Physics 1

UNIVERSITÄT SIEGEN
FESTKÖRPERPHYSIK

Lecture 3: Rotation and angular momentum

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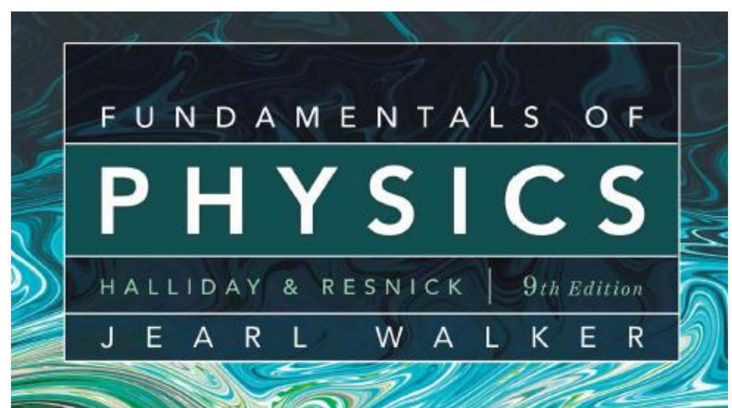
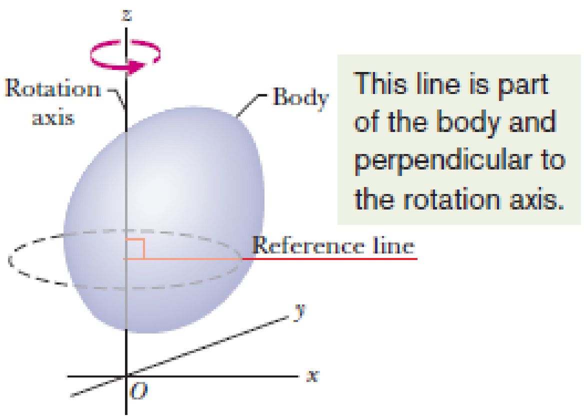
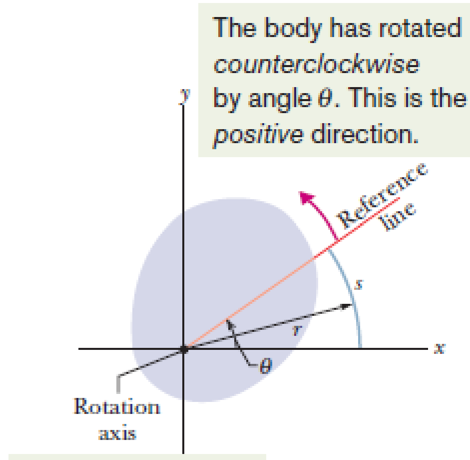
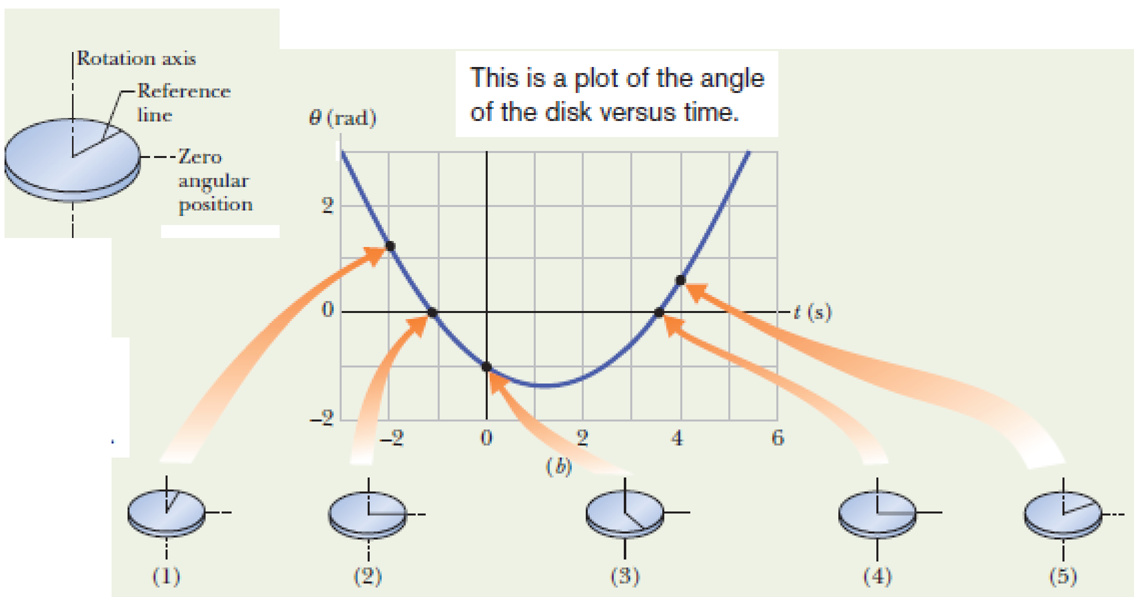


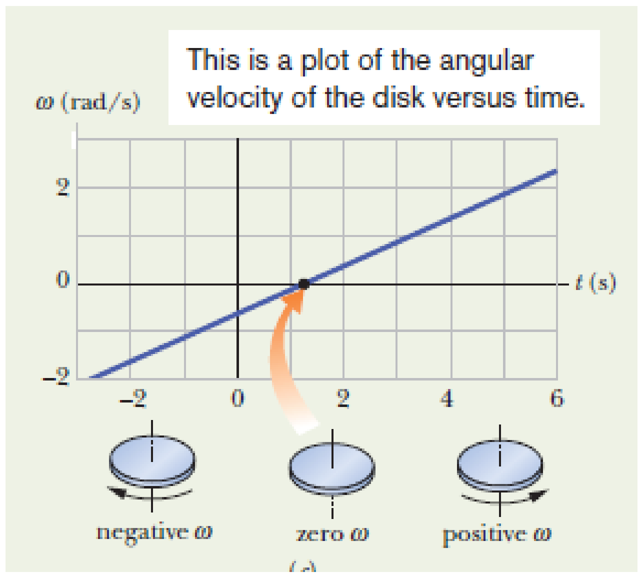
Angle, Θ, angular velocity, ω





$$\omega = d\Theta/dt$$





Angular velocity derived from angular acceleration

A child's top is spun with angular acceleration

$$\alpha = 5t^3 - 4t,$$

with t in seconds and α in radians per second-squared. At $t = 0$, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2$ rad.

(a) Obtain an expression for the angular velocity $\omega(t)$ of the top. That is, find an expression that explicitly indicates how the angular velocity depends on time. (We can tell that there *is* such a dependence because the top is undergoing an angular acceleration, which means that its angular velocity *is* changing.)

KEY IDEA

By definition, $\alpha(t)$ is the derivative of $\omega(t)$ with respect to time. Thus, we can find $\omega(t)$ by integrating $\alpha(t)$ with respect to time.

Calculations: Equation 10-8 tells us

$$d\omega = \alpha\, dt,$$

so

$$\int d\omega = \int \alpha\, dt.$$

From this we find

$$\omega = \int (5t^3 - 4t)\, dt = \tfrac{5}{4}t^4 - \tfrac{4}{2}t^2 + C.$$

To evaluate the constant of integration C, we note that $\omega = 5$ rad/s at $t = 0$. Substituting these values in our expression for ω yields

$$5 \text{ rad/s} = 0 - 0 + C,$$

so $C = 5$ rad/s. Then

$$\omega = \tfrac{5}{4}t^4 - 2t^2 + 5. \qquad \text{(Answer)}$$

(b) Obtain an expression for the angular position $\theta(t)$ of the top.

KEY IDEA

By definition, $\omega(t)$ is the derivative of $\theta(t)$ with respect to time. Therefore, we can find $\theta(t)$ by integrating $\omega(t)$ with respect to time.

Calculations: Since Eq. 10-6 tells us that

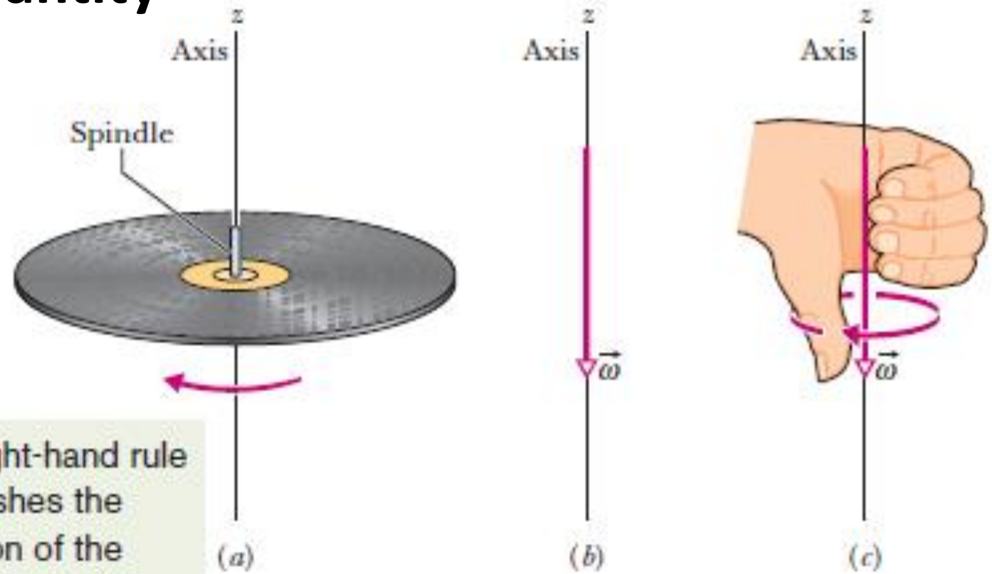
$$d\theta = \omega\, dt,$$

we can write

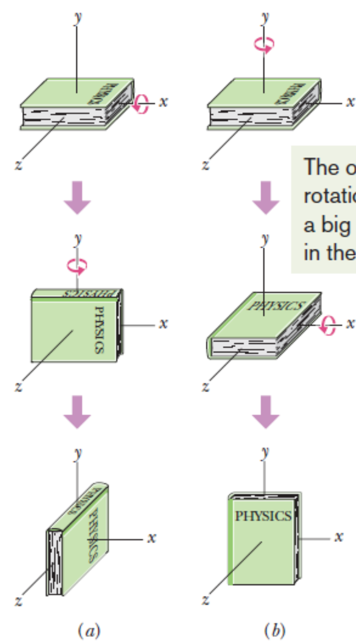
$$\begin{aligned}\theta &= \int \omega\, dt = \int (\tfrac{5}{4}t^4 - 2t^2 + 5)\, dt \\ &= \tfrac{1}{4}t^5 - \tfrac{2}{3}t^3 + 5t + C' \\ &= \tfrac{1}{4}t^5 - \tfrac{2}{3}t^3 + 5t + 2, \qquad \text{(Answer)}\end{aligned}$$

where C' has been evaluated by noting that $\theta = 2$ rad at $t = 0$.

Vector quantity

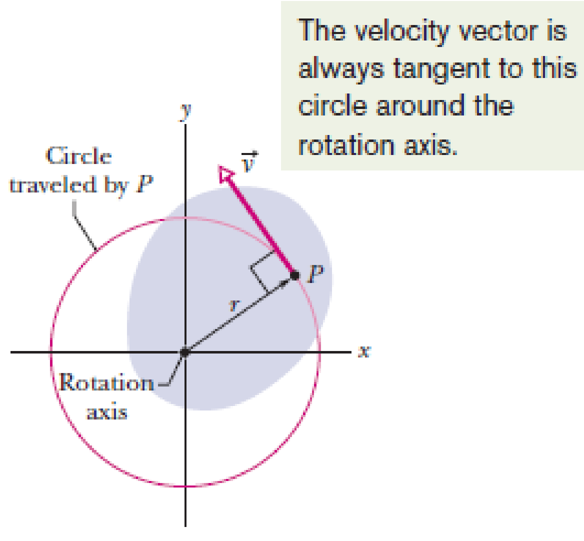


This right-hand rule establishes the direction of the angular velocity vector.

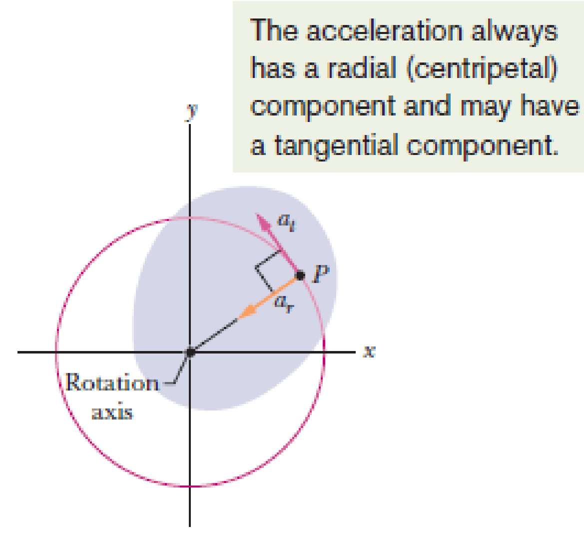


The order of the rotations makes a big difference in the result.

Relation between linear and angular variables



(a)



(b)

The Position

$s = \theta r$ (radian measure).

The Speed

$$\frac{ds}{dt} = \frac{d\theta}{dt} r.$$

$v = \omega r$ (radian measure).

period of revolution

$$T = \frac{2\pi r}{v}.$$

$$T = \frac{2\pi}{\omega}$$ (radian measure).

The Acceleration

$$\frac{dv}{dt} = \frac{d\omega}{dt} r.$$

tangential acceleration

$a_t = \alpha r$ (radian measure),

radial acceleration

$a_r = \frac{v^2}{r} = \omega^2 r$ (radian measure).

In spite of the extreme care taken in engineering a roller coaster, an unlucky few of the millions of people who ride roller coasters each year end up with a medical condition called *roller-coaster headache*. Symptoms, which might not appear for several days, include vertigo and headache, both severe enough to require medical treatment.

Let's investigate the probable cause by designing the track for our own *induction roller coaster* (which can be accelerated by magnetic forces even on a horizontal track). To create an initial thrill, we want each passenger to leave the loading point with acceleration g along the horizontal track. To increase the thrill, we also want that first section of track to form a circular arc (Fig. 10-10), so that the passenger also experiences a centripetal acceleration. As the passenger accelerates along the arc, the magnitude of this centripetal acceleration increases alarmingly. When the magnitude a of the net acceleration reaches $4g$ at some point P and angle θ_P along the arc, we want the passenger then to move in a straight line, along a tangent to the arc.

(a) What angle θ_P should the arc subtend so that a is $4g$ at point P?

KEY IDEAS

(1) At any given time, the passenger's net acceleration $\vec{a}$ is the vector sum of the tangential acceleration $\vec{a}_t$ along the track and the radial acceleration $\vec{a}_r$ toward the arc's center of curvature (as in Fig. 10-9*b*). (2) The value of a_r at any given time depends on the angular speed ω according to Eq. 10-23 ($a_r = \omega^2 r$, where r is the radius of the circular arc). (3) An angular acceleration α around the arc is associated with the tangential acceleration a_t along the track according to Eq. 10-22 ($a_t = \alpha r$). (4) Because a_t and r are constant, so is α and thus we can use the constant angular-acceleration equations.

Calculations: Because we are trying to determine a value for angular position θ, let's choose Eq. 10-14 from among the constant angular-acceleration equations:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0). \tag{10-24}$$

For the angular acceleration α, we substitute from Eq. 10-22:

$$\alpha = \frac{a_t}{r}. \tag{10-25}$$

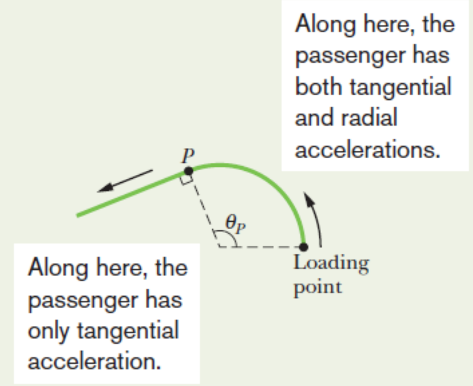


Fig. 10-10 An overhead view of a horizontal track for a roller coaster. The track begins as a circular arc at the loading point and then, at point P, continues along a tangent to the arc.

We also substitute $\omega_0 = 0$ and $\theta_0 = 0$, and we find

$$\omega^2 = \frac{2a_t\theta}{r}. \tag{10-26}$$

Substituting this result for ω^2 into

$$a_r = \omega^2 r \tag{10-27}$$

gives a relation between the radial acceleration, the tangential acceleration, and the angular position θ:

$$a_r = 2a_t\theta. \tag{10-28}$$

Because $\vec{a}_t$ and $\vec{a}_r$ are perpendicular vectors, their sum has the magnitude

$$a = \sqrt{a_t^2 + a_r^2}. \tag{10-29}$$

Substituting for a_r from Eq. 10-28 and solving for θ lead to

$$\theta = \tfrac{1}{2}\sqrt{\frac{a^2}{a_t^2} - 1}. \tag{10-30}$$

When a reaches the design value of $4g$, angle θ is the angle θ_P we want. Substituting $a = 4g$, $\theta = \theta_P$, and $a_t = g$ into Eq. 10-30, we find

$$\theta_P = \tfrac{1}{2}\sqrt{\frac{(4g)^2}{g^2} - 1} = 1.94 \text{ rad} = 111°. \qquad \text{(Answer)}$$

(b) What is the magnitude a of the passenger's net acceleration at point P and after point P?

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Rotational Inertia

$$I = \Sigma\, m_i\, r_i^2 = \int r^2\, dm$$

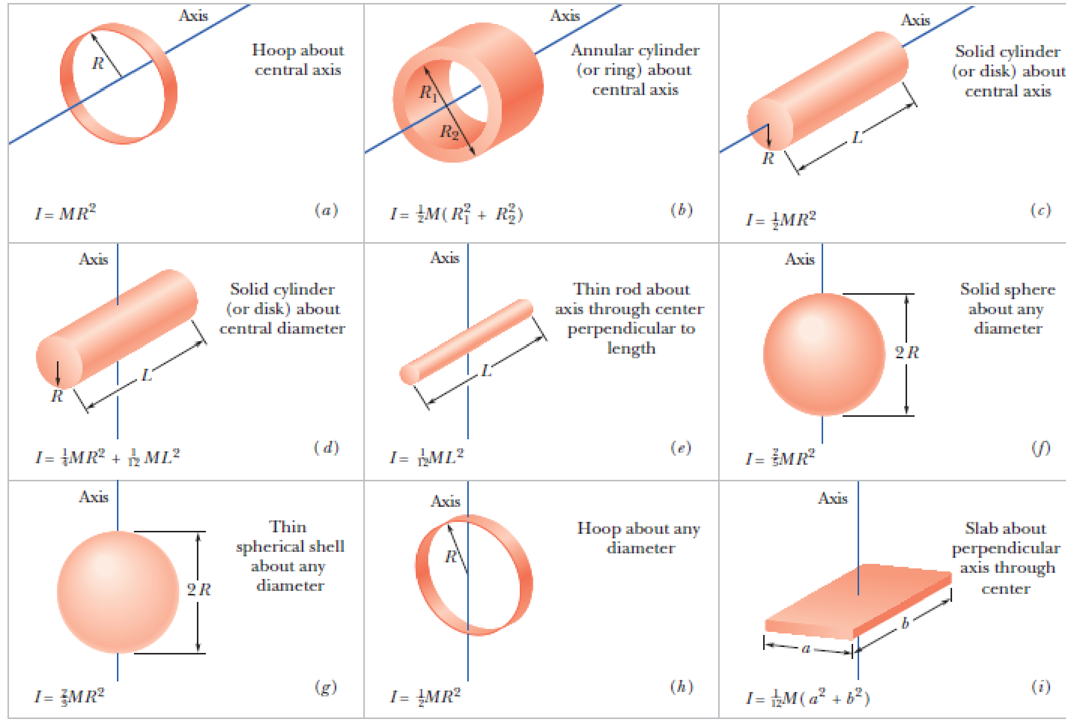
$$K = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}m_3v_3^2 + \cdots$$
$$= \sum \tfrac{1}{2}m_iv_i^2,$$

$$v = \omega r$$

$$K = \sum \tfrac{1}{2}m_i(\omega r_i)^2 = \tfrac{1}{2}\left(\sum m_ir_i^2\right)\omega^2,$$

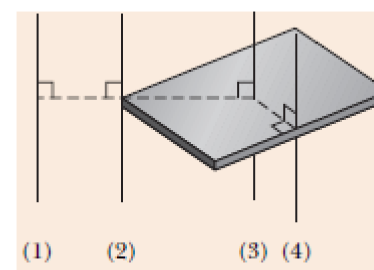
kinetic energy : $K = \frac{1}{2} I \omega^2$

Some Rotational Inertias



$$I = \sum m_ir_i^2 \quad \text{(rotational inertia)}$$

$$I = \int r^2\, dm \quad \text{(rotational inertia, continuous body).}$$



Rotational inertia of a two-particle system

Figure 10-13*a* shows a rigid body consisting of two particles of mass m connected by a rod of length L and negligible mass.

(a) What is the rotational inertia I_{com} about an axis through the center of mass, perpendicular to the rod as shown?

KEY IDEA

Because we have only two particles with mass, we can find the body's rotational inertia I_{com} by using Eq. 10-33 rather than by integration.

Calculations: For the two particles, each at perpendicular distance $\frac{1}{2}L$ from the rotation axis, we have

$$I = \sum m_i r_i^2 = (m)(\tfrac{1}{2}L)^2 + (m)(\tfrac{1}{2}L)^2$$
$$= \tfrac{1}{2}mL^2. \qquad \text{(Answer)}$$

(b) What is the rotational inertia I of the body about an axis through the left end of the rod and parallel to the first axis (Fig. 10-13*b*)?

KEY IDEAS

This situation is simple enough that we can find I using either of two techniques. The first is similar to the one used in part (a). The other, more powerful one is to apply the parallel-axis theorem.

First technique: We calculate I as in part (a), except here the perpendicular distance r_i is zero for the particle on the left and L for the particle on the right. Now Eq. 10-33 gives us

$$I = m(0)^2 + mL^2 = mL^2. \qquad \text{(Answer)}$$

Second technique: Because we already know I_{com} about an axis through the center of mass and because the axis here is parallel to that "com axis," we can apply the parallel-axis theorem (Eq. 10-36). We find

$$I = I_{com} + Mh^2 = \tfrac{1}{2}mL^2 + (2m)(\tfrac{1}{2}L)^2$$
$$= mL^2. \qquad \text{(Answer)}$$

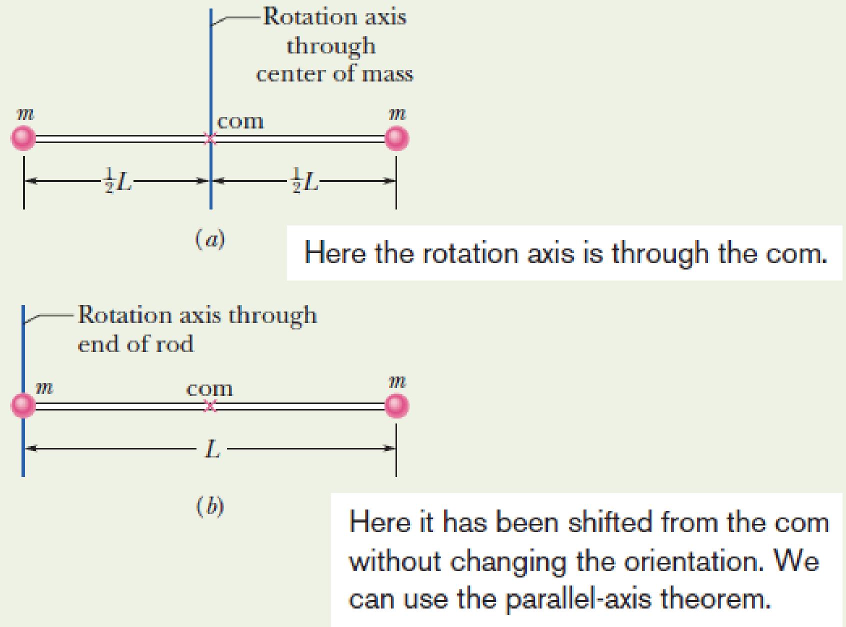


Fig. 10-13 A rigid body consisting of two particles of mass m joined by a rod of negligible mass.

Rotational inertia of a uniform rod, integration

Figure 10-14 shows a thin, uniform rod of mass M and length L, on an x axis with the origin at the rod's center.

(a) What is the rotational inertia of the rod about the perpendicular rotation axis through the center?

KEY IDEAS

(1) Because the rod is uniform, its center of mass is at its center. Therefore, we are looking for I_{com}. (2) Because the rod is a continuous object, we must use the integral of Eq. 10-35,

$$I = \int r^2\, dm, \qquad (10\text{-}38)$$

to find the rotational inertia.

Calculations: We want to integrate with respect to coordinate x (not mass m as indicated in the integral), so we must relate the mass dm of an element of the rod to its length dx along the rod. (Such an element is shown in Fig. 10-14.) Because the rod is uniform, the ratio of mass to length is the same for all the elements and for the rod as a whole. Thus, we can write

$$\frac{\text{element's mass } dm}{\text{element's length } dx} = \frac{\text{rod's mass } M}{\text{rod's length } L}$$

or

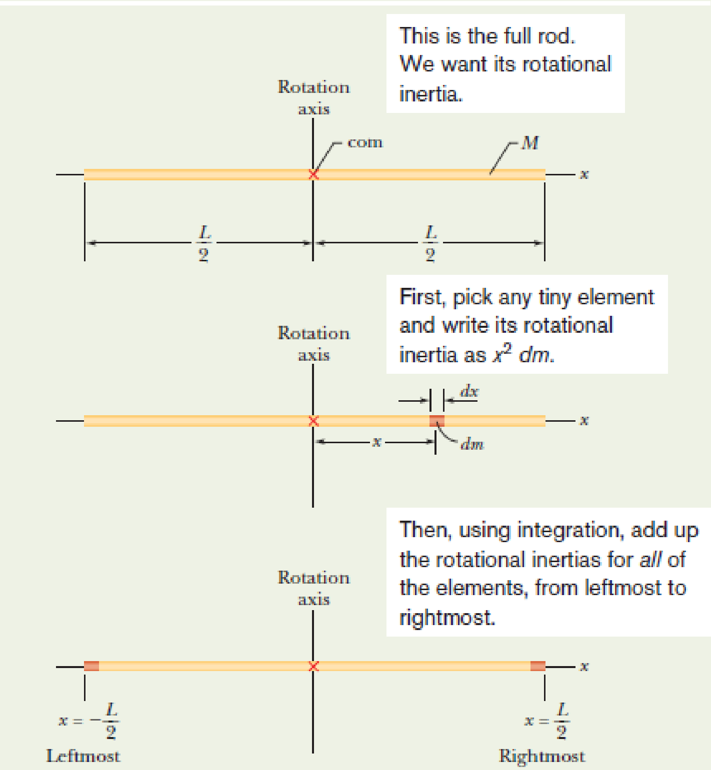
$$dm = \frac{M}{L}\, dx.$$

We can now substitute this result for dm and x for r in Eq. 10-38. Then we integrate from end to end of the rod (from $x = -L/2$ to $x = L/2$) to include all the elements. We find

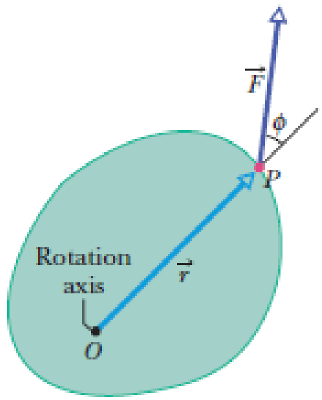
$$I = \int_{x=-L/2}^{x=+L/2} x^2 \left(\frac{M}{L}\right) dx$$

$$= \frac{M}{3L}\left[x^3\right]_{-L/2}^{+L/2} = \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$

$$= \tfrac{1}{12} ML^2. \qquad \text{(Answer)}$$

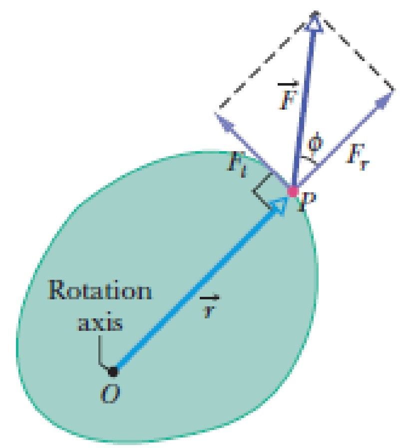


Torque

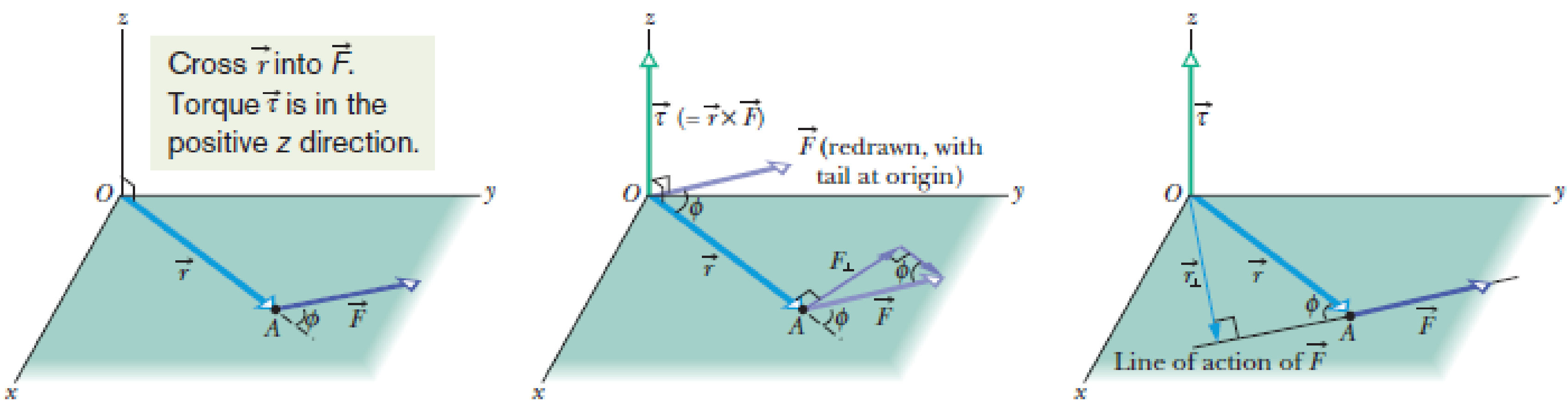


$$\vec{\tau} = \vec{r} \times \vec{F}$$

(a)



$$\tau = r\,F\,\sin\phi$$



(a) (b) (c)

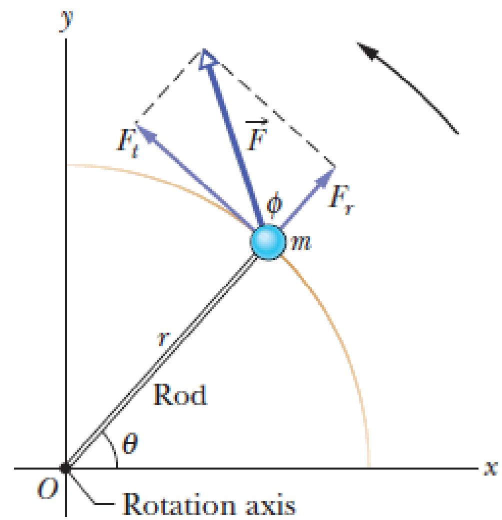
Newton's Second Law for Rotation

Newton´s 2nd law $F_t = ma_t.$

torque acting on the particle $\tau = F_t r = ma_t r.$

$a_t = \alpha r$ $\tau = m(\alpha r)r = (mr^2)\alpha.$

$\tau = I\alpha$ (radian measure).



Work and Rotational Kinetic Energy

$$\Delta K = K_f - K_i = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = W$$ (work–kinetic energy theorem).

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$ (work, rotation about fixed axis),

$$W = \tau(\theta_f - \theta_i)$$ (work, constant torque).

$$P = \frac{dW}{dt} = \tau\omega$$ (power, rotation about fixed axis).

Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

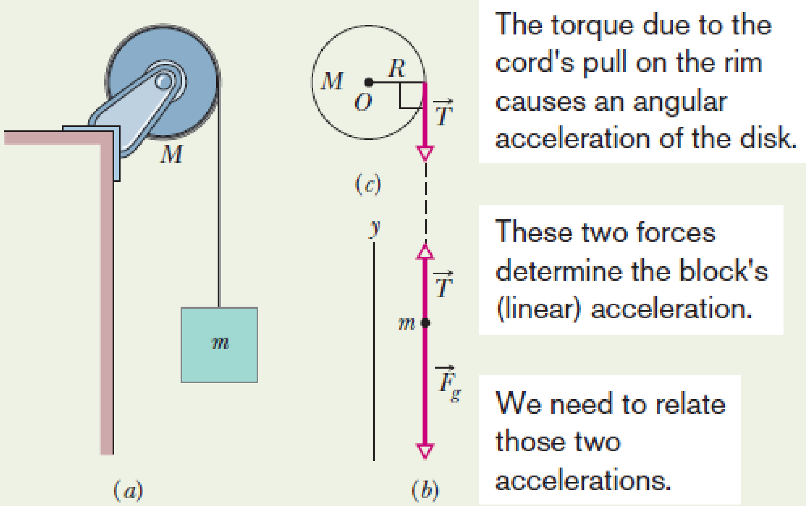


Fig. 10-18 (*a*) The falling block causes the disk to rotate. (*b*) A free-body diagram for the block. (*c*) An incomplete free-body diagram for the disk.

KEY IDEA

We can find K with Eq. 10-34 ($K = \frac{1}{2}I\omega^2$). We already know that $I = \frac{1}{2}MR^2$, but we do not yet know ω at $t = 2.5$ s. However, because the angular acceleration α has the constant value of $-24\ \text{rad/s}^2$, we can apply the equations for constant angular acceleration in Table 10-1.

Calculations: Because we want ω and know α and ω_0 (= 0), we use Eq. 10-12:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t.$$

Substituting $\omega = \alpha t$ and $I = \frac{1}{2}MR^2$ into Eq. 10-34, we find

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(\tfrac{1}{2}MR^2)(\alpha t)^2 = \tfrac{1}{4}M(R\alpha t)^2$$
$$= \tfrac{1}{4}(2.5\ \text{kg})[(0.20\ \text{m})(-24\ \text{rad/s}^2)(2.5\ \text{s})]^2$$
$$= 90\ \text{J}. \qquad \text{(Answer)}$$

KEY IDEA

We can also get this answer by finding the disk's kinetic energy from the work done on the disk.

Let the disk in Fig. 10-18 start from rest at time $t = 0$ and also let the tension in the massless cord be 6.0 N and the angular acceleration of the disk be $-24\ \text{rad/s}^2$. What is its rotational kinetic energy K at $t = 2.5$ s?

etic energy, torque, disk

Calculations: First, we relate the *change* in the kinetic energy of the disk to the net work W done on the disk, using the work–kinetic energy theorem of Eq. 10-52 ($K_f - K_i = W$). With K substituted for K_f and 0 for K_i, we get

$$K = K_i + W = 0 + W = W. \qquad (10\text{-}60)$$

Next we want to find the work W. We can relate W to the torques acting on the disk with Eq. 10-53 or 10-54. The only torque causing angular acceleration and doing work is the torque due to force $\vec{T}$ on the disk from the cord, which is equal to $-TR$. Because α is constant, this torque also must be constant. Thus, we can use Eq. 10-54 to write

$$W = \tau(\theta_f - \theta_i) = -TR(\theta_f - \theta_i). \qquad (10\text{-}61)$$

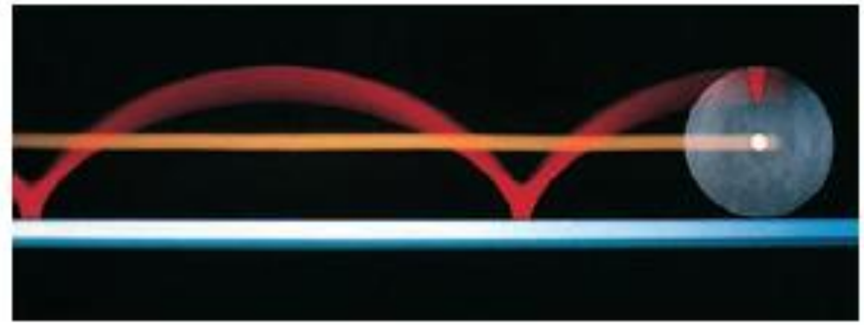
Because α is constant, we can use Eq. 10-13 to find $\theta_f - \theta_i$. With $\omega_i = 0$, we have

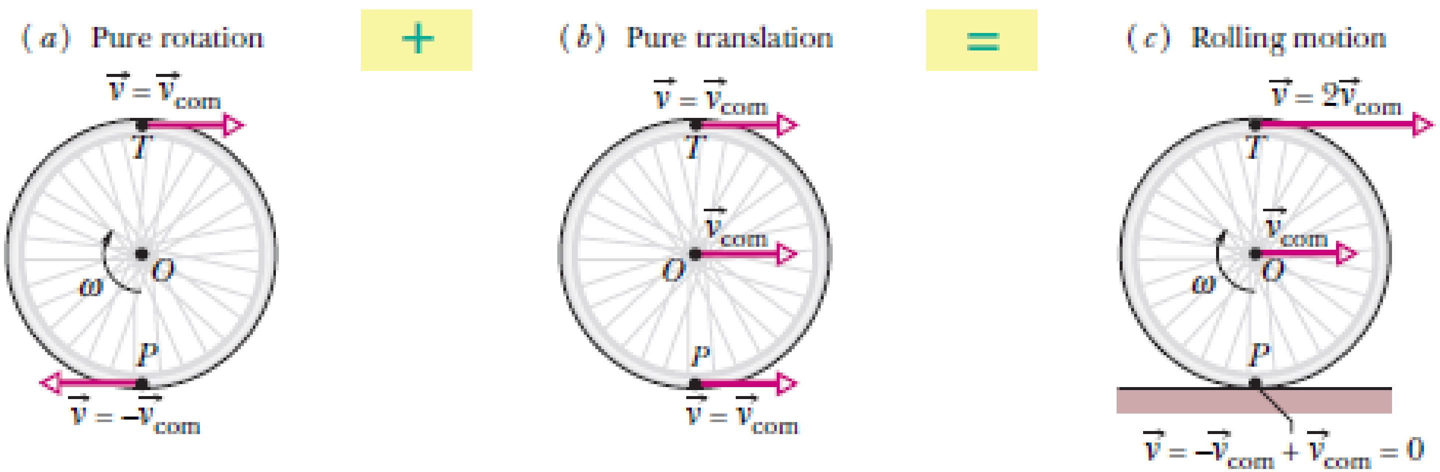
$$\theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\alpha t^2.$$

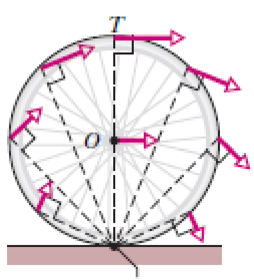
Now we substitute this into Eq. 10-61 and then substitute the result into Eq. 10-60. Inserting the given values $T = 6.0$ N and $\alpha = -24\ \text{rad/s}^2$, we have

$$K = W = -TR(\theta_f - \theta_i) = -TR(\tfrac{1}{2}\alpha t^2) = -\tfrac{1}{2}TR\alpha t^2$$
$$= -\tfrac{1}{2}(6.0\ \text{N})(0.20\ \text{m})(-24\ \text{rad/s}^2)(2.5\ \text{s})^2$$
$$= 90\ \text{J}. \qquad \text{(Answer)}$$

Rolling + Translation







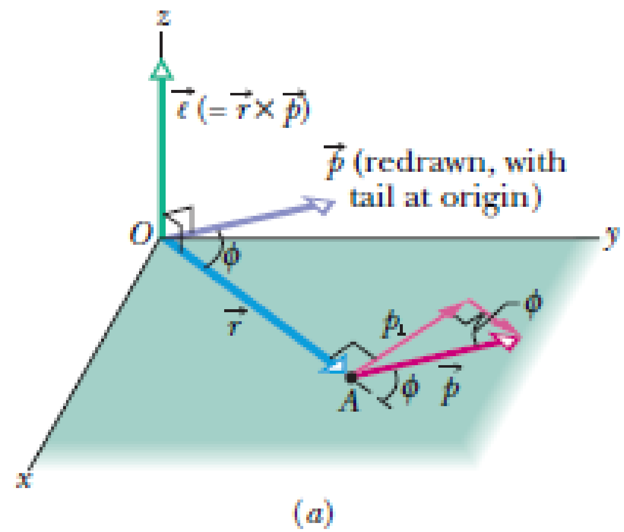
$$K = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$$

Angular momentum

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$\ell = rmv \sin \phi,$$

$$\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3 + \cdots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i.$$



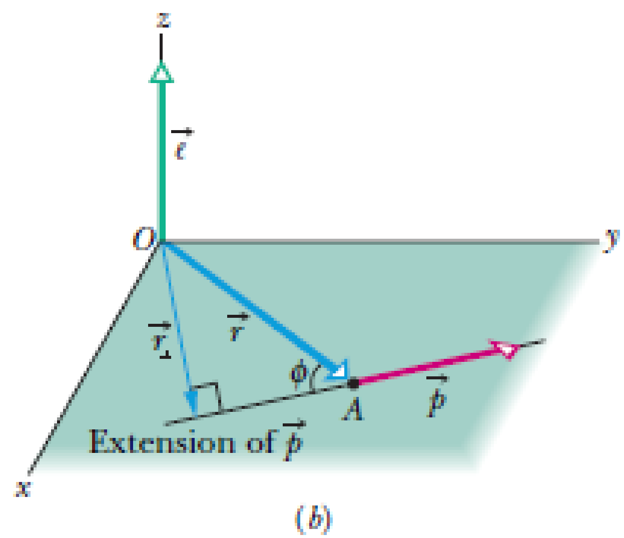
Angular momentum of a rigid body

$l = r\, p \sin 90° = r\, \Delta m\, v$

$l_z = l \sin \Theta = (r \sin \Theta)(\Delta m\, v)$

$L_z = \omega\ (\Sigma\, \Delta m_i\, r_\perp^2)$

$$L = I\, \omega$$



Newton´s 2nd law

$$\vec{\ell} = m(\vec{r} \times \vec{v}),$$

$$\frac{d\vec{\ell}}{dt} = m\left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v}\right).$$

$$\frac{d\vec{\ell}}{dt} = m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v}).$$

$$\frac{d\vec{\ell}}{dt} = m(\vec{r} \times \vec{a}) = \vec{r} \times m\vec{a}.$$

$$\frac{d\vec{\ell}}{dt} = \vec{r} \times \vec{F}_{\text{net}} = \sum(\vec{r} \times \vec{F}).$$

$$\boxed{\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt}.}$$

Angular momentum of a two-particle system

Figure 11-13 shows an overhead view of two particles moving at constant momentum along horizontal paths. Particle 1, with momentum magnitude $p_1 = 5.0\ \text{kg}\cdot\text{m/s}$, has position vector $\vec{r}_1$ and will pass 2.0 m from point O. Particle 2, with momentum magnitude $p_2 = 2.0\ \text{kg}\cdot\text{m/s}$, has position vector $\vec{r}_2$ and will pass 4.0 m from point O. What are the magnitude and direction of the net angular momentum $\vec{L}$ about point O of the two-particle system?

KEY IDEA

To find $\vec{L}$, we can first find the individual angular momenta $\vec{\ell}_1$ and $\vec{\ell}_2$ and then add them. To evaluate their magnitudes, we can use any one of Eqs. 11-18 through 11-21. However, Eq. 11-21 is easiest, because we are given the perpendicular distances $r_{1\perp}$ (= 2.0 m) and $r_{2\perp}$ (= 4.0 m) and the momentum magnitudes p_1 and p_2.

Calculations: For particle 1, Eq. 11-21 yields

$$\ell_1 = r_{\perp 1}p_1 = (2.0\ \text{m})(5.0\ \text{kg}\cdot\text{m/s}) = 10\ \text{kg}\cdot\text{m}^2/\text{s}.$$

To find the direction of vector $\vec{\ell}_1$, we use Eq. 11-18 and the right-hand rule for vector products. For $\vec{r}_1 \times \vec{p}_1$, the vector product is out of the page, perpendicular to the plane of Fig. 11-13. This is the positive direction, consistent with the counterclockwise rotation of the particle's position vector $\vec{r}_1$ around O as particle 1 moves. Thus, the angular momentum vector for particle 1 is

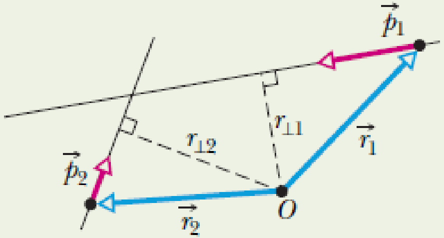


Fig. 11-13 Two particles pass near point O.

$$\ell_1 = +10\ \text{kg}\cdot\text{m}^2/\text{s}.$$

Similarly, the magnitude of $\vec{\ell}_2$ is

$$\ell_2 = r_{\perp 2}p_2 = (4.0\ \text{m})(2.0\ \text{kg}\cdot\text{m/s}) = 8.0\ \text{kg}\cdot\text{m}^2/\text{s},$$

and the vector product $\vec{r}_2 \times \vec{p}_2$ is into the page, which is the negative direction, consistent with the clockwise rotation of $\vec{r}_2$ around O as particle 2 moves. Thus, the angular momentum vector for particle 2 is

$$\ell_2 = -8.0\ \text{kg}\cdot\text{m}^2/\text{s}.$$

The net angular momentum for the two-particle system is

$$L = \ell_1 + \ell_2 = +10\ \text{kg}\cdot\text{m}^2/\text{s} + (-8.0\ \text{kg}\cdot\text{m}^2/\text{s}) = +2.0\ \text{kg}\cdot\text{m}^2/\text{s}. \qquad \text{(Answer)}$$

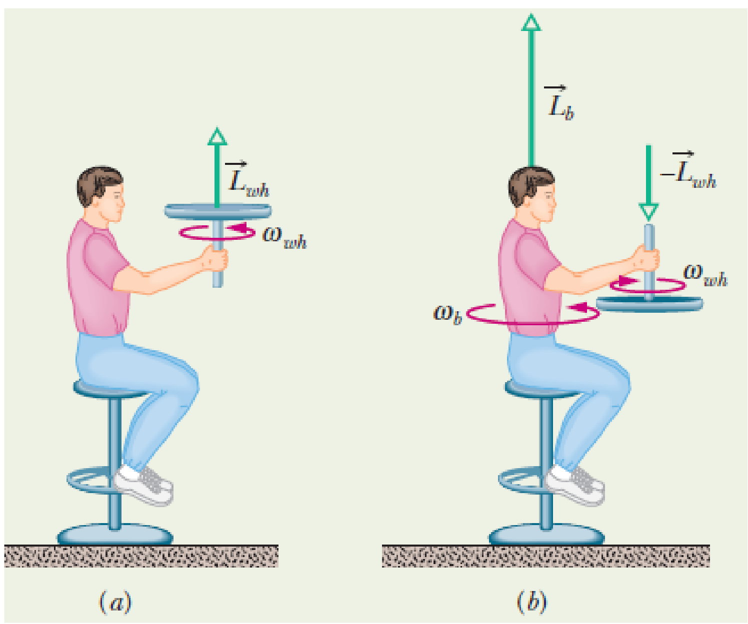
The plus sign means that the system's net angular momentum about point O is out of the page.

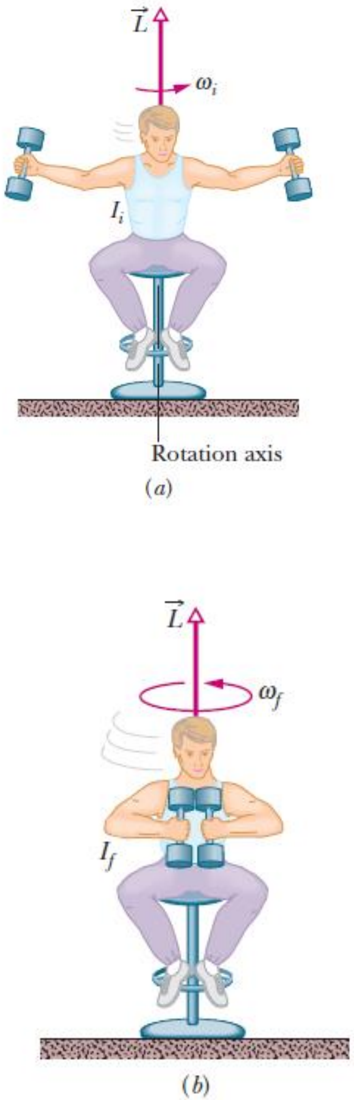
Conservation of Angular Momentum

$$\vec{L} = \text{a constant} \qquad \text{(isolated system).}$$

$$\begin{pmatrix}\text{net angular momentum} \\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{net angular momentum} \\ \text{at some later time } t_f\end{pmatrix},$$

$$\vec{L}_i = \vec{L}_f \qquad \text{(isolated system).}$$





Conservation of angular momentum, cockroach on disk

In Fig. 11-21, a cockroach with mass m rides on a disk of mass $6.00m$ and radius R. The disk rotates like a merry-go-round around its central axis at angular speed $\omega_i = 1.50$ rad/s. The cockroach is initially at radius $r = 0.800R$, but then it crawls out to the rim of the disk. Treat the cockroach as a particle. What then is the angular speed?

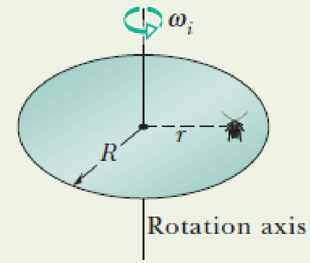


Fig. 11-21 A cockroach rides at radius r on a disk rotating like a merry-go-round.

KEY IDEAS

(1) The cockroach's crawl changes the mass distribution (and thus the rotational inertia) of the cockroach–disk system. (2) The angular momentum of the system does not change because there is no external torque to change it. (The forces and torques due to the cockroach's crawl are internal to the system.) (3) The magnitude of the angular momentum of a rigid body or a particle is given by Eq. 11-31 ($L = I\omega$).

Calculations: We want to find the final angular speed. Our key is to equate the final angular momentum L_f to the initial angular momentum L_i, because both involve angular speed. They also involve rotational inertia I. So, let's start by finding the rotational inertia of the system of cockroach and disk before and after the crawl.

The rotational inertia of a disk rotating about its central axis is given by Table 10-2*c* as $\frac{1}{2}MR^2$. Substituting $6.00m$ for the mass M, our disk here has rotational inertia

$$I_d = 3.00mR^2. \tag{11-36}$$

(We don't have values for m and R, but we shall continue with physics courage.)

From Eq. 10-33, we know that the rotational inertia of the cockroach (a particle) is equal to mr^2. Substituting the cockroach's initial radius ($r = 0.800R$) and final radius ($r = R$), we find that its initial rotational inertia about the rotation axis is

$$I_{ci} = 0.64mR^2 \tag{11-37}$$

and its final rotational inertia about the rotation axis is

$$I_{cf} = mR^2. \tag{11-38}$$

So, the cockroach–disk system initially has the rotational inertia

$$I_i = I_d + I_{ci} = 3.64mR^2, \tag{11-39}$$

and finally has the rotational inertia

$$I_f = I_d + I_{cf} = 4.00mR^2. \tag{11-40}$$

Next, we use Eq. 11-31 ($L = I\omega$) to write the fact that the system's final angular momentum L_f is equal to the system's initial angular momentum L_i:

$$I_f\omega_f = I_i\omega_i$$

or

$$4.00mR^2\omega_f = 3.64mR^2(1.50 \text{ rad/s}).$$

After canceling the unknowns m and R, we come to

$$\omega_f = 1.37 \text{ rad/s}. \quad \text{(Answer)}$$

Note that the angular speed decreased because part of the mass moved outward from the rotation axis, thus increasing the rotational inertia of the system.

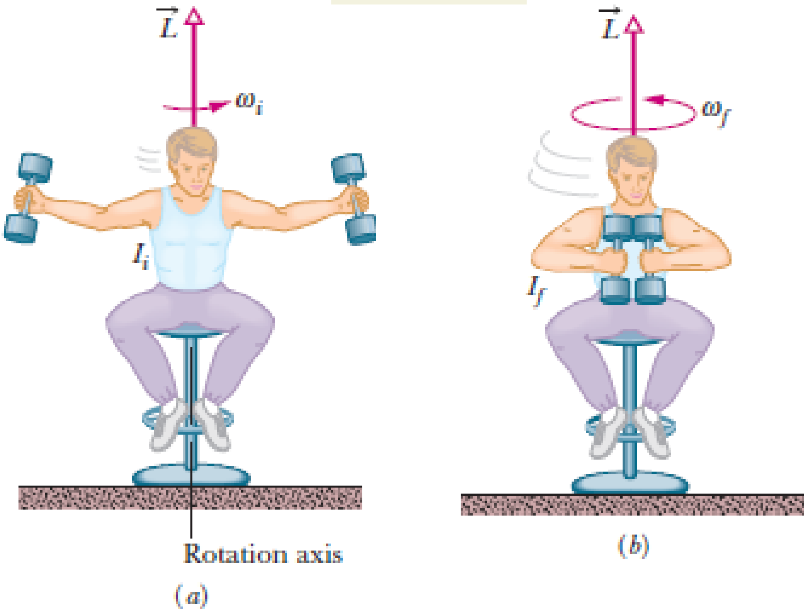
More Corresponding Variables and Relations for Translational and Rotational Motion[a]

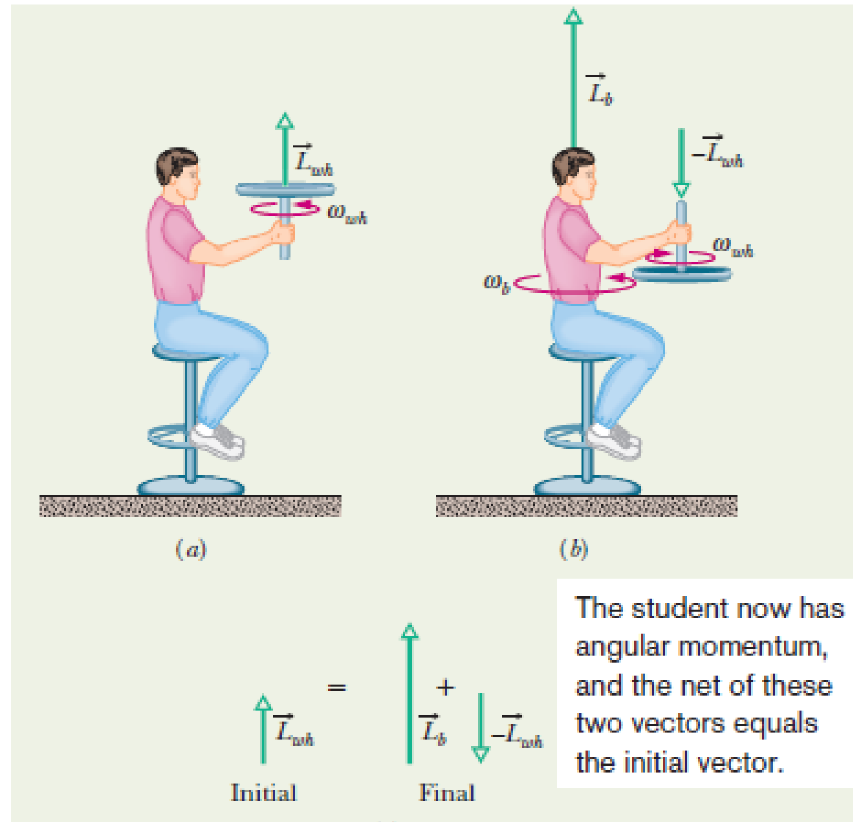
Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\ (= \vec{r} \times \vec{F})$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\ (= \vec{r} \times \vec{p})$
Linear momentum[b]	$\vec{P}\ (= \Sigma \vec{p}_i)$	Angular momentum[b]	$\vec{L}\ (= \Sigma \vec{\ell}_i)$
Linear momentum[b]	$\vec{P} = M\vec{v}_{com}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\vec{F}_{net} = \dfrac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}$ = a constant	Conservation law[d]	$\vec{L}$ = a constant

Conservation of angular momentum

$$\vec{L} = \text{a constant} \qquad \text{(isolated system).}$$

$$\vec{L}_i = \vec{L}_f$$





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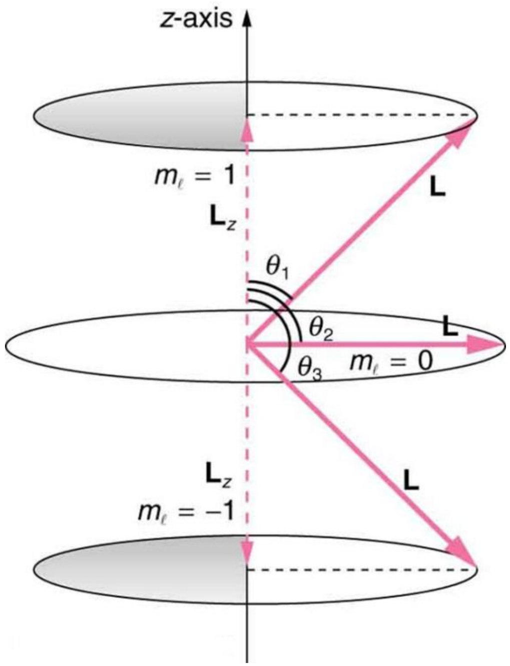
412, see script

Angular momentum in quantum mechanics



the magnitude of angular momentum, L

$$L = \sqrt{l(l+1)}h/2\pi$$

can have only the values, *l* = 0,1..,n-1, where n is principal quantum number. Different projections of L with respect to z-axis define the magnetic quantum number , m_l *: - l,…0,..+l,*

compared to classical physics only discrete values of L are possible in QM