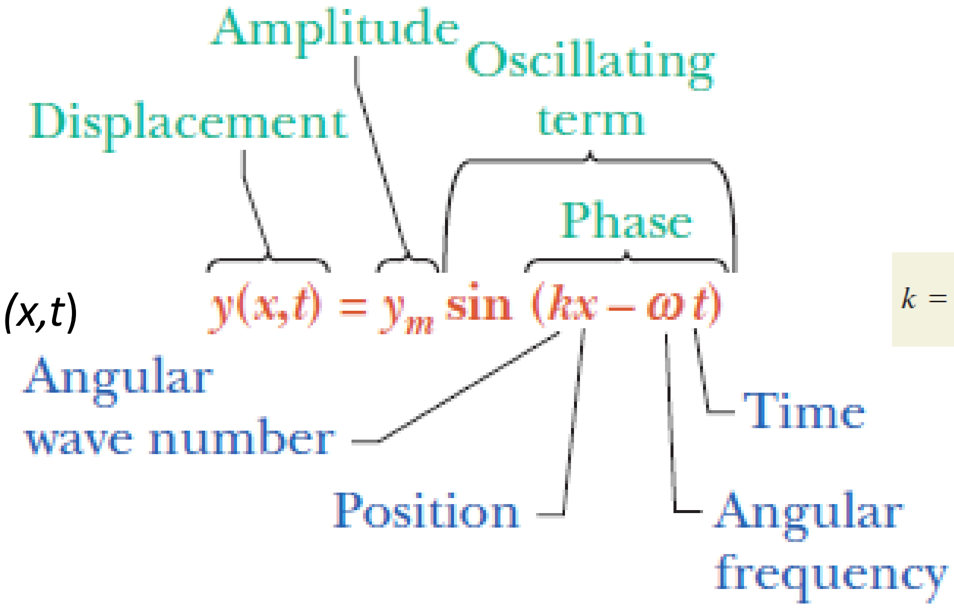
The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

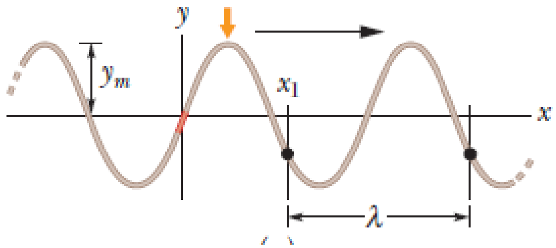
transversal and longitudinal wave



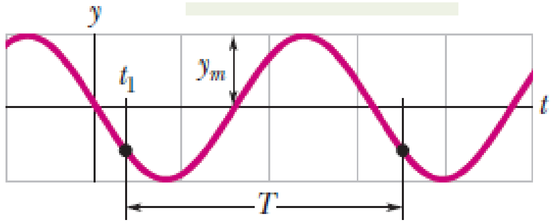


$y=h(x,t)$





$$k = \frac{2\pi}{\lambda} \quad \text{(angular wave number).}$$



$$\omega = \frac{2\pi}{T} \quad \text{(angular frequency).}$$

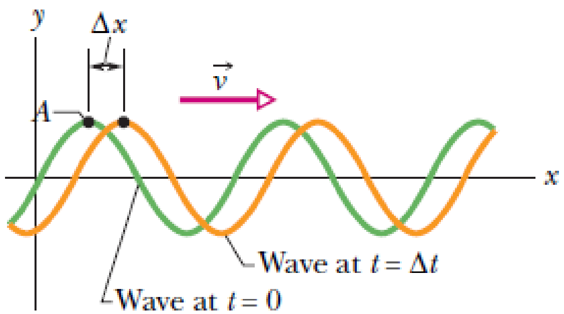
$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \text{(frequency).}$$

Speed of travelling wave

$$kx - \omega t = \text{a constant.}$$

$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = v = \frac{\omega}{k}.$$



$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \text{(wave speed).}$$

Transverse wave, amplitude, wavelength, period, velocity

A wave traveling along a string is described by

$$y(x, t) = 0.00327 \sin(72.1x - 2.72t), \tag{16-18}$$

in which the numerical constants are in SI units (0.00327 m, 72.1 rad/m, and 2.72 rad/s).

(a) What is the amplitude of this wave?

KEY IDEA

Equation 16-18 is of the same form as Eq. 16-2,

$$y = y_m \sin(kx - \omega t), \tag{16-19}$$

so we have a sinusoidal wave. By comparing the two equations, we can find the amplitude.

Calculation: We see that

$$y_m = 0.00327 \text{ m} = 3.27 \text{ mm}. \quad \text{(Answer)}$$

(b) What are the wavelength, period, and frequency of this wave?

Calculations: By comparing Eqs. 16-18 and 16-19, we see that the angular wave number and angular frequency are

$$k = 72.1 \text{ rad/m} \quad \text{and} \quad \omega = 2.72 \text{ rad/s}.$$

We then relate wavelength λ to k via Eq. 16-5:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{72.1 \text{ rad/m}}$$
$$= 0.0871 \text{ m} = 8.71 \text{ cm}. \quad \text{(Answer)}$$

Next, we relate T to ω with Eq. 16-8:

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{2.72 \text{ rad/s}} = 2.31 \text{ s}, \quad \text{(Answer)}$$

and from Eq. 16-9 we have

$$f = \frac{1}{T} = \frac{1}{2.31 \text{ s}} = 0.433 \text{ Hz}. \quad \text{(Answer)}$$

(c) What is the velocity of this wave?

Calculation: The speed of the wave is given by Eq. 16-13:

$$v = \frac{\omega}{k} = \frac{2.72 \text{ rad/s}}{72.1 \text{ rad/m}} = 0.0377 \text{ m/s}$$
$$= 3.77 \text{ cm/s}. \quad \text{(Answer)}$$

Because the phase in Eq. 16-18 contains the position variable x, the wave is moving along the x axis. Also, because the wave equation is written in the form of Eq. 16-2, the *minus* sign in front of the ωt term indicates that the wave is moving in the *positive* direction of the x axis. (Note that the quantities calculated in (b) and (c) are independent of the amplitude of the wave.)

(d) What is the displacement y of the string at $x = 22.5$ cm and $t = 18.9$ s?

Calculation: Equation 16-18 gives the displacement as a function of position x and time t. Substituting the given values into the equation yields

$$y = 0.00327 \sin(72.1 \times 0.225 - 2.72 \times 18.9)$$
$$= (0.00327 \text{ m}) \sin(-35.1855 \text{ rad})$$
$$= (0.00327 \text{ m})(0.588)$$
$$= 0.00192 \text{ m} = 1.92 \text{ mm}. \quad \text{(Answer)}$$

Thus, the displacement is positive. (Be sure to change your calculator mode to radians before evaluating the sine. Also, note that we do *not* round off the sine's argument before evaluating the sine. Also note that both terms in the argument are properly in radians, a dimensionless quantity.)

In the preceding sample problem, we showed that at $t = 18.9$ s the transverse displacement y of the element of the string at $x = 22.5$ cm due to the wave of Eq. 16-18 is 1.92 mm.

(a) What is u, the transverse velocity of the same element of the string, at that time? (This velocity, which is associated with the transverse oscillation of an element of the string, is in the y direction. Do not confuse it with v, the constant velocity at which the *wave form* travels along the x axis.)

KEY IDEAS

The transverse velocity u is the rate at which the displacement y of the element is changing. In general, that displacement is given by

$$y(x, t) = y_m \sin(kx - \omega t). \tag{16-20}$$

For an element at a certain location x, we find the rate of change of y by taking the derivative of Eq. 16-20 with respect to t while treating x as a constant. A derivative taken while one (or more) of the variables is treated as a constant is called a *partial derivative* and is represented by the symbol $\partial/\partial x$ rather than d/dx.

Calculations: Here we have

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t). \tag{16-21}$$

Next, substituting numerical values from the preceding sample problem, we obtain

$$u = (-2.72 \text{ rad/s})(3.27 \text{ mm}) \cos(-35.1855 \text{ rad})$$
$$= 7.20 \text{ mm/s}. \qquad \text{(Answer)}$$

Thus, at $t = 18.9$ s, the element of the string at $x = 22.5$ cm is moving in the positive direction of y with a speed of 7.20 mm/s.

(b) What is the transverse acceleration a_y of the same element at that time?

KEY IDEA

The transverse acceleration a_y is the rate at which the transverse velocity of the element is changing.

Calculations: From Eq. 16-21, again treating x as a constant but allowing t to vary, we find

$$a_y = \frac{\partial u}{\partial t} = -\omega^2 y_m \sin(kx - \omega t).$$

Comparison with Eq. 16-20 shows that we can write this as

$$a_y = -\omega^2 y.$$

We see that the transverse acceleration of an oscillating string element is proportional to its transverse displacement but opposite in sign. This is completely consistent with the action of the element itself—namely, that it is moving transversely in simple harmonic motion. Substituting numerical values yields

$$a_y = -(2.72 \text{ rad/s})^2(1.92 \text{ mm})$$
$$= -14.2 \text{ mm/s}^2. \qquad \text{(Answer)}$$

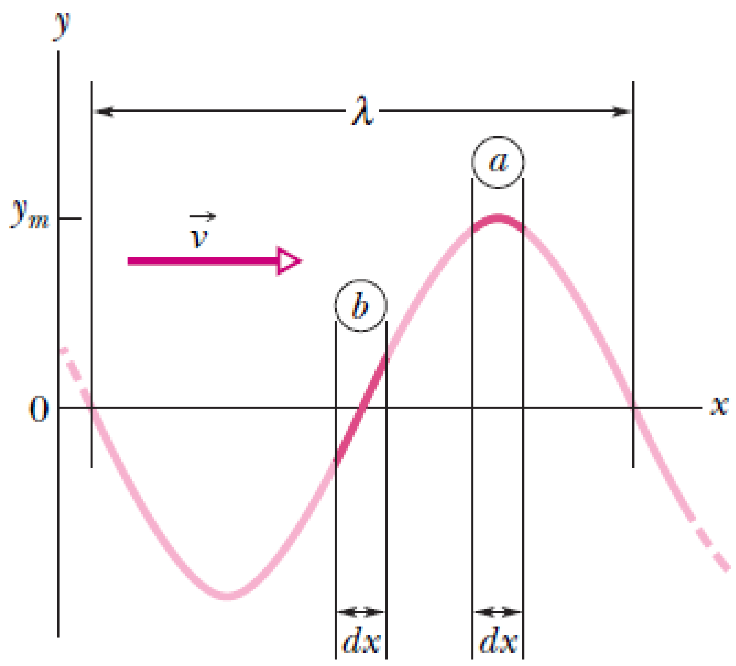
Thus, at $t = 18.9$ s, the element of string at $x = 22.5$ cm is displaced from its equilibrium position by 1.92 mm in the positive y direction and has an acceleration of magnitude 14.2 mm/s^2 in the negative y direction.

Energy transport of a travelling wave

$E_{kin} = E_{max}$ for y =0
max speed, max kin energy

$E_{pot} = E_{max}$ for y =0
Max stretch, max elastic potentianlenergie

Both E_{kin} and E_{pot} are min at y_m



$$dK = \tfrac{1}{2}\, dm\, u^2, \qquad dm = \mu\, dx,$$

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t).$$

$$dK = \tfrac{1}{2}(\mu\, dx)(-\omega y_m)^2 \cos^2(kx - \omega t).$$

$$\frac{dK}{dt} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx - \omega t).$$

Rate of energy transmission

$$\left(\frac{dK}{dt}\right)_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \,[\cos^2(kx - \omega t)]_{avg}$$

$$(dK/dt)_{ave} = \tfrac{1}{4}\, \mu v \omega^2 y_m^2$$

Wave equation

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

Ansatz : $y = y_m \sin(\omega t - kx)$

$dy/dt = \omega\ y_m \cos(\omega t - kx)$
$d^2y/dt^2 = -\omega^2\ y_m \sin(\omega t - kx)$
$dy/dx = -k\ y_m \cos(\omega t - kx)$
$d^2y/dt^2 = -k^2\ y_m \sin(\omega t - kx)$

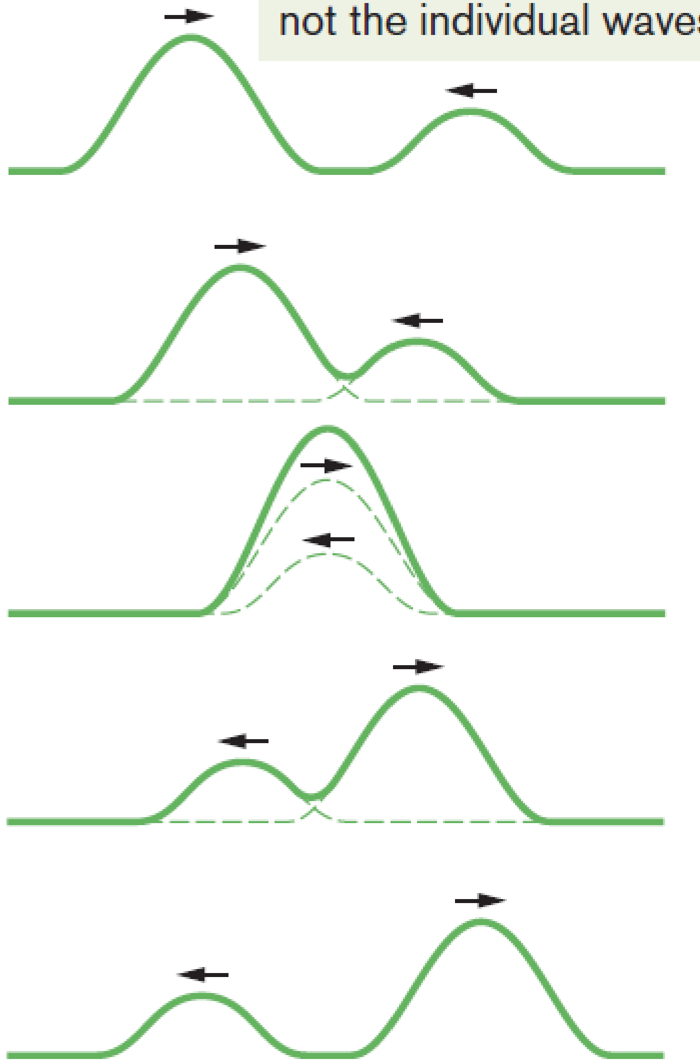
$-\omega^2\ y_m \sin(\omega t - kx) = 1/v^2\ (-k^2\ y_m \sin(\omega t - kx))$
$-\omega^2 = 1/v^2\ (-k^2)$

$v = \lambda f$

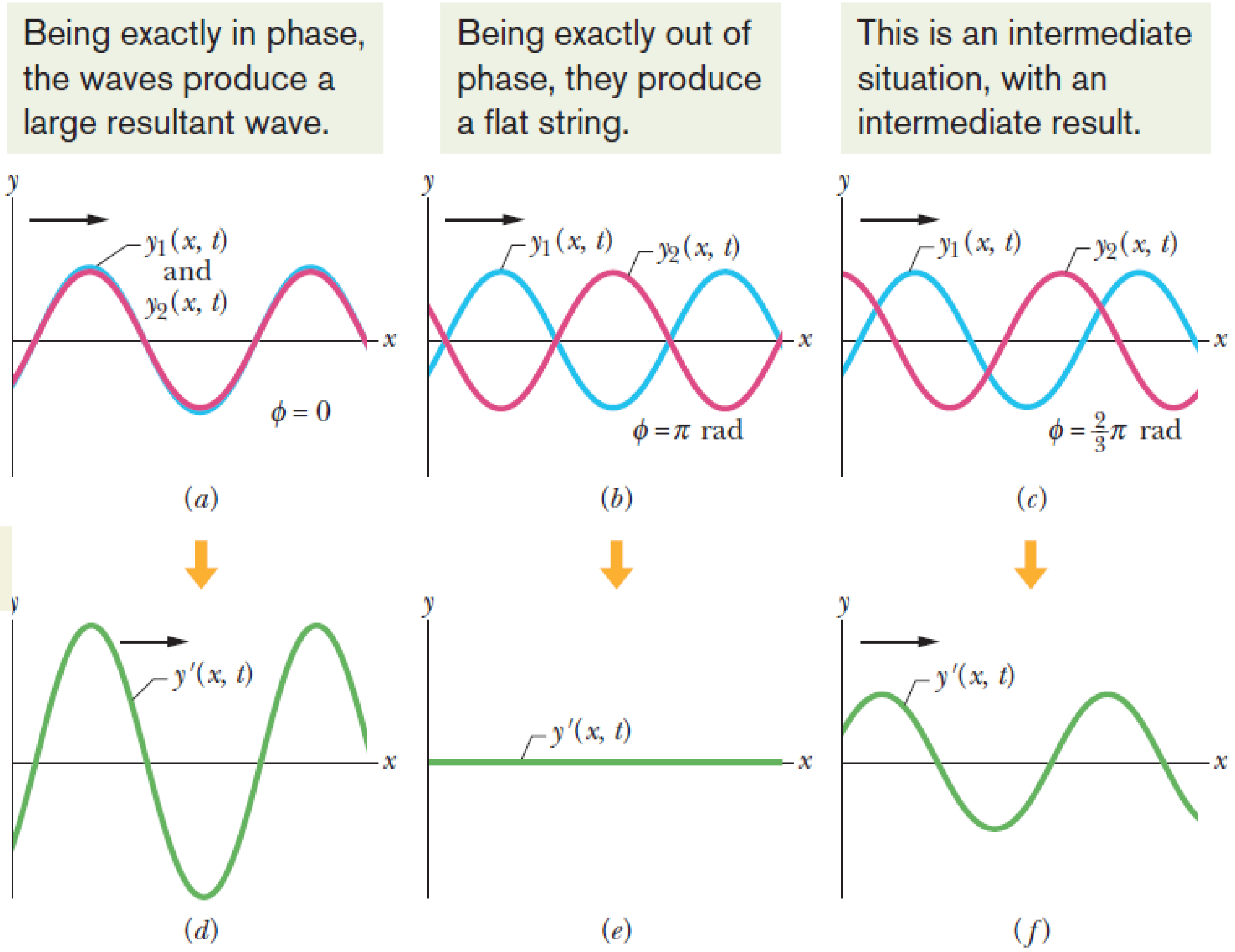
Superposition of waves

$$y'(x,t) = y_1(x,t) + y_2(x,t).$$

When two waves overlap, we see the resultant wave, not the individual waves.



Interference



$$y'(x,t) = y_1(x,t) + y_2(x,t).$$

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$
$$= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi).$$

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta).$$

$$y'(x,t) = [2y_m \cos\tfrac{1}{2}\phi]\sin(kx - \omega t + \tfrac{1}{2}\phi).$$

$$\underbrace{y'(x,t)}_{\text{Displacement}} = \underbrace{[2y_m \cos\tfrac{1}{2}\phi]}_{\text{Magnitude gives amplitude}}\ \underbrace{\sin(kx - \omega t + \tfrac{1}{2}\phi)}_{\text{Oscillating term}}$$

Interference of two waves, same direction, same amplitude

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100°.

(a) What is the amplitude y'_m of the resultant wave due to the interference, and what is the type of this interference?

KEY IDEA

These are identical sinusoidal waves traveling in the *same direction* along a string, so they interfere to produce a sinusoidal traveling wave.

Calculations: Because they are identical, the waves have the *same amplitude*. Thus, the amplitude y'_m of the resultant wave is given by Eq. 16-52:

$$y'_m = |2y_m \cos \tfrac{1}{2}\phi| = |(2)(9.8\text{ mm}) \cos(100°/2)|$$
$$= 13\text{ mm.} \qquad \text{(Answer)}$$

We can tell that the interference is *intermediate* in two ways. The phase difference is between 0 and 180°, and, correspondingly, the amplitude y'_m is between 0 and $2y_m$ (= 19.6 mm).

(b) What phase difference, in radians and wavelengths, will give the resultant wave an amplitude of 4.9 mm?

Calculations: Now we are given y'_m and seek ϕ. From Eq. 16-52,

$$y'_m = |2y_m \cos \tfrac{1}{2}\phi|,$$

we now have

$$4.9\text{ mm} = (2)(9.8\text{ mm}) \cos \tfrac{1}{2}\phi,$$

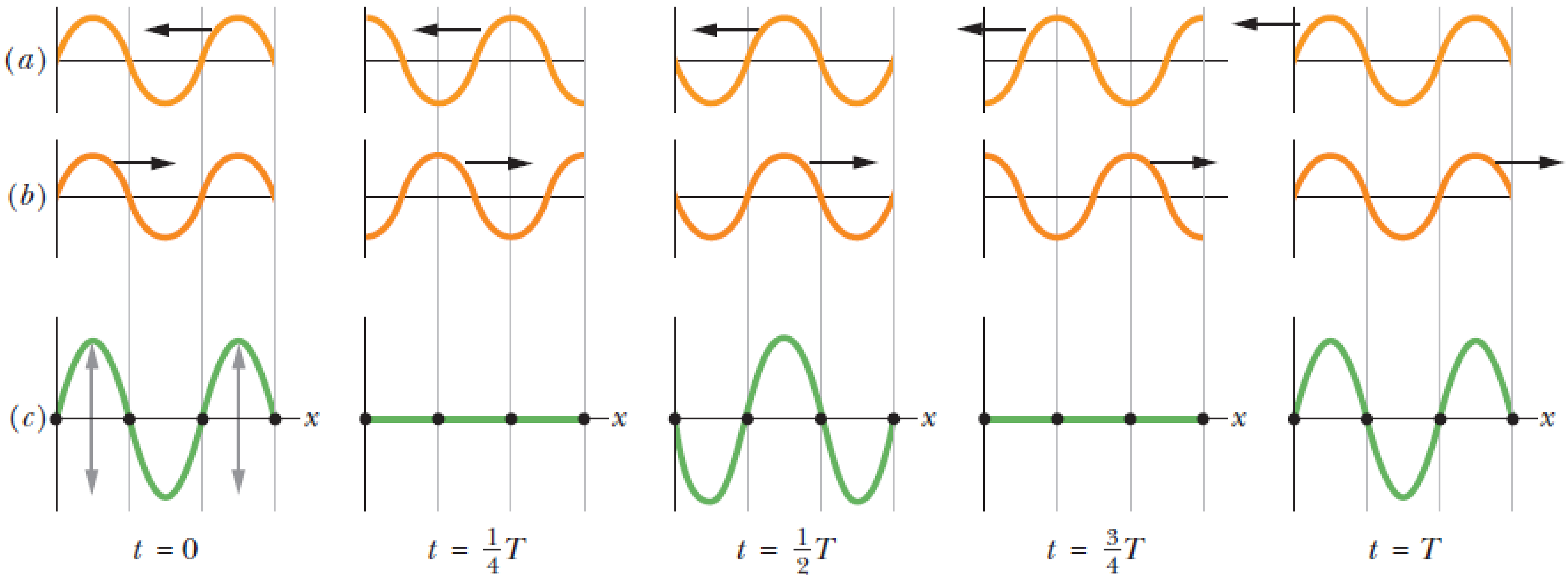
which gives us (with a calculator in the radian mode)

$$\phi = 2 \cos^{-1} \frac{4.9\text{ mm}}{(2)(9.8\text{ mm})}$$
$$= \pm 2.636\text{ rad} \approx \pm 2.6\text{ rad.} \qquad \text{(Answer)}$$

There are two solutions because we can obtain the same resultant wave by letting the first wave *lead* (travel ahead of) or *lag* (travel behind) the second wave by 2.6 rad. In wavelengths, the phase difference is

$$\frac{\phi}{2\pi\text{ rad/wavelength}} = \frac{\pm 2.636\text{ rad}}{2\pi\text{ rad/wavelength}}$$
$$= \pm 0.42\text{ wavelength.} \qquad \text{(Answer)}$$

Standing waves

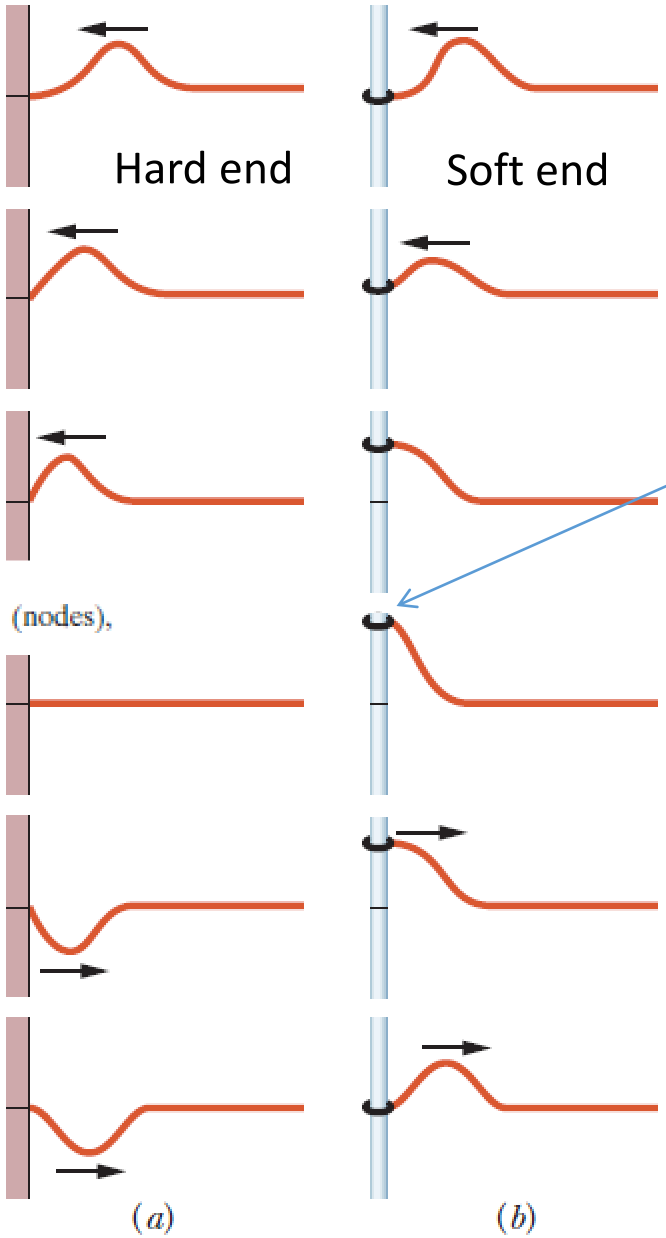


$y_1\ (x,t) = y_m \sin(kx - \omega t)$; $y_2\ (x,t) = y_m \sin(kx + \omega t)$

$\sin a + \sin b = 2 \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)$

$y'(x,t) = 2y_m \sin(kx) \cos(\omega t)$

Phase change at reflection



$kx = n\pi, \quad \text{for } n = 0, 1, 2, \ldots .$

$x = n\dfrac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, \ldots \quad \text{(nodes)},$

node

antinode

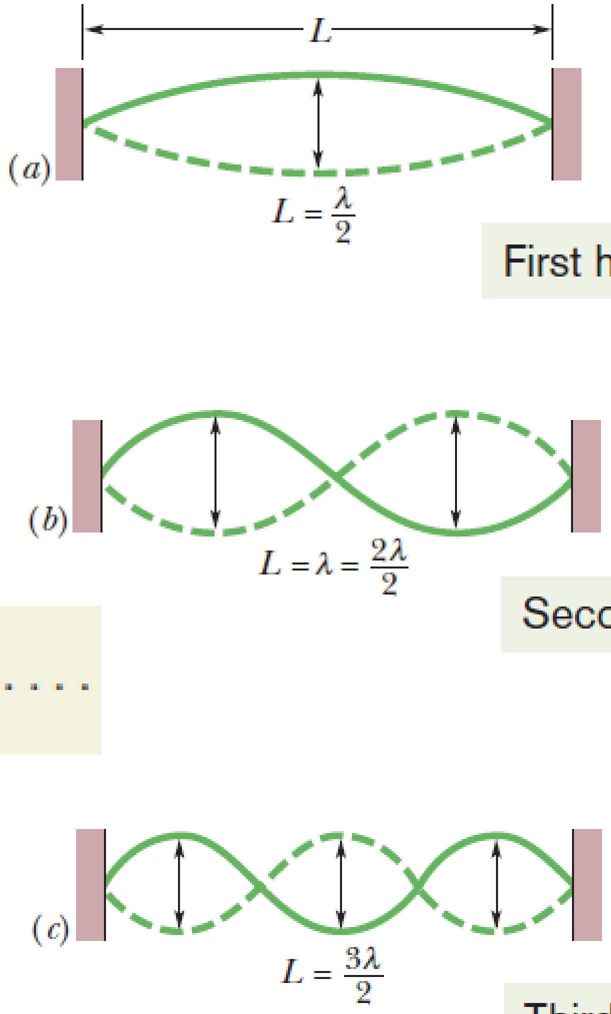
$kx = \tfrac{1}{2}\pi, \tfrac{3}{2}\pi, \tfrac{5}{2}\pi, \ldots$

$x = \left(n + \dfrac{1}{2}\right)\dfrac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, \ldots \quad \text{(antinodes)},$

Harmonics

$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots .$$



First harmonic

Second harmonic

Third harmonic

Resonance of transverse wav

Figure 16-22 shows a pattern of resonant oscillation of a string of mass $m = 2.500$ g and length $L = 0.800$ m and that is under tension $\tau = 325.0$ N. What is the wavelength λ of the transverse waves producing the standing-wave pattern, and what is the harmonic number n? What is the frequency f of the transverse waves and of the oscillations of the moving string elements? What is the maximum magnitude of the transverse velocity u_m of the element oscillating at coordinate $x = 0.180$ m (note the x axis in the figure)? At what point

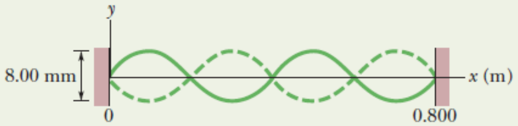


Fig. 16-22 Resonant oscillation of a string under tension.

KEY IDEAS

(1) The traverse waves that produce a standing-wave pattern must have a wavelength such that an integer number n of half-wavelengths fit into the length L of the string. (2) The frequency of those waves and of the oscillations of the string elements is given by Eq. 16-66 ($f = nv/2L$). (3) The displacement of a string element as a function of position x and time t is given by Eq. 16-60:

$$y'(x,t) = [2y_m \sin kx] \cos \omega t. \tag{16-67}$$

Wavelength and harmonic number: In Fig. 16-22, the solid line, which is effectively a snapshot (or freeze frame) of the oscillations, reveals that 2 full wavelengths fit into the length $L = 0.800$ m of the string. Thus, we have

$$2\lambda = L,$$

or

$$\lambda = \frac{L}{2}. \tag{16-68}$$

$$= \frac{0.800 \text{ m}}{2} = 0.400 \text{ m}. \quad \text{(Answer)}$$

By counting the number of loops (or half-wavelengths) in Fig. 16-22, we see that the harmonic number is

$$n = 4. \quad \text{(Answer)}$$

We reach the same conclusion by comparing Eqs. 16-68 and 16-65 ($\lambda = 2L/n$). Thus, the string is oscillating in its fourth harmonic.

Frequency: We can get the frequency f of the transverse waves from Eq. 16-13 ($v = \lambda f$) if we first find the speed v of the waves. That speed is given by Eq. 16-26, but we must substitute m/L for the unknown linear density μ. We obtain

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\tau}{m/L}} = \sqrt{\frac{\tau L}{m}}$$

$$= \sqrt{\frac{(325 \text{ N})(0.800 \text{ m})}{2.50 \times 10^{-3} \text{ kg}}} = 322.49 \text{ m/s}.$$

After rearranging Eq. 16-13, we write

$$f = \frac{v}{\lambda} = \frac{322.49 \text{ m/s}}{0.400 \text{ m}}$$

$$= 806.2 \text{ Hz} \approx 806 \text{ Hz}. \quad \text{(Answer)}$$

Note that we get the same answer by substituting into Eq. 16-66:

$$f = n\frac{v}{2L} = 4\frac{322.49 \text{ m/s}}{2(0.800 \text{ m})}$$

$$= 806 \text{ Hz}. \quad \text{(Answer)}$$

Now note that this 806 Hz is not only the frequency of the waves producing the fourth harmonic but also it is said to *be* the fourth harmonic, as in the statement, "The fourth harmonic of this oscillating string is 806 Hz." It is also the frequency of the string elements as they oscillate vertically in the figure in simple harmonic motion, just as a block on a vertical spring would oscillate in simple harmonic motion. Finally, it is also the frequency of the sound you would hear from the string as the oscillating string elements periodically push against the air, sending out sound waves.

Transverse velocity: The displacement y' of the string element located at coordinate x is given by Eq. 16-67 as a function of time t. The term $\cos \omega t$ contains the dependence on time and thus provides the "motion" of the standing wave. The term $2y_m \sin kx$ sets the extent of the motion—that is, the amplitude. The greatest amplitude occurs at an antinode, where $\sin kx$ is $+1$ or -1 and thus the greatest amplitude is $2y_m$. From Fig. 16-22, we see that $2y_m = 4.00$ mm, which tells us that $y_m = 2.00$ mm.

We want the transverse velocity—the velocity of a string element parallel to the y axis. To find it, we take the time derivative of Eq. 16-67:

$$u(x,t) = \frac{\partial y'}{\partial t} = \frac{\partial}{\partial t}[(2y_m \sin kx) \cos \omega t]$$

$$= [-2y_m\omega \sin kx] \sin \omega t. \tag{16-69}$$

Here the term $\sin \omega t$ provides the variation with time and the term $-2y_m\omega \sin kx$ provides the extent of that variation. We want the absolute magnitude of that extent:

$$u_m = |-2y_m\omega \sin kx|.$$

To evaluate this for the element at $x = 0.180$ m, we first note that $y_m = 2.00$ mm, $k = 2\pi/\lambda = 2\pi/(0.400 \text{ m})$, and $\omega = 2\pi f = 2\pi(806.2 \text{ Hz})$. Then the maximum speed of the element at $x = 0.180$ m is

$$u_m = \left| -2(2.00 \times 10^{-3} \text{ m})(2\pi)(806.2 \text{ Hz}) \times \sin\left(\frac{2\pi}{0.400 \text{ m}}(0.180 \text{ m})\right) \right|$$

$$= 6.26 \text{ m/s}. \quad \text{(Answer)}$$

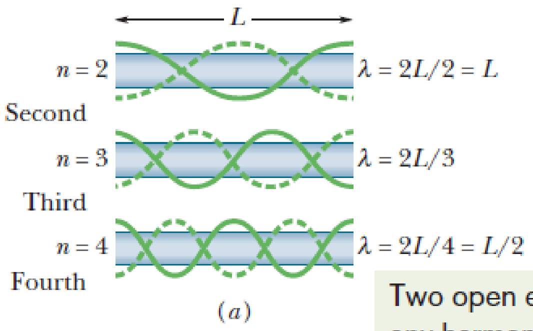
To determine when the string element has this maximum speed, we could investigate Eq. 16-69. However, a little thought can save a lot of work. The element is undergoing simple harmonic motion and must come to a momentary stop at its extreme upward position and extreme downward position. It has the greatest speed as it zips through the midpoint of its oscillation, just as a block does in a block–spring oscillator.

Resonance frequency of a pipe of length L

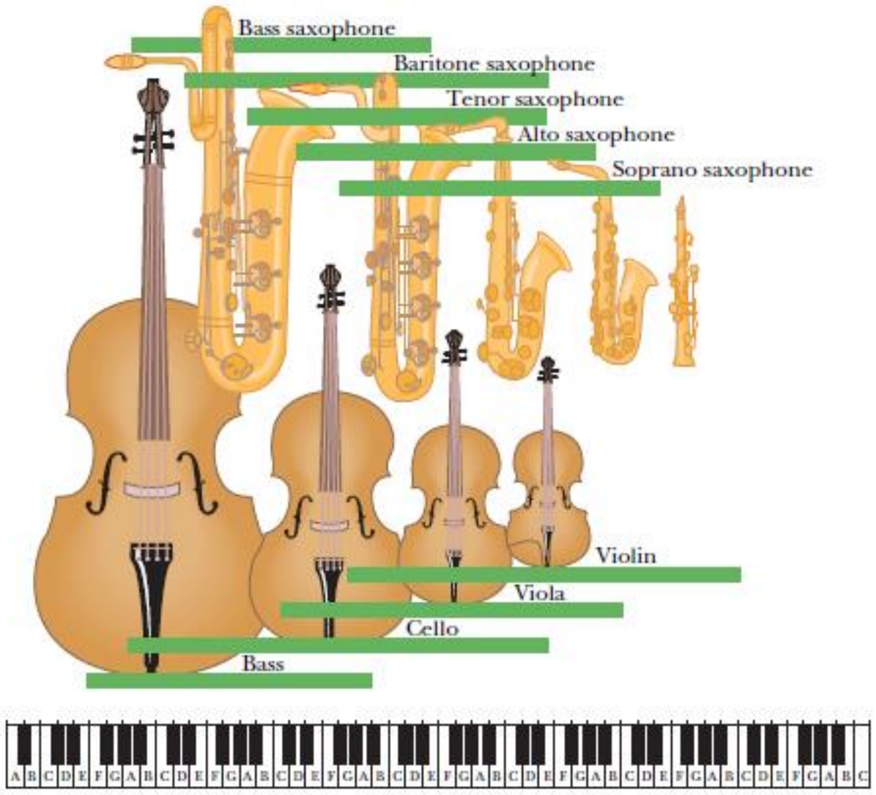
$$f = \frac{v}{\lambda} = \frac{nv}{2L}, \quad \text{for } n = 1, 2, 3, \ldots \quad \text{(pipe, two open ends).}$$

Corresponds to wave length λ

$$\lambda = \frac{2L}{n}, \quad \text{for } n = 1, 2, 3, \ldots,$$

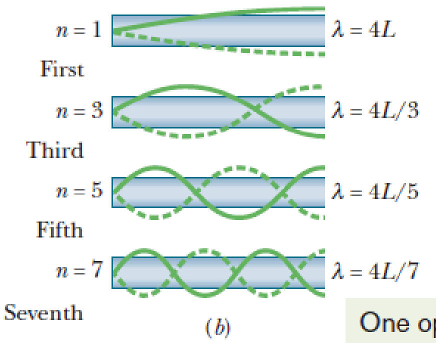


Two open ends—any harmonic



$$f = \frac{v}{\lambda} = \frac{nv}{4L}, \quad \text{for } n = 1, 3, 5, \ldots \quad \text{(pipe, one open end).}$$

$$\lambda = \frac{4L}{n}, \quad \text{for } n = 1, 3, 5, \ldots,$$



One open end—only *odd* harmonics

Sound resonance in double-open pipe and single-open pipe

Weak background noises from a room set up the fundamental standing wave in a cardboard tube of length $L = 67.0$ cm with two open ends. Assume that the speed of sound in the air within the tube is 343 m/s.

(a) What frequency do you hear from the tube?

KEY IDEA

With both pipe ends open, we have a symmetric situation in which the standing wave has an antinode at each end of the tube. The standing wave pattern (in string wave style) is that of Fig. 17-13*b*.

Calculation: The frequency is given by Eq. 17-39 with $n = 1$ for the fundamental mode:

$$f = \frac{nv}{2L} = \frac{(1)(343\text{ m/s})}{2(0.670\text{ m})} = 256\text{ Hz.} \qquad \text{(Answer)}$$

If the background noises set up any higher harmonics, such as the second harmonic, you also hear frequencies that are *integer* multiples of 256 Hz. (Thus, the lowest frequency is this fundamental frequency of 256 Hz.)

(b) If you jam your ear against one end of the tube, what fundamental frequency do you hear from the tube?

KEY IDEA

With your ear effectively closing one end of the tube, we have an asymmetric situation—an antinode still exists at the open end, but a node is now at the other (closed) end. The standing wave pattern is the top one in Fig. 17-14*b*.

Calculation: The frequency is given by Eq. 17-41 with $n = 1$ for the fundamental mode:

$$f = \frac{nv}{4L} = \frac{(1)(343\text{ m/s})}{4(0.670\text{ m})} = 128\text{ Hz.} \qquad \text{(Answer)}$$

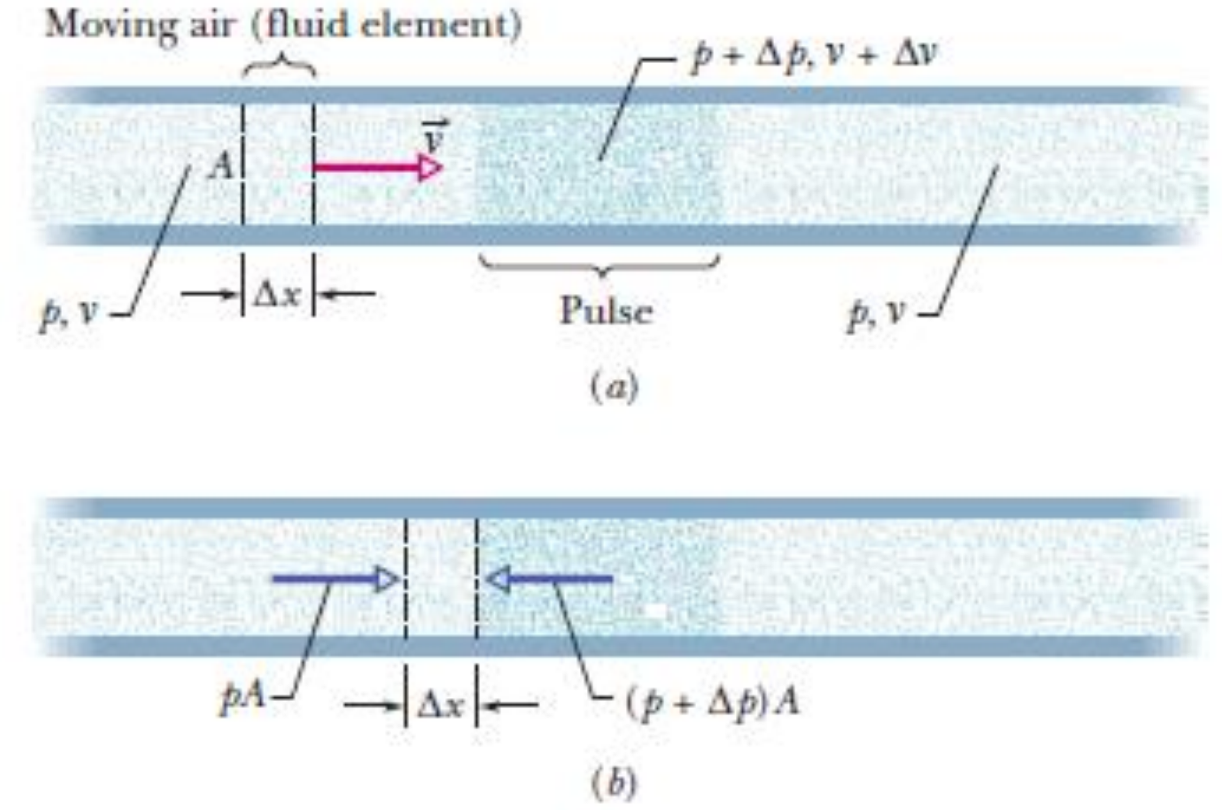
If the background noises set up any higher harmonics, they will be *odd* multiples of 128 Hz. That means that the frequency of 256 Hz (which is an even multiple) cannot now occur.

Sound waves

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

$$v = \sqrt{\frac{B}{\rho}} \quad \text{(speed of sound)}$$

$$B = -\frac{\Delta p}{\Delta V/V}$$



The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

Traveling sound waves

$$\Delta p = -B\frac{\Delta V}{V}.$$

$$V = A\,\Delta x.$$

$$\Delta V = A\,\Delta s.$$

$$\Delta p = -B\frac{\Delta s}{\Delta x} = -B\frac{\partial s}{\partial x}.$$

$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x}[s_m \cos(kx - \omega t)] = -ks_m \sin(kx - \omega t).$$

$$\Delta p = Bks_m \sin(kx - \omega t).$$

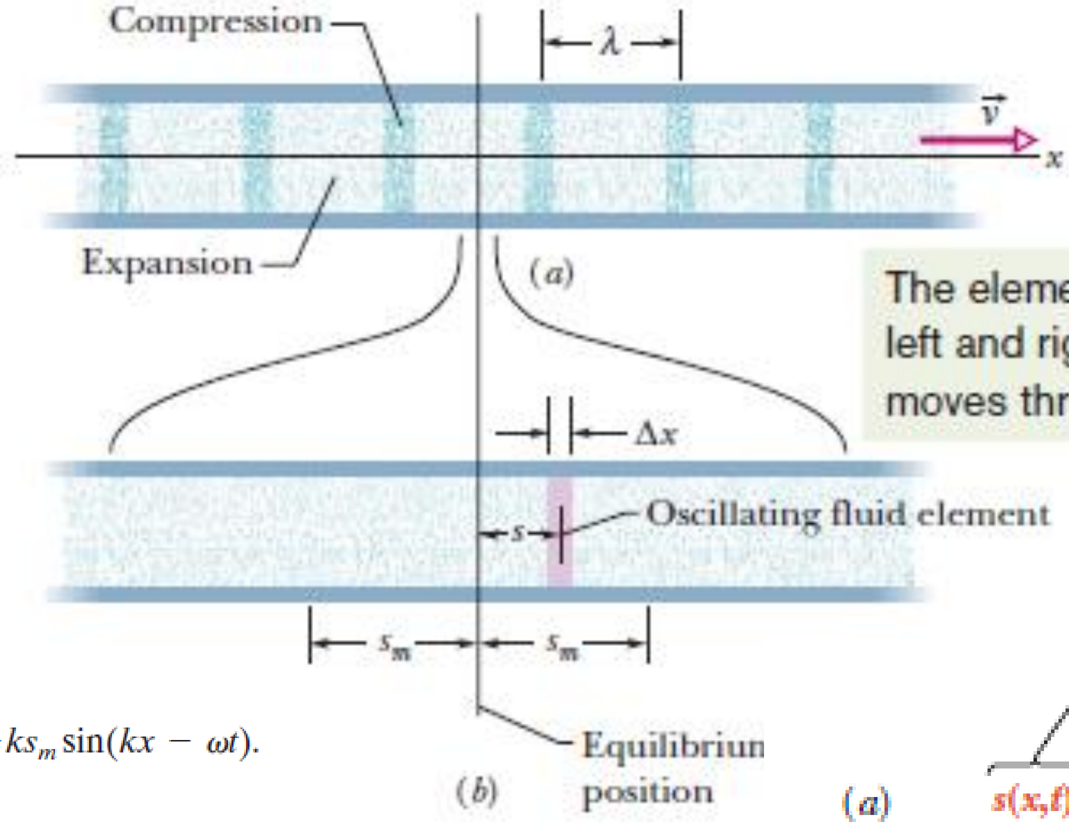
$$\Delta p_m = (Bk)s_m = (v^2\rho k)s_m.$$

$$v = \sqrt{\frac{B}{\rho}}$$

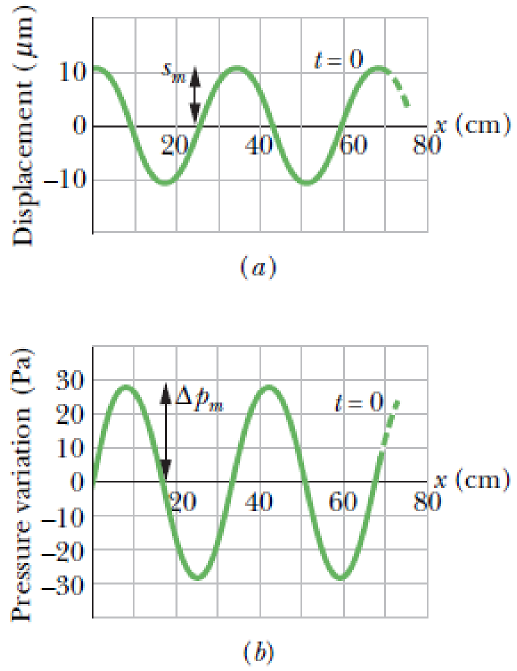
$B = v^2 \rho$

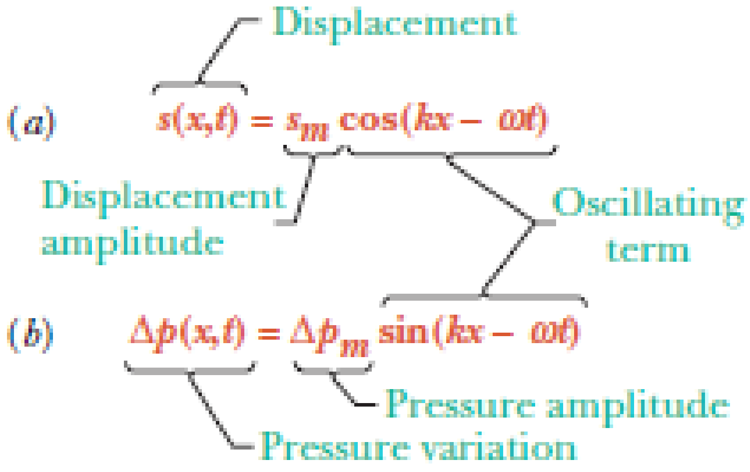
$\omega = vk$

$$\Delta p_m = (v\rho\omega)s_m.$$



The element oscillates left and right as the wave moves through it.





Pressure amplitude, displacement amplitude

The maximum pressure amplitude Δp_m that the human ear can tolerate in loud sounds is about 28 Pa (which is very much less than the normal air pressure of about 10^5 Pa). What is the displacement amplitude s_m for such a sound in air of density $\rho = 1.21\ \text{kg/m}^3$, at a frequency of 1000 Hz and a speed of 343 m/s?

KEY IDEA

The displacement amplitude s_m of a sound wave is related to the pressure amplitude Δp_m of the wave according to Eq. 17-14.

Calculations: Solving that equation for s_m yields

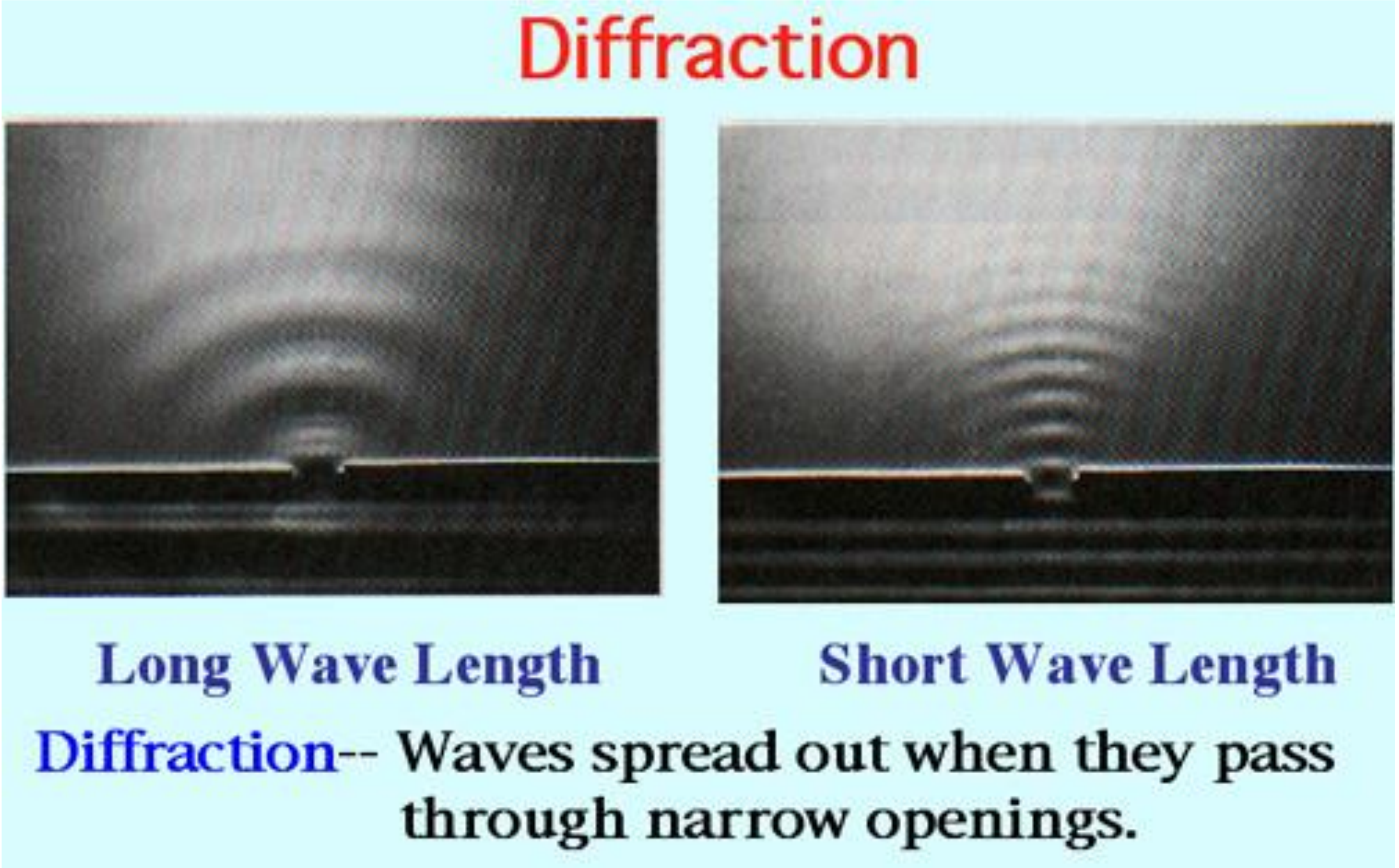
$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi f)}.$$

Substituting known data then gives us

$$s_m = \frac{28\ \text{Pa}}{(343\ \text{m/s})(1.21\ \text{kg/m}^3)(2\pi)(1000\ \text{Hz})}$$
$$= 1.1 \times 10^{-5}\ \text{m} = 11\ \mu\text{m}. \qquad \text{(Answer)}$$

That is only about one-seventh the thickness of a book page. Obviously, the displacement amplitude of even the loudest sound that the ear can tolerate is very small. Temporary exposure to such loud sound produces temporary hearing loss, probably due to a decrease in blood supply to the inner ear. Prolonged exposure produces permanent damage.

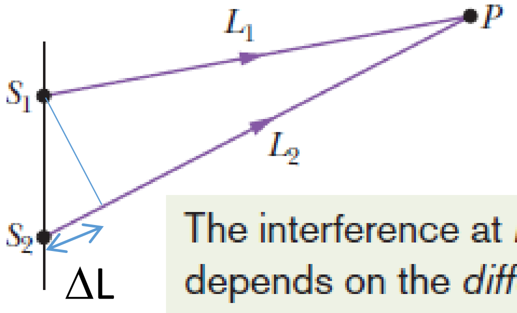
The pressure amplitude Δp_m for the *faintest* detectable sound at 1000 Hz is 2.8×10^{-5} Pa. Proceeding as above leads to $s_m = 1.1 \times 10^{-11}$ m or 11 pm, which is about one-tenth the radius of a typical atom. The ear is indeed a sensitive detector of sound waves.



Each narrow slit is source of a speherical wave

Interference

$$\Delta L = |L_2 - L_1|.$$



The interference at *P* depends on the *difference* in the path lengths to reach *P*.

$$\frac{\phi}{2\pi} = \frac{\Delta L}{\lambda},$$

$$\phi = \frac{\Delta L}{\lambda} 2\pi.$$

Interference

Interference -- Constructive & Destructive

$$\phi = m(2\pi), \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(fully constructive interference).}$$

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots$$

$$\phi = (2m + 1)\pi, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(fully destructive interference).}$$

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots$$

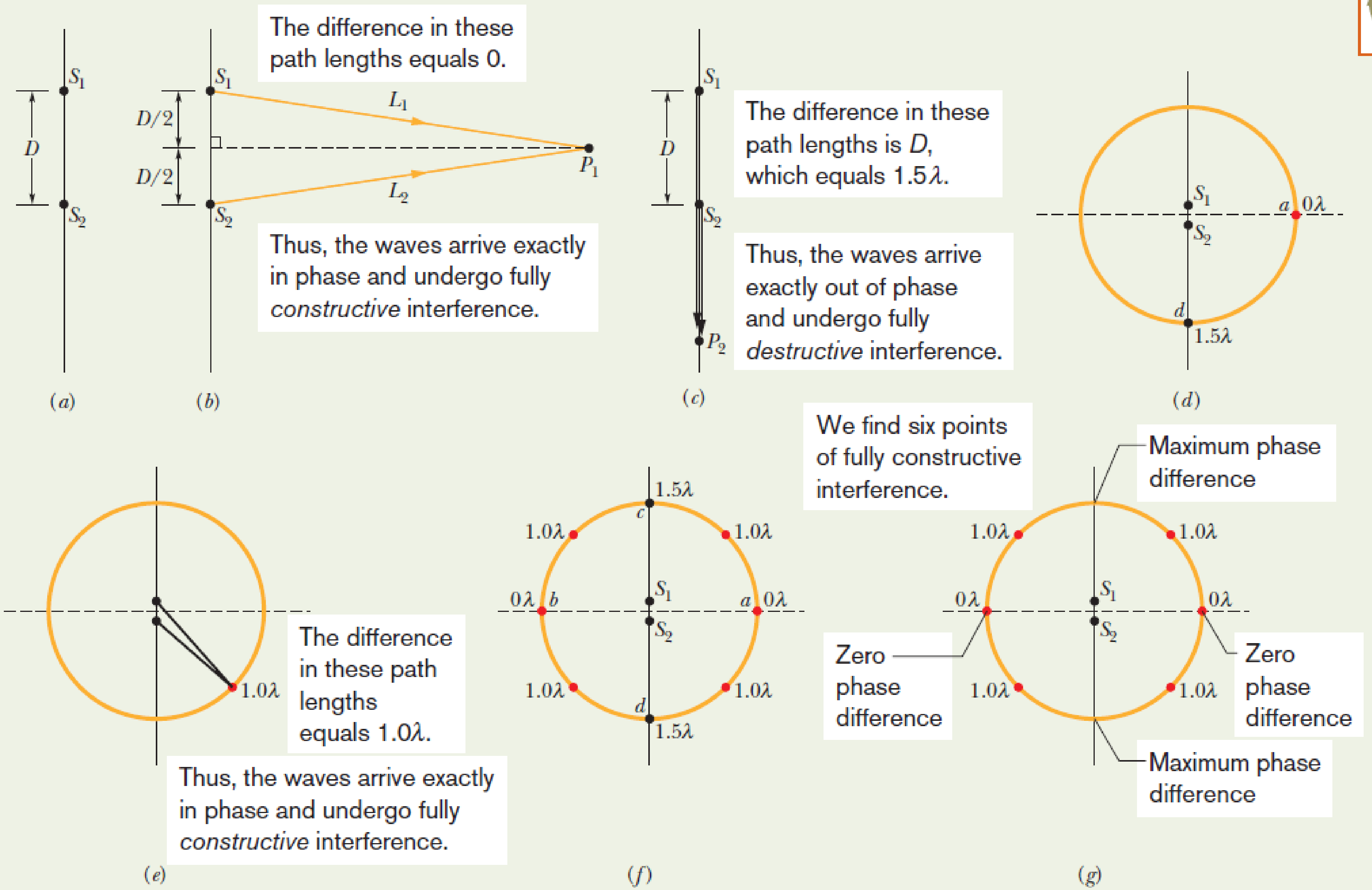
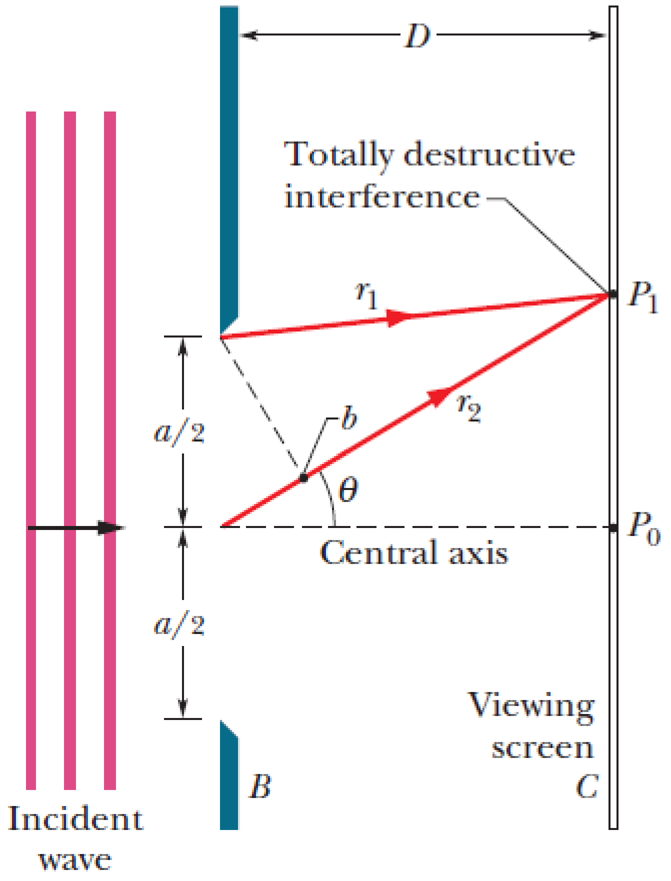


Fig. 17-8 (*a*) Two point sources S_1 and S_2, separated by distance D, emit spherical sound waves in phase. (*b*) The waves travel equal distances to reach point P_1. (*c*) Point P_2 is on the line extending through S_1 and S_2. (*d*) We move around a large circle. (*e*) Another point of fully constructive interference. (*f*) Using symmetry to determine other points. (*g*) The six points of fully constructive interference.

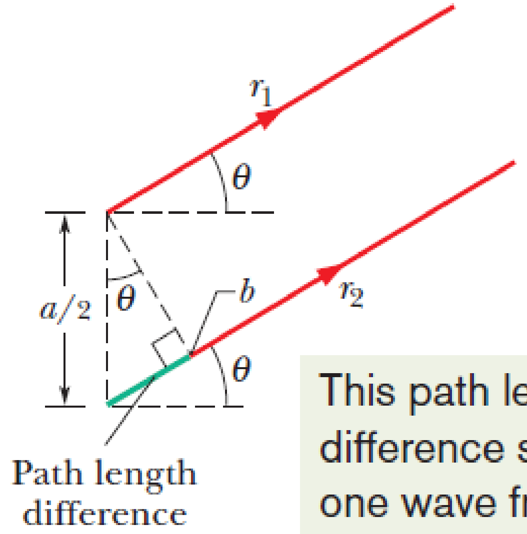
Single slit experiment

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$$

$$a\sin\theta = \lambda \qquad \text{(first minimum).}$$

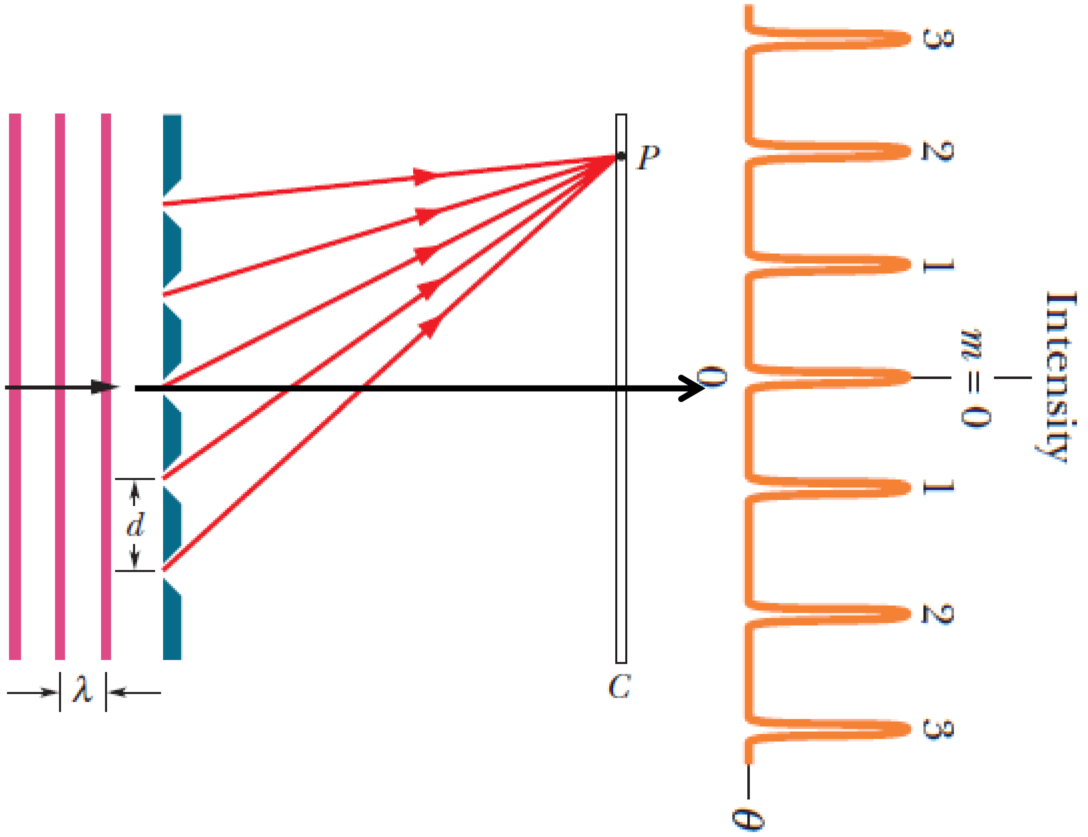


This pair of rays cancel each other at P_1. So do all such pairings.



This path length difference shifts one wave from the other, which determines the interference.

Multiple slit experiment



$$d \sin \theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \qquad \text{(maxima—lines)},$$

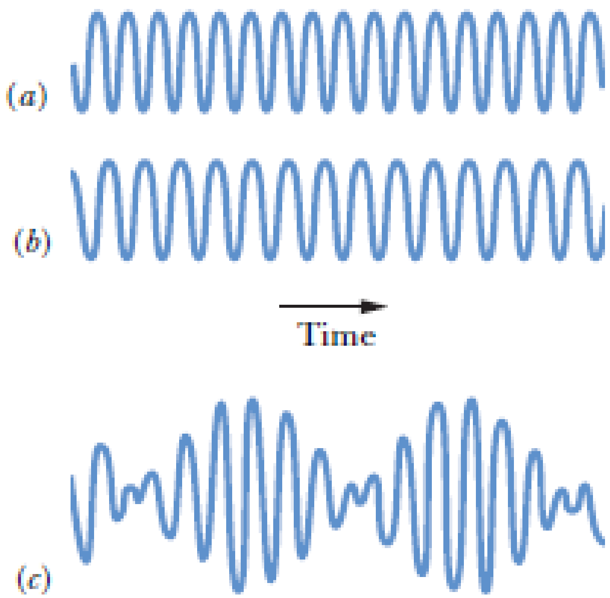
Interference of sound waves

$$s = s_1 + s_2 = s_m(\cos \omega_1 t + \cos \omega_2 t).$$

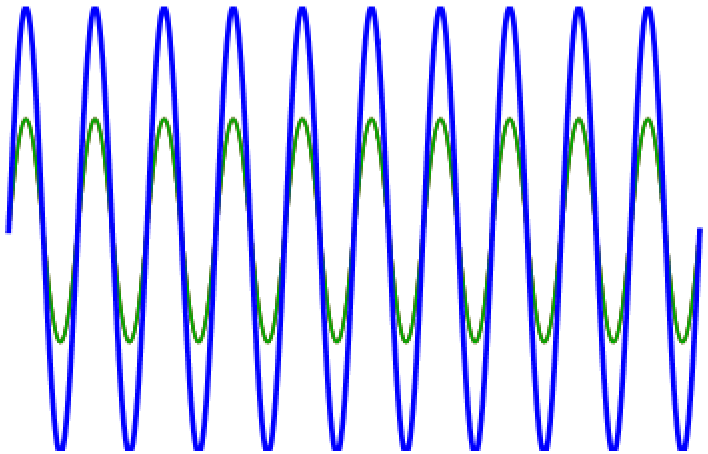
$$s = 2s_m \cos[\tfrac{1}{2}(\omega_1 - \omega_2)t] \cos[\tfrac{1}{2}(\omega_1 + \omega_2)t].$$

$$\omega_{\text{beat}} = 2\omega' = (2)(\tfrac{1}{2})(\omega_1 - \omega_2) = \omega_1 - \omega_2.$$

$$\omega = 2\pi f,$$



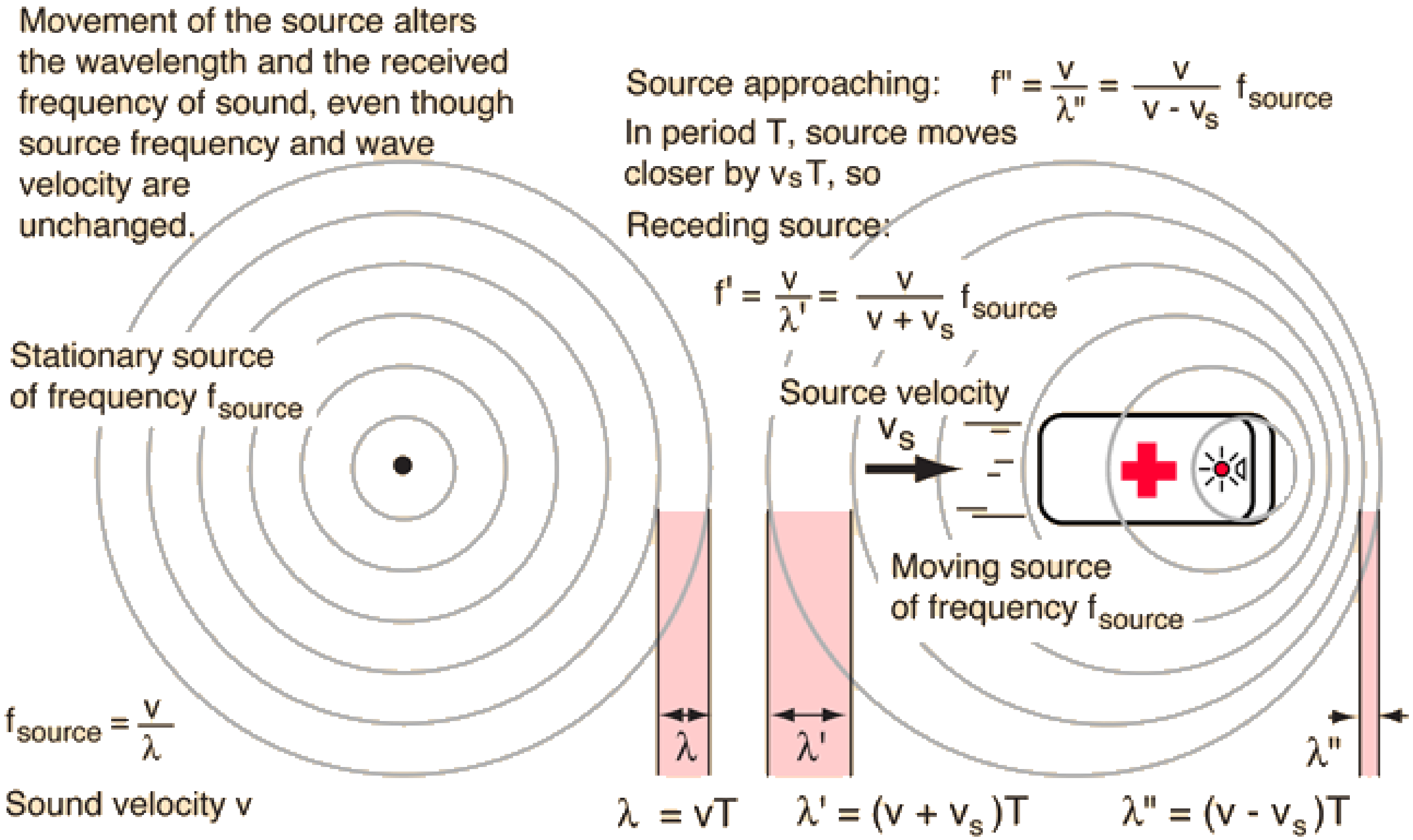
$$f_{\text{beat}} = f_1 - f_2 \quad \text{(beat frequency).}$$



Doppler effect

$$f' = f\frac{v \pm v_D}{v \pm v_S}$$

When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from the other, the sign on its speed must give a downward shift in frequency.



Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than can be heard by a human. Suppose a bat emits ultrasound at frequency $f_{be} = 82.52$ kHz while flying with velocity $\vec{v}_b = (9.00 \text{ m/s})\hat{\text{i}}$ as it chases a moth that flies with velocity $\vec{v}_m = (8.00 \text{ m/s})\hat{\text{i}}$. What frequency f_{md} does the moth detect? What frequency f_{bd} does the bat detect in the returning echo from the moth?

KEY IDEAS

The frequency is shifted by the relative motion of the bat and moth. Because they move along a single axis, the shifted frequency is given by Eq. 17-47 for the general Doppler effect. Motion *toward* tends to shift the frequency *up*, and motion *away* tends to shift the frequency *down.*

Detection by moth: The general Doppler equation is

$$f' = f\frac{v \pm v_D}{v \pm v_S}. \qquad (17\text{-}56)$$

Here, the detected frequency f' that we want to find is the frequency f_{md} detected by the moth. On the right side of the equation, the emitted frequency f is the bat's emission frequency $f_{be} = 82.52$ kHz, the speed of sound is $v = 343$ m/s, the speed v_D of the detector is the moth's speed $v_m = 8.00$ m/s, and the speed v_S of the source is the bat's speed $v_b = 9.00$ m/s.

These substitutions into Eq. 17-56 are easy to make. However, the decisions about the plus and minus signs can be tricky. Think in terms of *toward* and *away*. We have the speed of the moth (the detector) in the numerator of Eq. 17-56. The moth moves *away* from the bat, which tends to lower the detected frequency. Because the speed is in the numerator, we choose the minus sign to meet that tendency (the numerator becomes smaller). These reasoning steps are shown in Table 17-3.

We have the speed of the bat in the denominator of Eq. 17-56. The bat moves *toward* the moth, which tends to increase the detected frequency. Because the speed is in the denominator, we choose the minus sign to meet that tendency (the denominator becomes smaller).

With these substitutions and decisions, we have

$$f_{md} = f_{be}\frac{v - v_m}{v - v_b}$$
$$= (82.52 \text{ kHz})\frac{343 \text{ m/s} - 8.00 \text{ m/s}}{343 \text{ m/s} - 9.00 \text{ m/s}}$$
$$= 82.767 \text{ kHz} \approx 82.8 \text{ kHz}. \qquad \text{(Answer)}$$

Detection of echo by bat: In the echo back to the bat, the moth acts as a source of sound, emitting at the frequency f_{md} we just calculated. So now the moth is the source (moving *away*) and the bat is the detector (moving *toward*). The reasoning steps are shown in Table 17-3. To find the frequency f_{bd} detected by the bat, we write Eq. 17-56 as

$$f_{bd} = f_{md}\frac{v + v_b}{v + v_m}$$
$$= (82.767 \text{ kHz})\frac{343 \text{ m/s} + 9.00 \text{ m/s}}{343 \text{ m/s} + 8.00 \text{ m/s}}$$
$$= 83.00 \text{ kHz} \approx 83.0 \text{ kHz}. \qquad \text{(Answer)}$$

Some moths evade bats by "jamming" the detection system with ultrasonic clicks.

Supersonic Speeds, Shock Waves

$$\sin\theta = \frac{vt}{v_S t} = \frac{v}{v_S} \qquad \text{(Mach cone angle).}$$