Physics 1

UNIVERSITÄT SIEGEN

FESTKÖRPERPHYSIK

Lecture 11b: Wave to Particle dualism

Prof. Dr. U. Pietsch

Electron diffraction

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E}}$$

Particles can show wave character

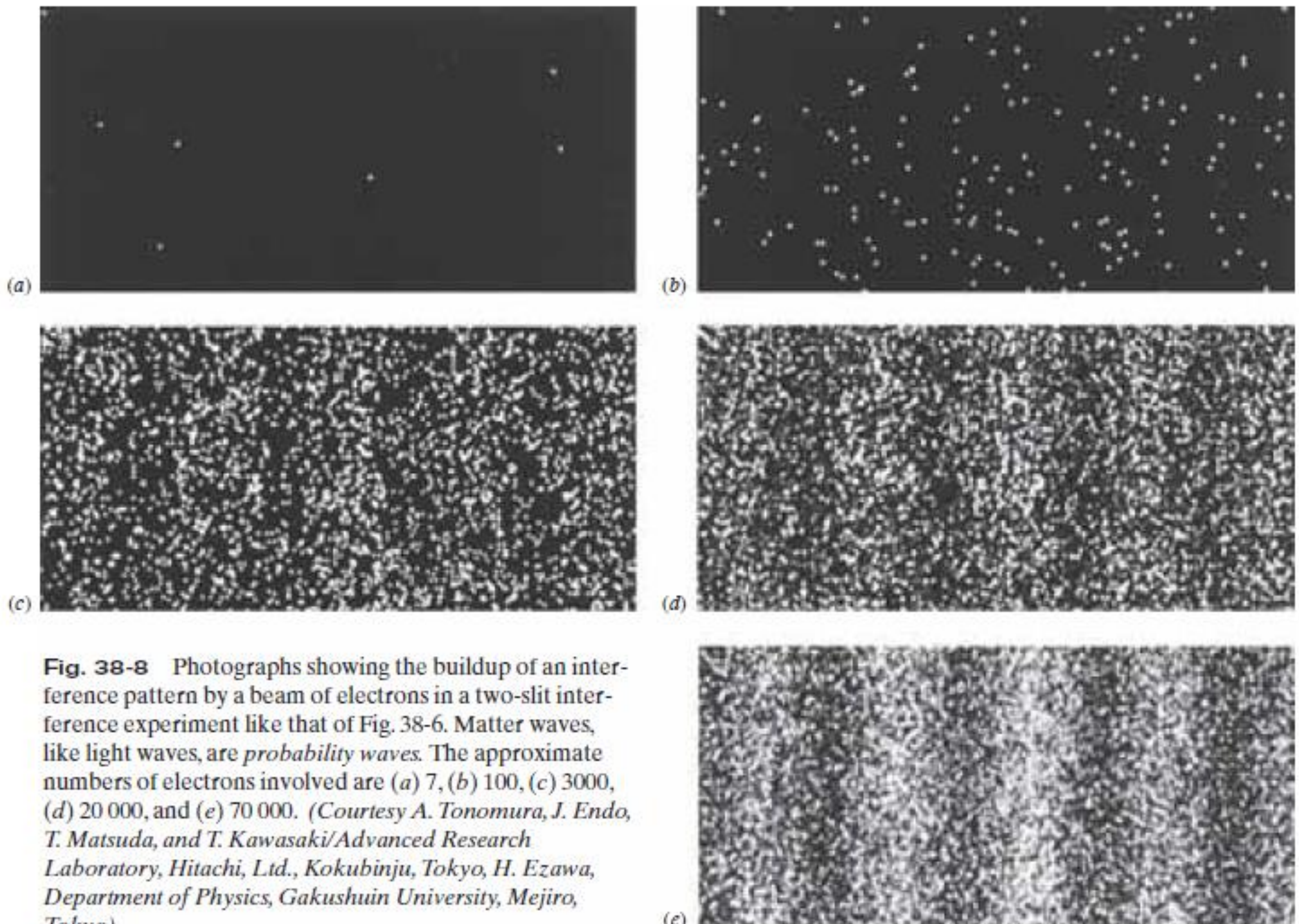


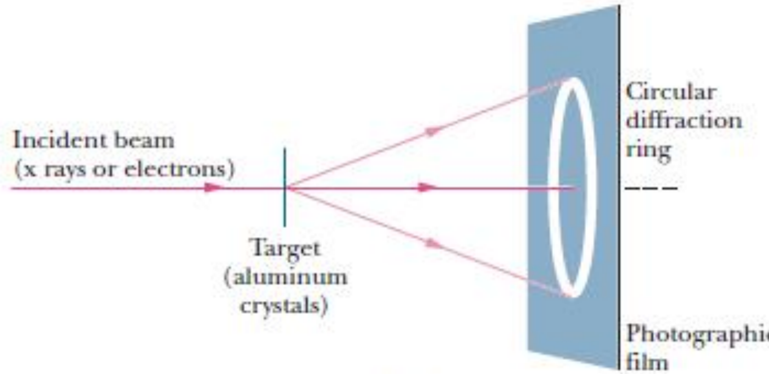
Fig. 38-8 Photographs showing the buildup of an interference pattern by a beam of electrons in a two-slit interference experiment like that of Fig. 38-6. Matter waves, like light waves, are *probability waves.* The approximate numbers of electrons involved are (*a*) 7, (*b*) 100, (*c*) 3000, (*d*) 20 000, and (*e*) 70 000. *(Courtesy A. Tonomura, J. Endo, T. Matsuda, and T. Kawasaki/Advanced Research Laboratory, Hitachi, Ltd., Kokubinju, Tokyo, H. Ezawa, Department of Physics, Gakushuin University, Mejiro, Tokyo)*

Wave length for e-beam (120eV)

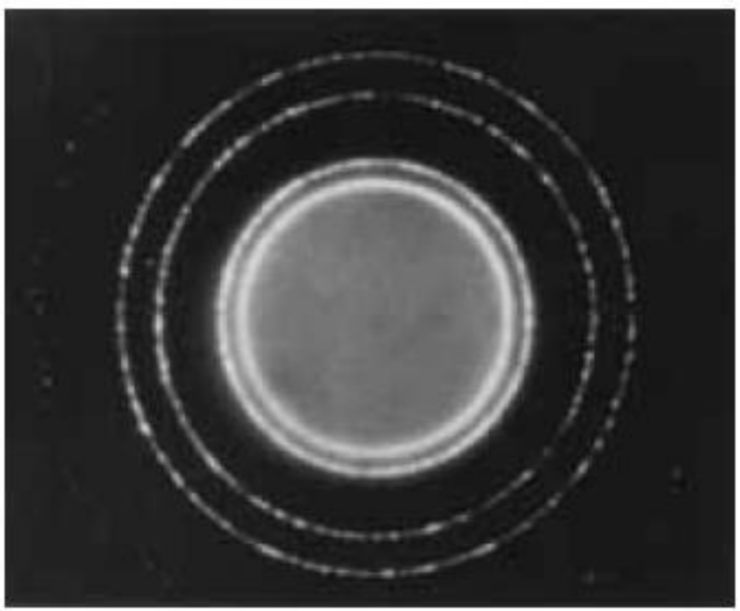
$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{5.91 \times 10^{-24}\ \text{kg}\cdot\text{m/s}} = 112\ \text{pm}.$$

e-beam vs light diffraction

Fig. 38-9 (*a*) An experimental arrangement used to demonstrate, by diffraction techniques, the wave-like character of the incident beam. Photographs of the diffraction patterns when the incident beam is (*b*) an x-ray beam (light wave) and (*c*) an electron beam (matter wave). Note that the two patterns are geometrically identical to each other. *(Photo (b) Cameca, Inc. Photo (c) from PSSC film "Matter Waves," courtesy Education Development Center, Newton, Massachusetts)*

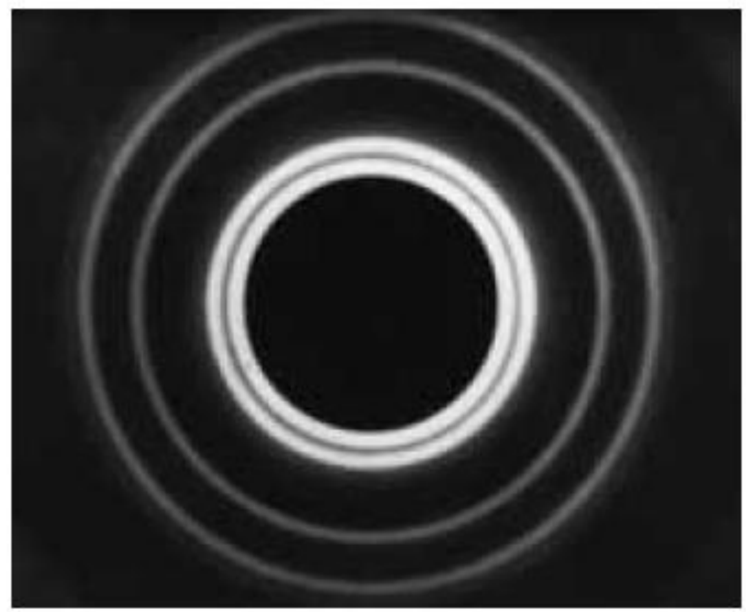


(*a*)



(*b*)

X-ray beam



(*c*)

e- beam

de Broglie wavelength of an electron

What is the de Broglie wavelength of an electron with a kinetic energy of 120 eV?

KEY IDEAS

(1) We can find the electron's de Broglie wavelength λ from Eq. 38-13 ($\lambda = h/p$) if we first find the magnitude of its momentum p. (2) We find p from the given kinetic energy K of the electron. That kinetic energy is much less than the rest energy of an electron (0.511 MeV, from Table 37-3). Thus, we can get by with the classical approximations for momentum p ($= mv$) and kinetic energy K ($= \frac{1}{2}mv^2$).

Calculations: We are given the value of the kinetic energy. So, in order to use the de Broglie relation, we first solve the kinetic energy equation for v and then substitute into the momentum equation, finding

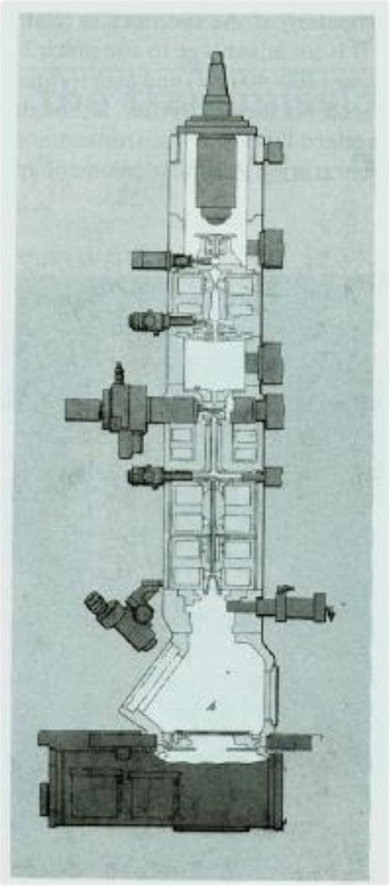
$$\begin{aligned} p &= \sqrt{2mK} \\ &= \sqrt{(2)(9.11 \times 10^{-31}\ \text{kg})(120\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})} \\ &= 5.91 \times 10^{-24}\ \text{kg} \cdot \text{m/s}. \end{aligned}$$

From Eq. 38-13 then

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{5.91 \times 10^{-24}\ \text{kg} \cdot \text{m/s}} \\ &= 1.12 \times 10^{-10}\ \text{m} = 112\ \text{pm}. \qquad \text{(Answer)} \end{aligned}$$

This wavelength associated with the electron is about the size of a typical atom. If we increase the electron's kinetic energy, the wavelength becomes even smaller.

Transmission Electron Microskop



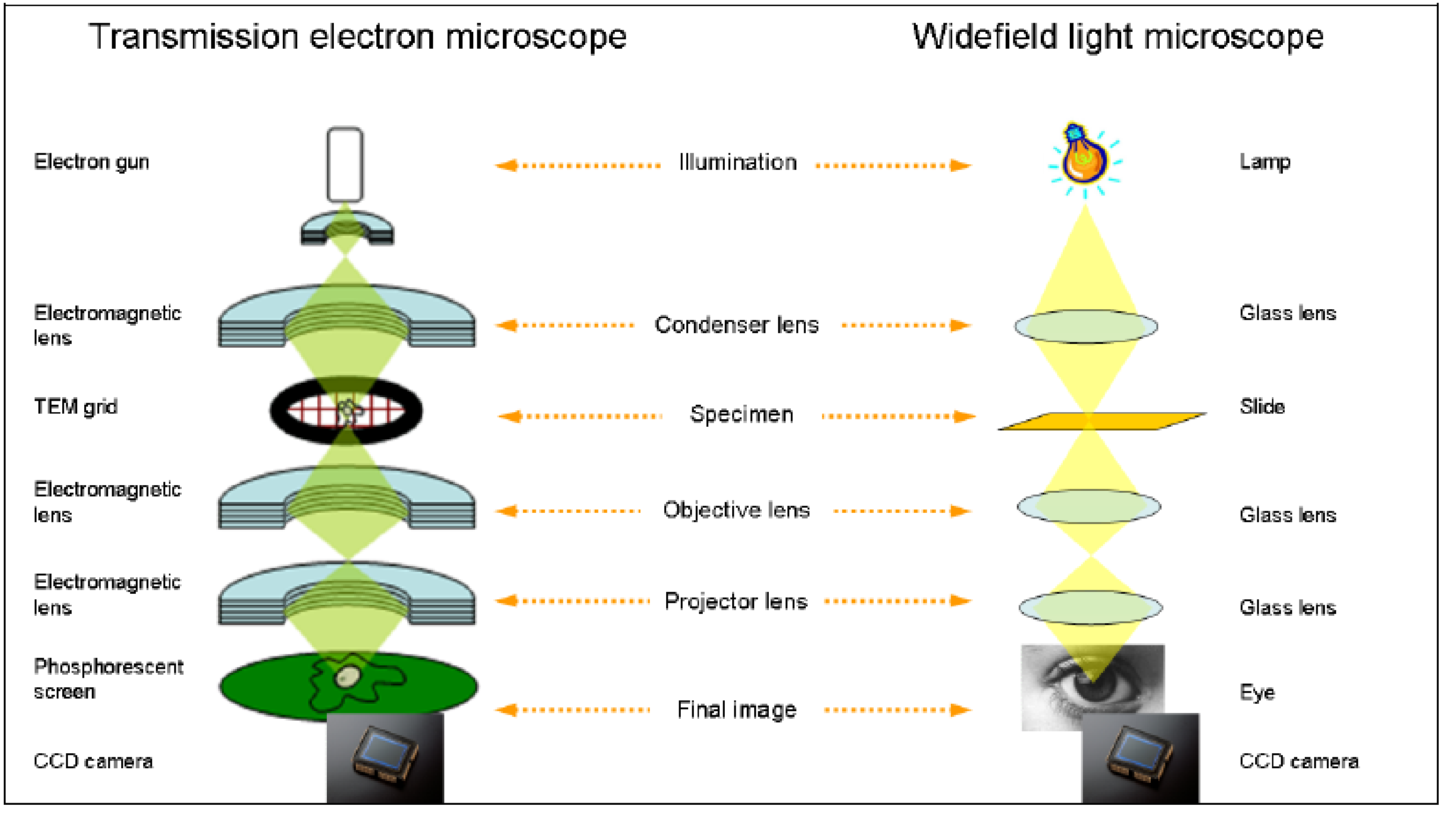


Fig. 2.1: Similarity of a transmission electron microscope with a wide field light microscope. An electron beam is formed at the tip of a heated filament. The electrons are accelerated with high voltages (60 – 1200 kV depending on the type of TEM) and are guided through the electron microscope column by electromagnetic lenses. The beam penetrates and interacts with the specimen and leads to an image. The image is monitored on a phosphorescent screen or specially designed CCD camera and recorded.

Electromagnetic lense

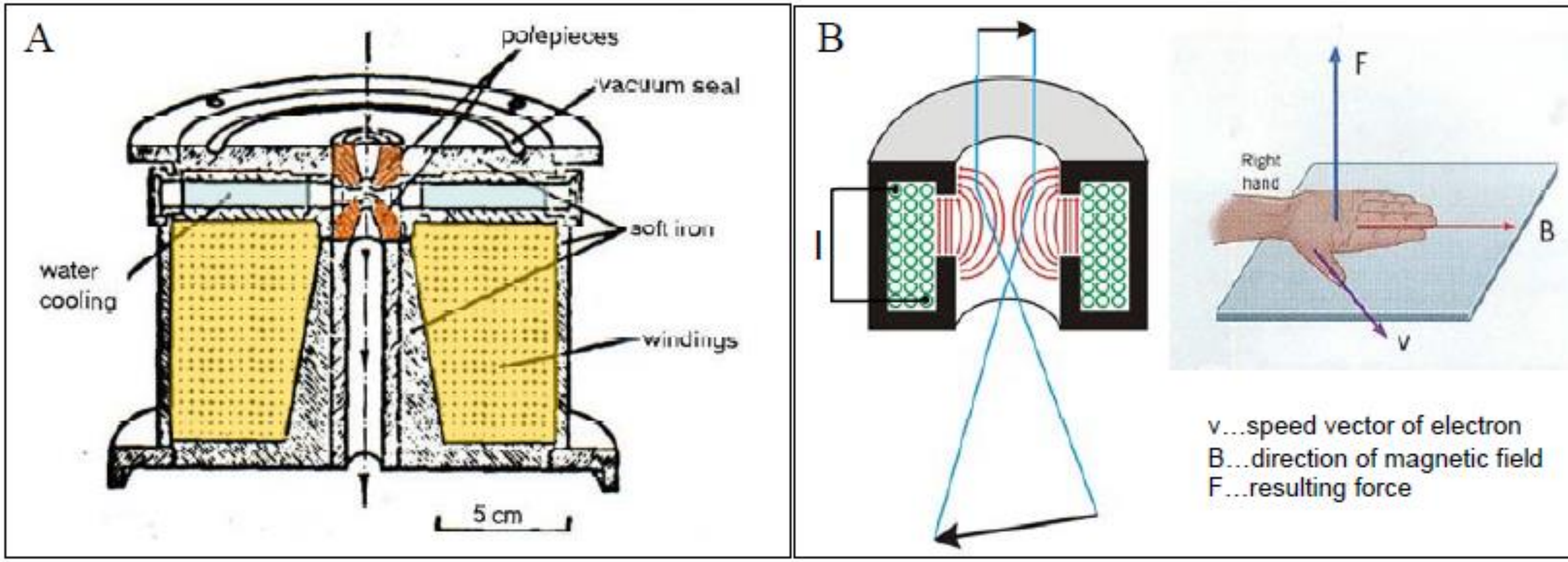


Fig. 2.5: Electromagnetic lens. A) The magnetic field is strongest in the area of the pole piece. The focal length of the lens is changed by the current trhough the windings. The lens is water cooled to maintain stability during operation. B) Electrons passing the magnetic field are deviated perpendicular to the plane defined by the magnetic field B and the velocity vector v. The black arrows demonstrate the rotation of the image.

Sample holder

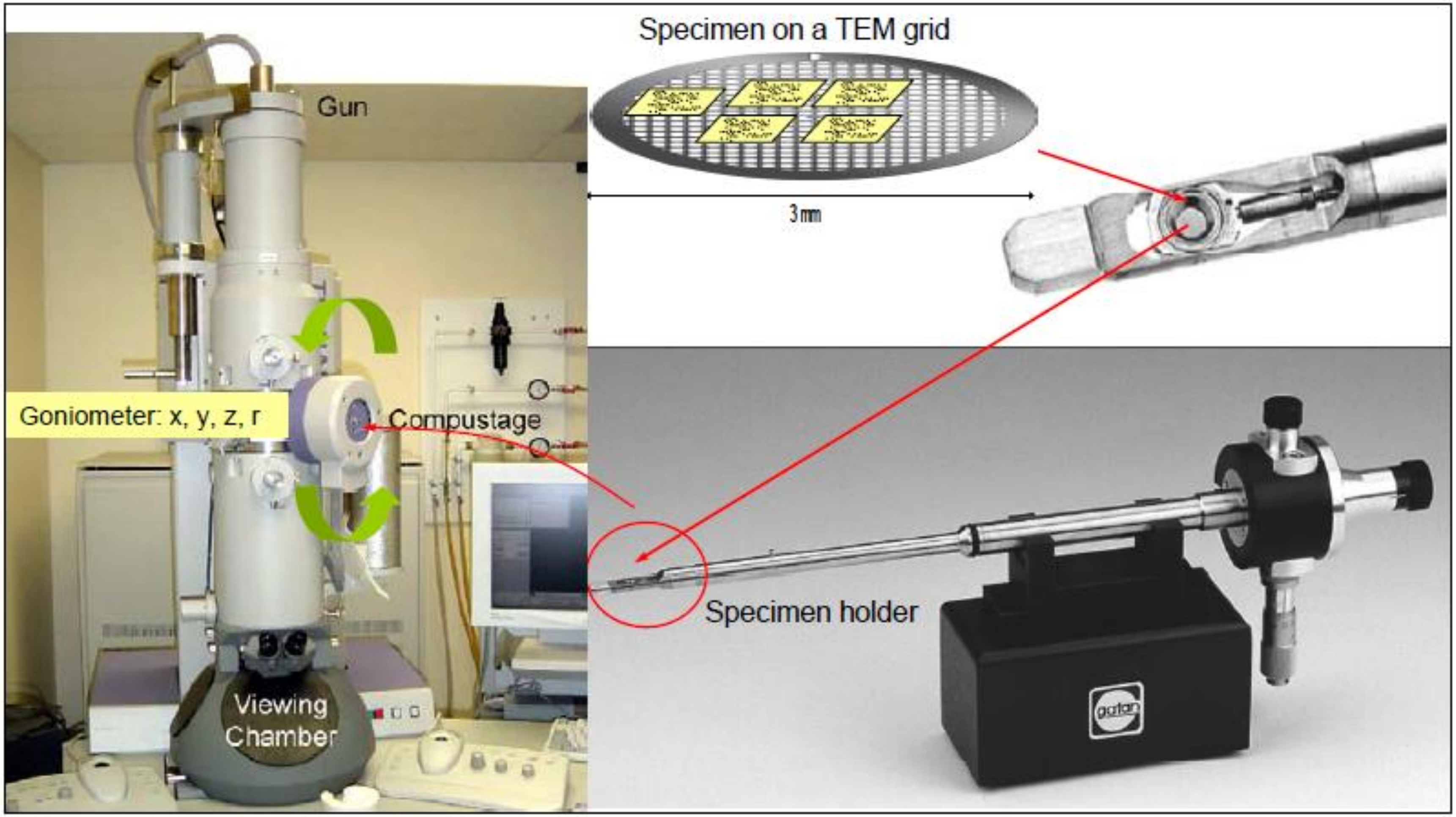


Fig. 2.7: Thin sections of a specimen on a TEM grid, holder tip and complete specimen holder, which is introduced into the goniometer of the TEM through a vacuum lock.

Electron – specimen interaction

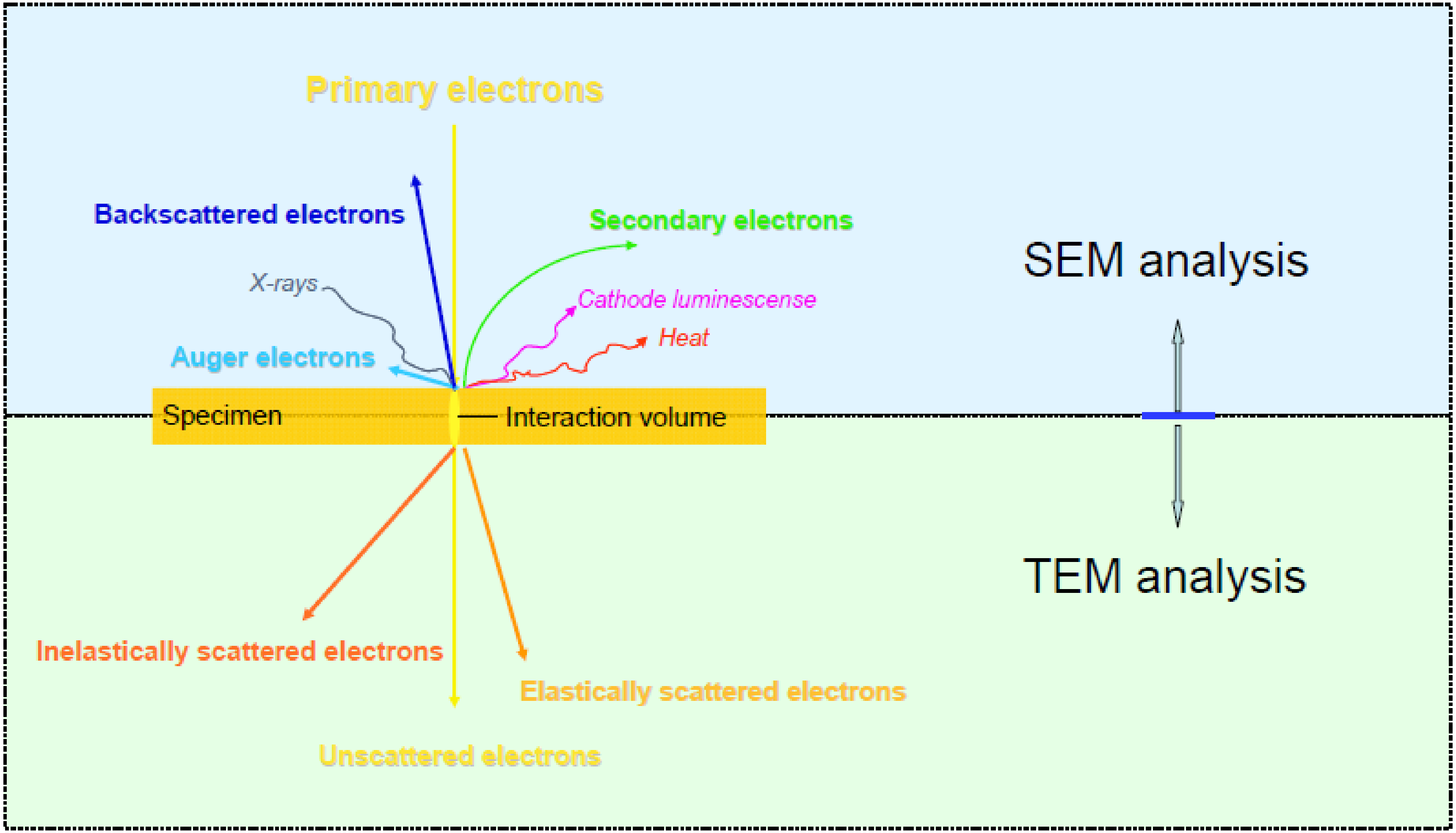


Fig 3.2: Interaction of electrons with specimen/matter, induced radiation and emission. Transmitted electrons like elastically scattered electrons are typically used for TEM imaging of biological specimens. Secondary electrons and backscattered electrons are mainly used for imaging in the SEM.

Contrast formation

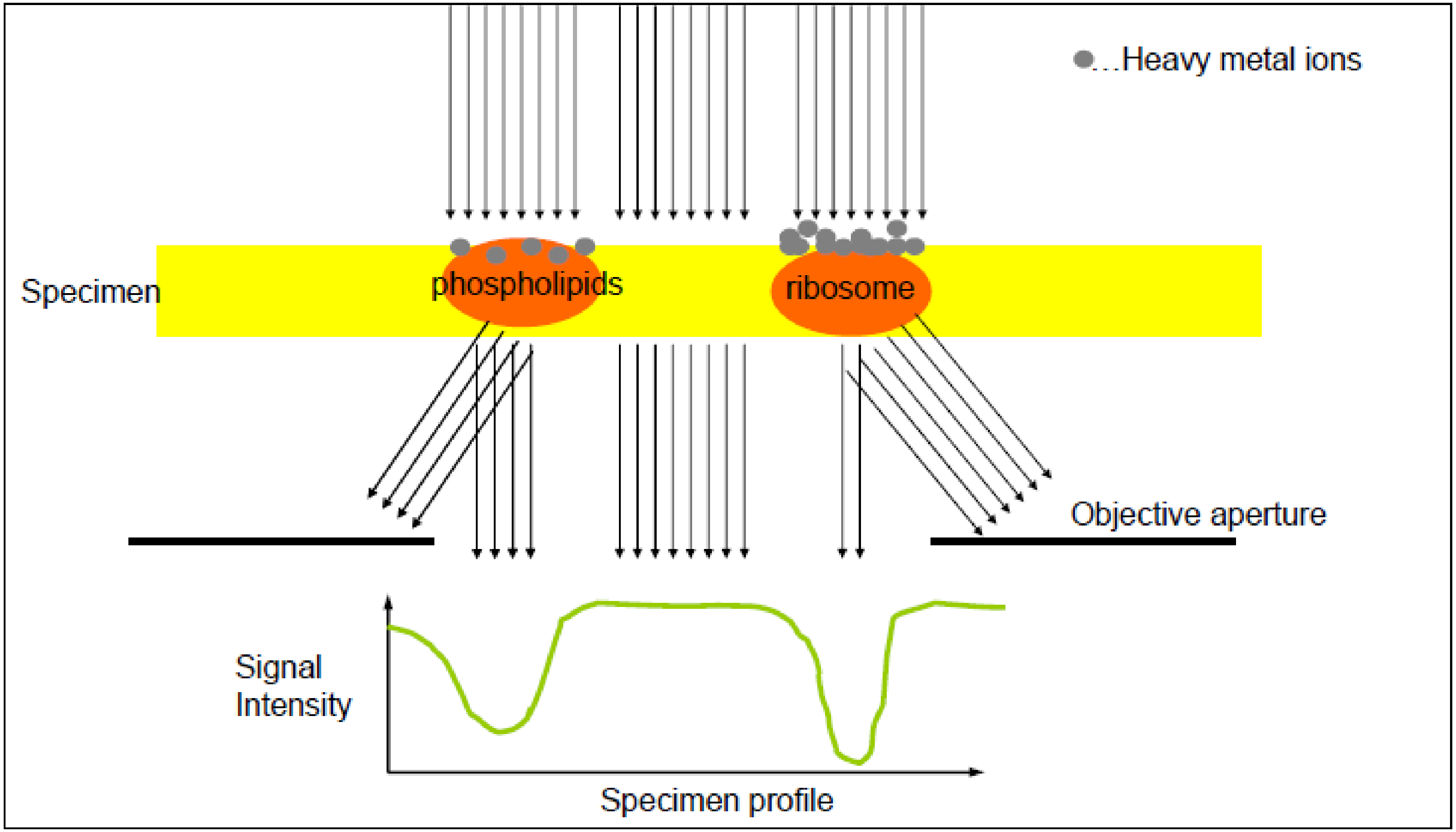
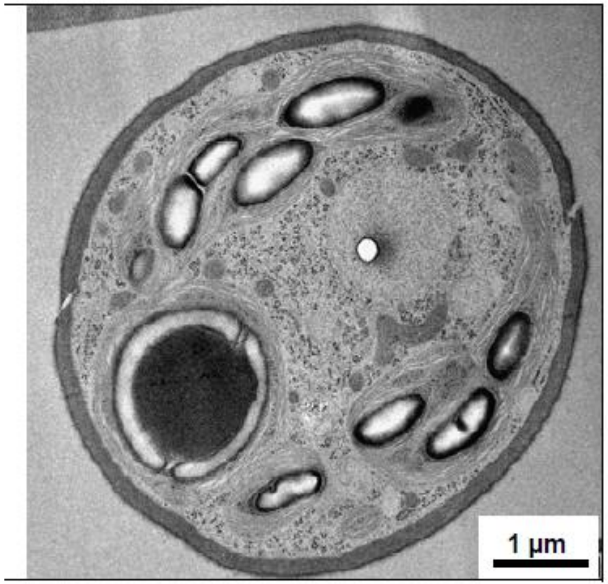
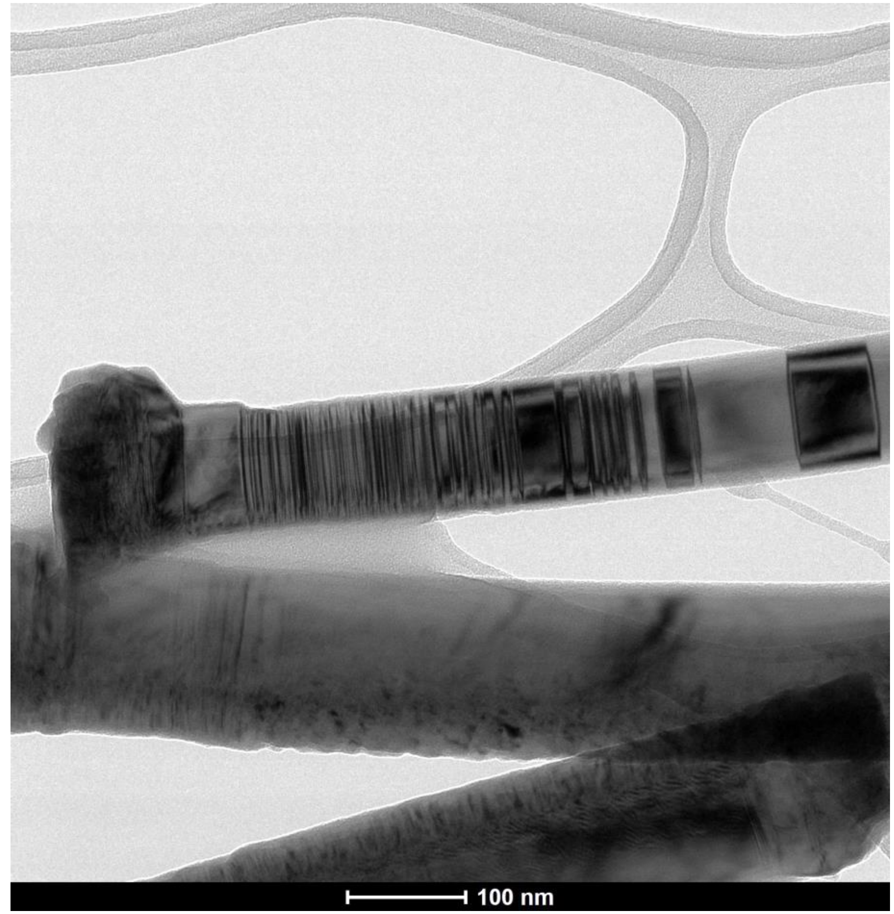


Fig. 4.1: Contrast formation in biological objects (simplified): A different number of heavy metal ions stick to the sample surface depending on the constituents, e.g. membranes (phospholipids), ribosomes, chromatin. Many diffracted/scattered electrons hit the objective aperture and do not reach the imaging device (CCD, viewing screen). Specimen spots with a lot of heavy metals will yield low signal intensity (electron dense, dark spots) whereas spots with no or few heavy metal ions yield high signal intensity (bright spots).

Transmission contrast





GaAs Nanowire

Visualization of dislocations

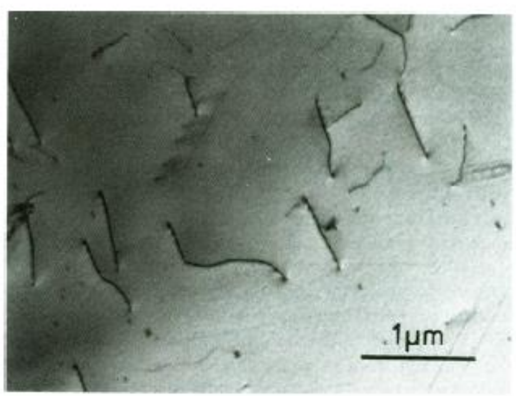
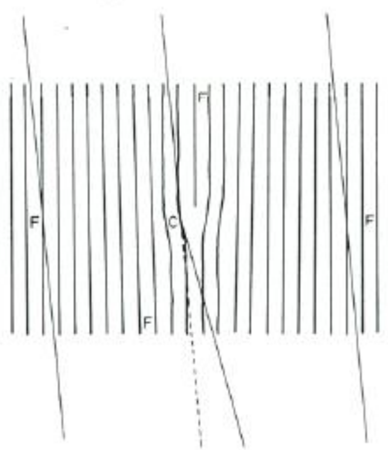
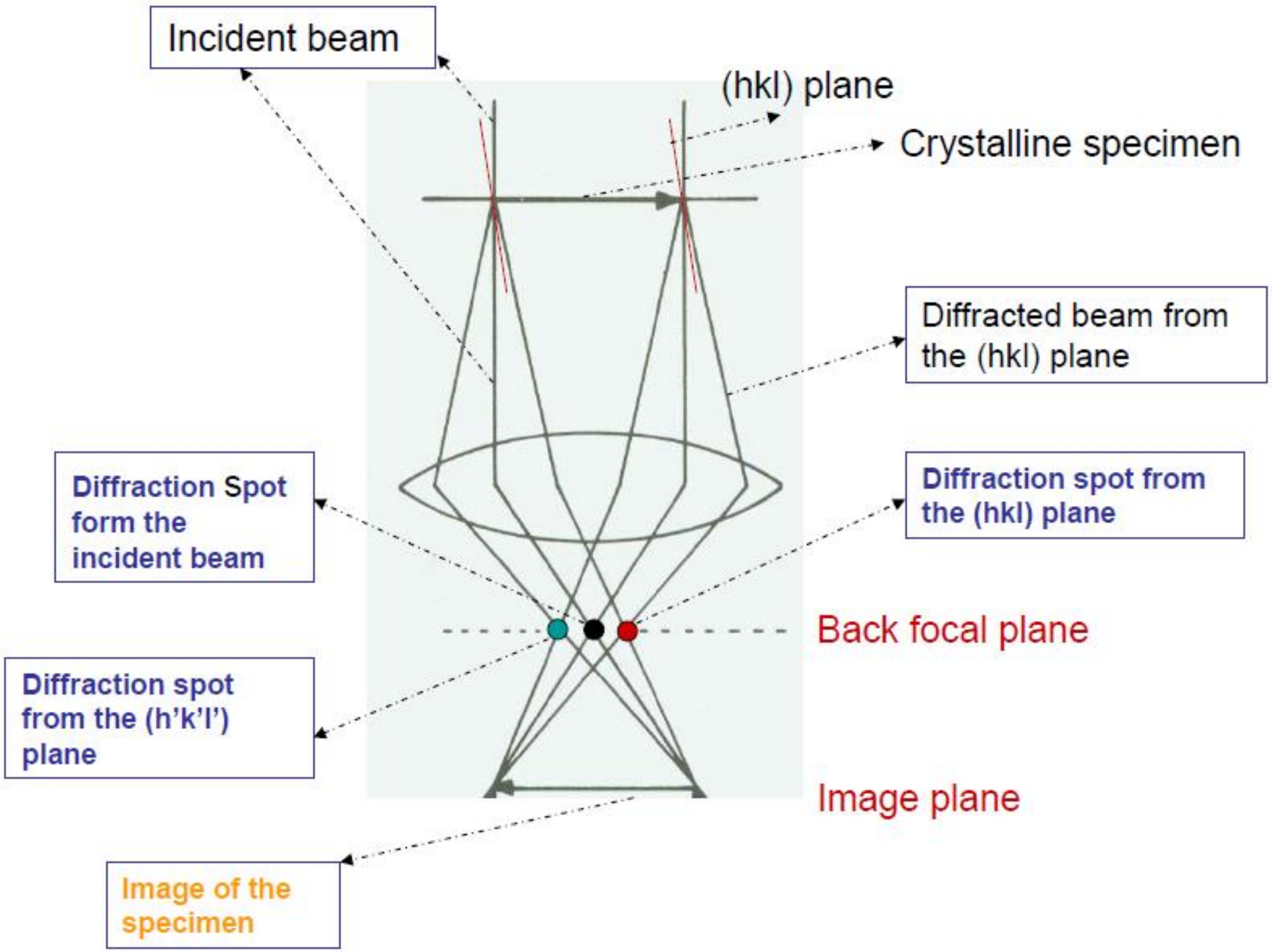


Figure 4.22 Dislocations in strong diffraction contrast in a metal foil.

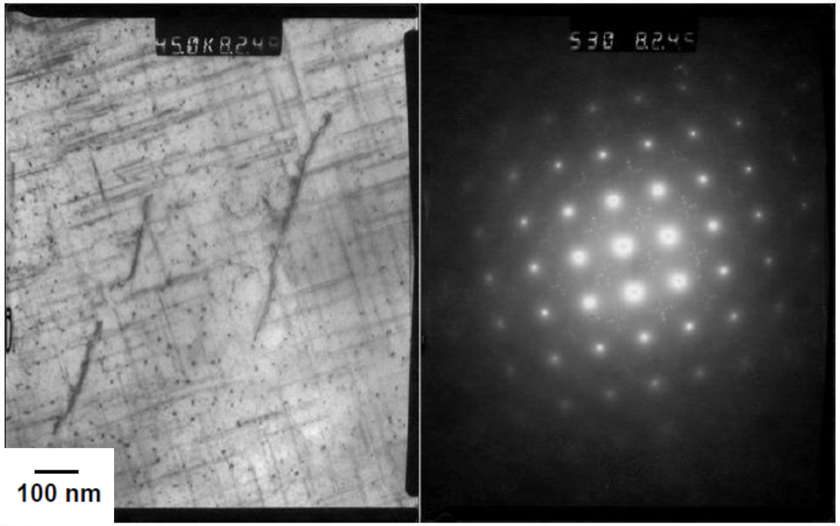


Formation of the diffraction pattern and the image in the TEM

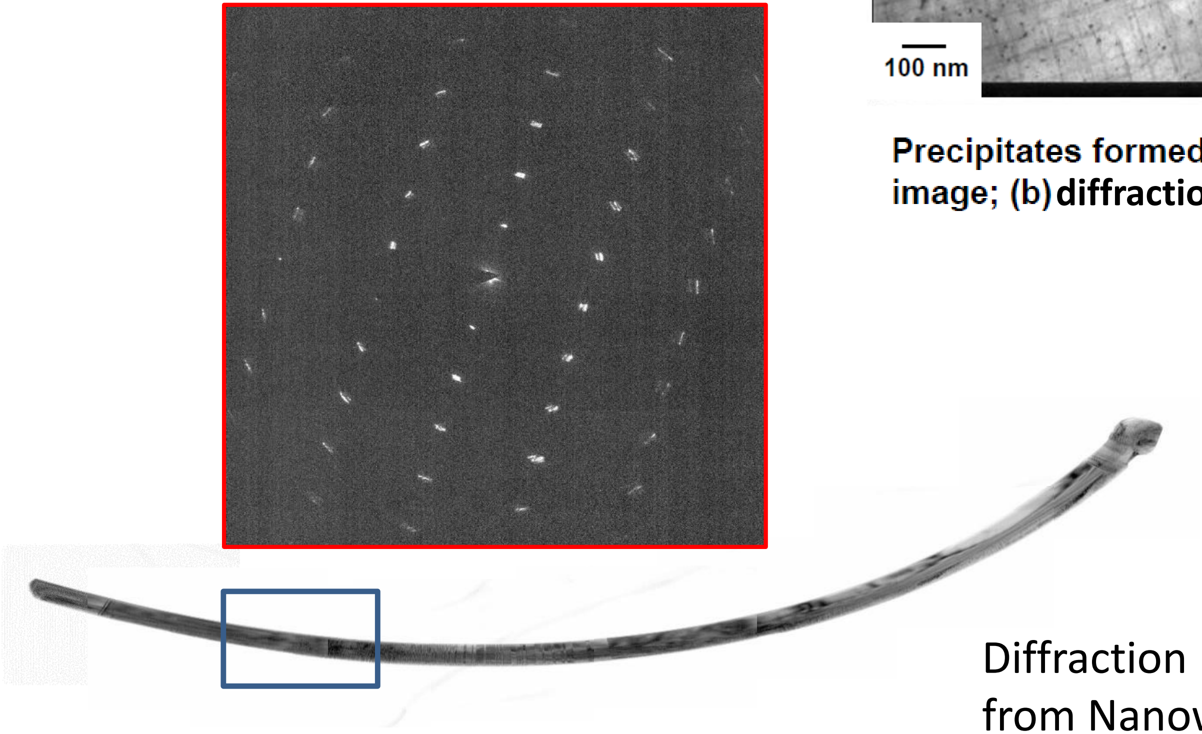


https://www.mah.se/upload/_upload/Electron%20microscopy.pdf

Electron diffraction pattern

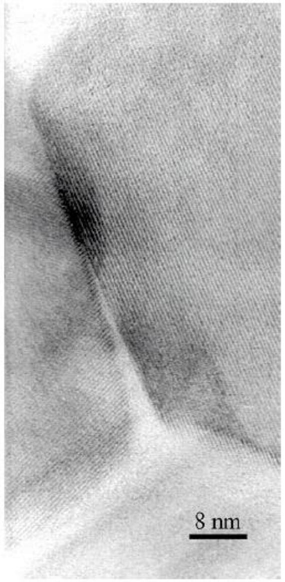


Precipitates formed in an Al alloy (a) bright field image; (b) **diffraction** pattern from the area in (a).

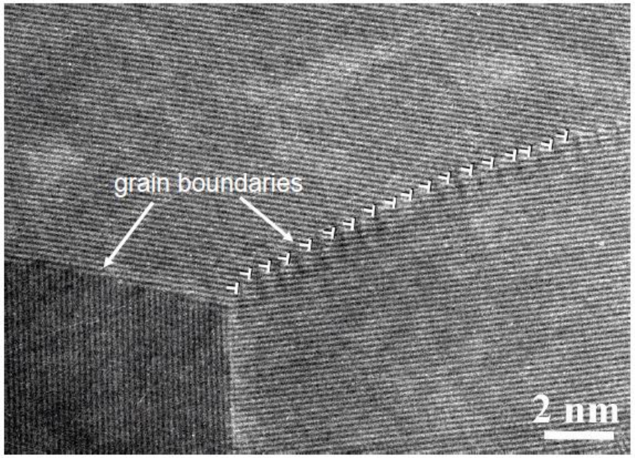


Diffraction pattern from Nanowire

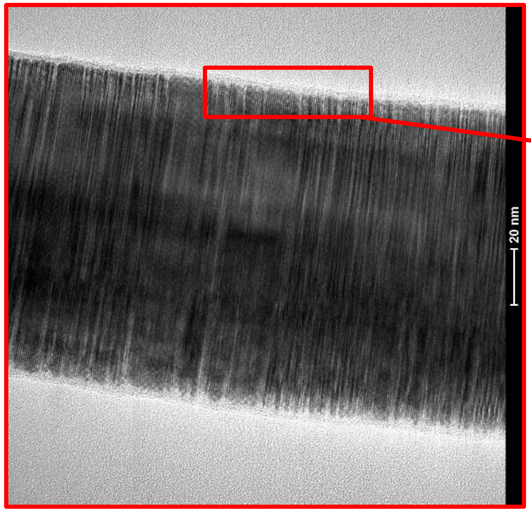
High resolution images formed in the TEM.

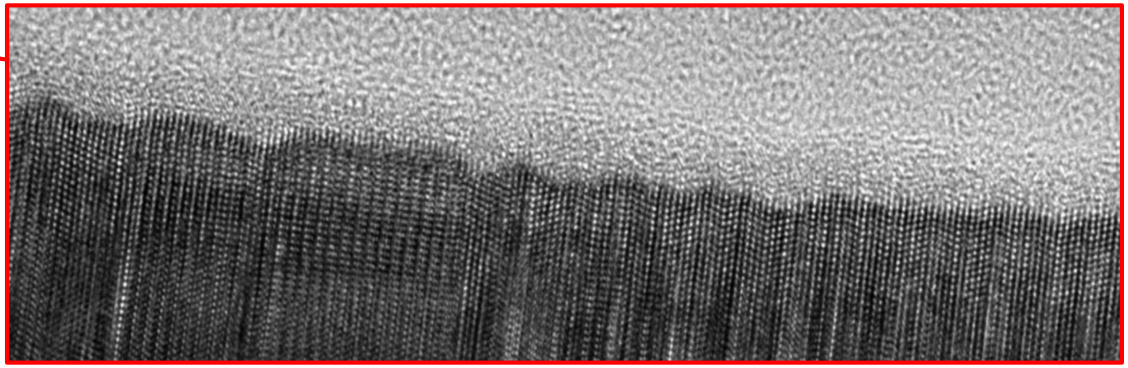


Metallic alloy



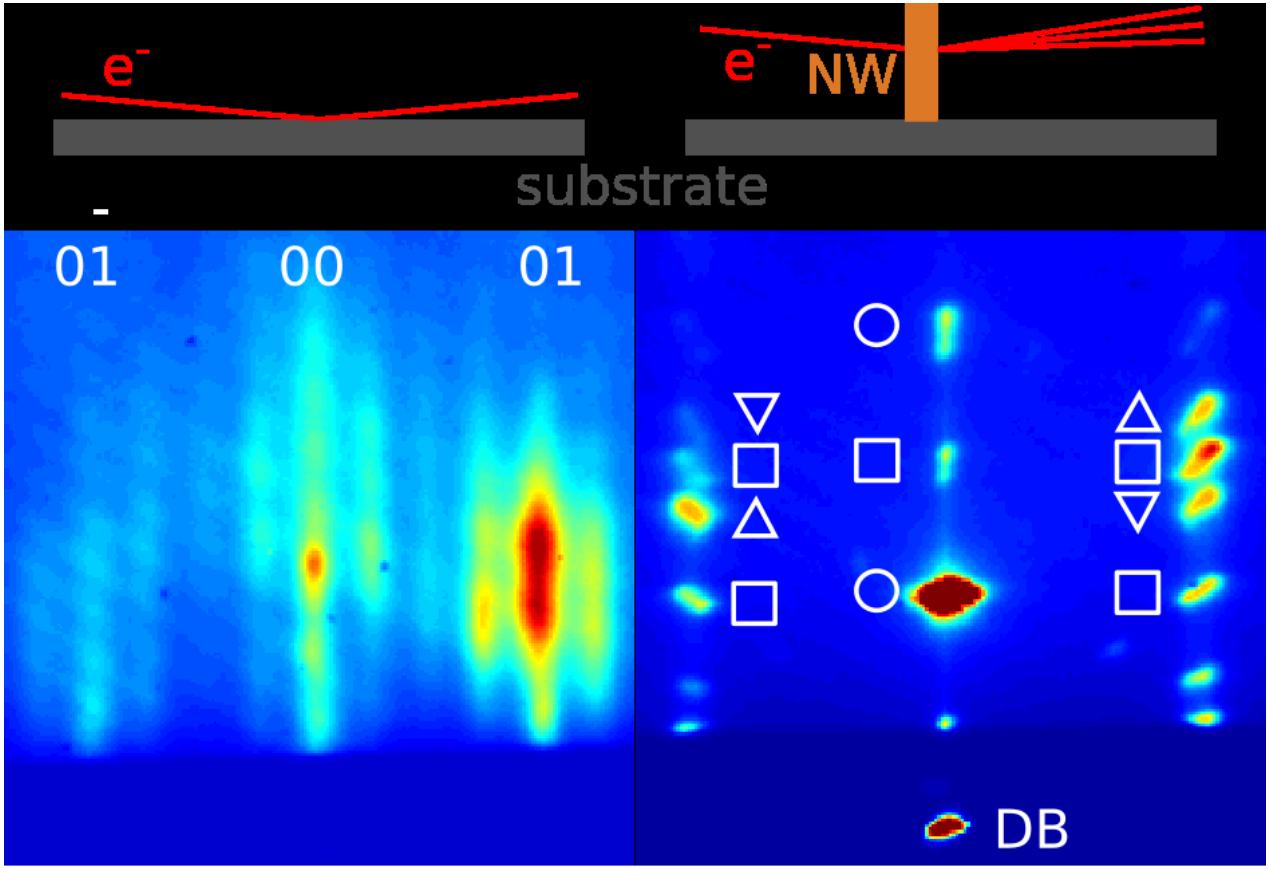
Semiconductor nanowire





Dr. Julian Müller – Uni Siegen

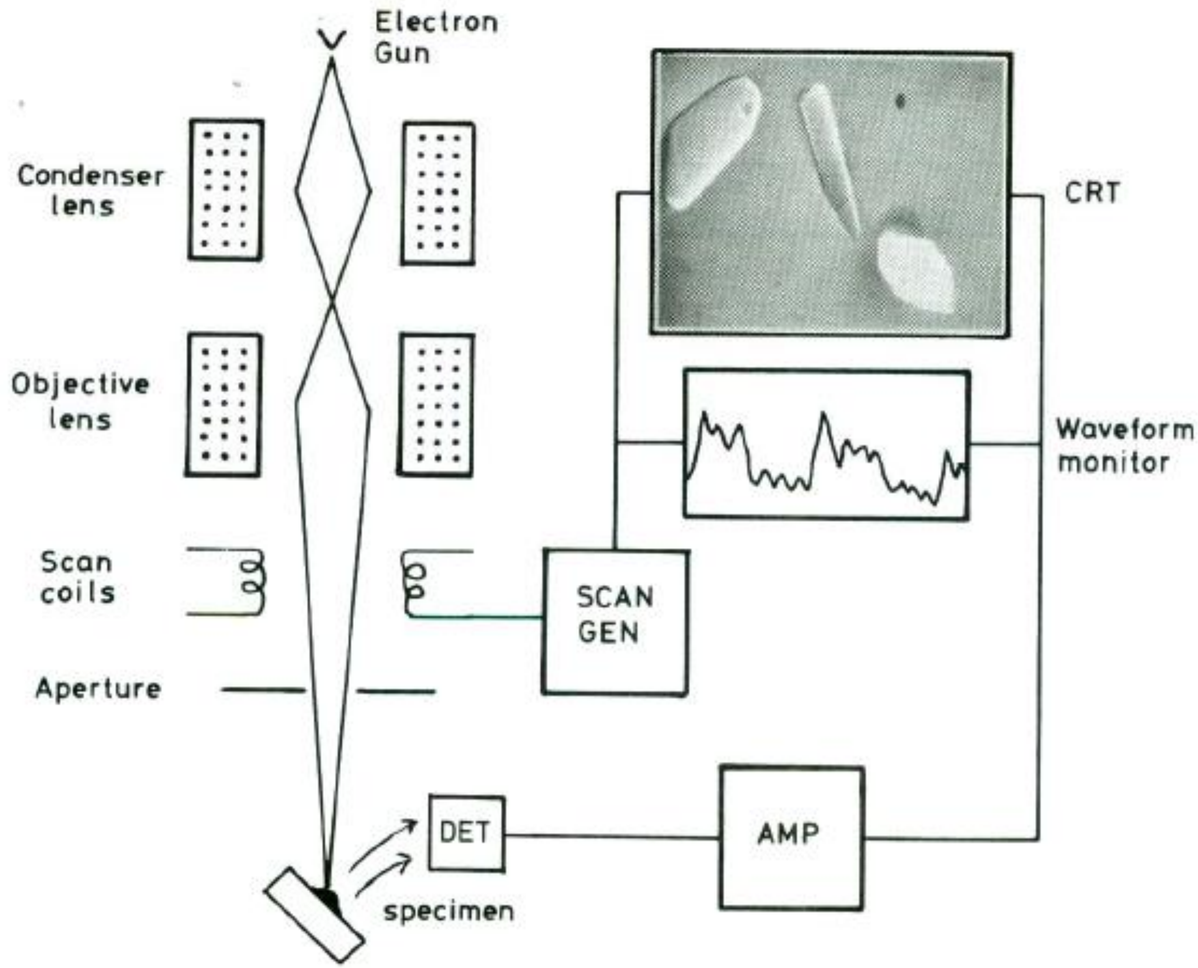
RHEED - Reflecting High energy electron diffraction



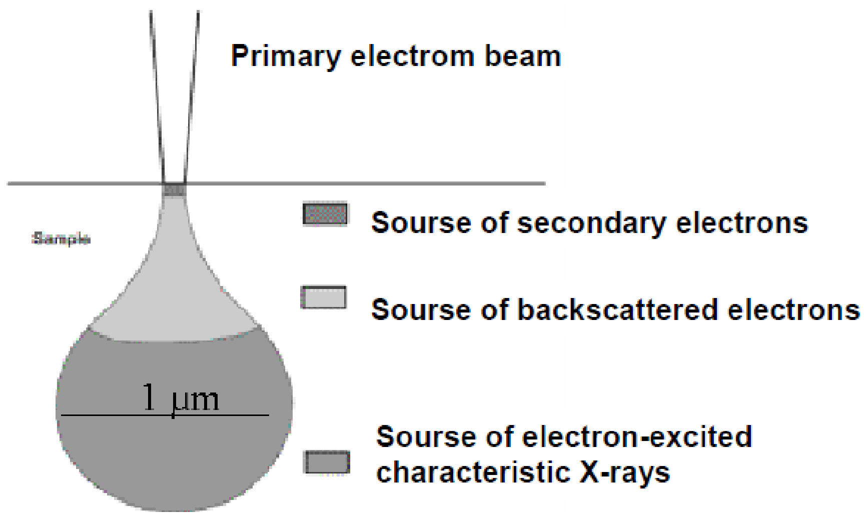
Surface sensitive electron diffraction at growing nanowires

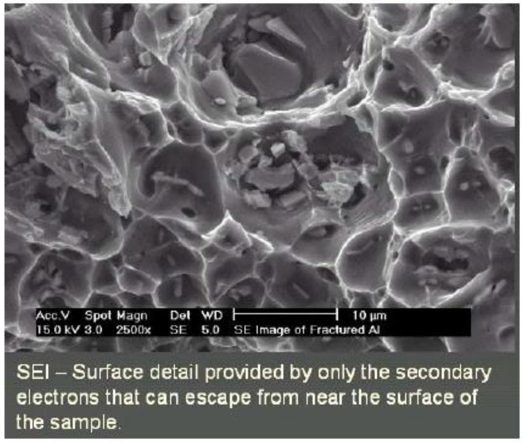
Julian Jakob - KIT

Scanning electron microscope



Beam – specimen interaction 20kV

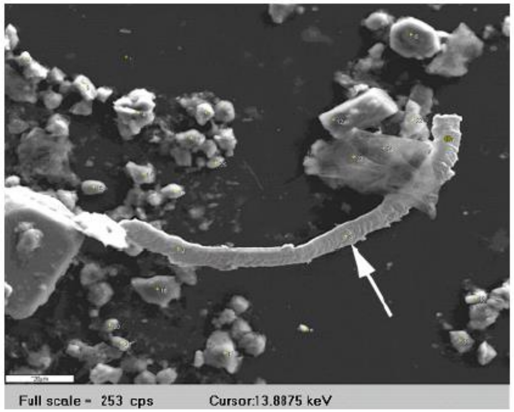


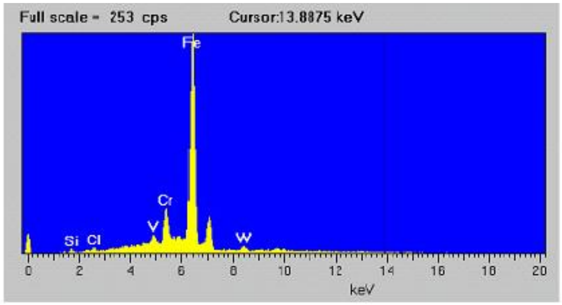


Secondary electron image (topographic image)

SEM

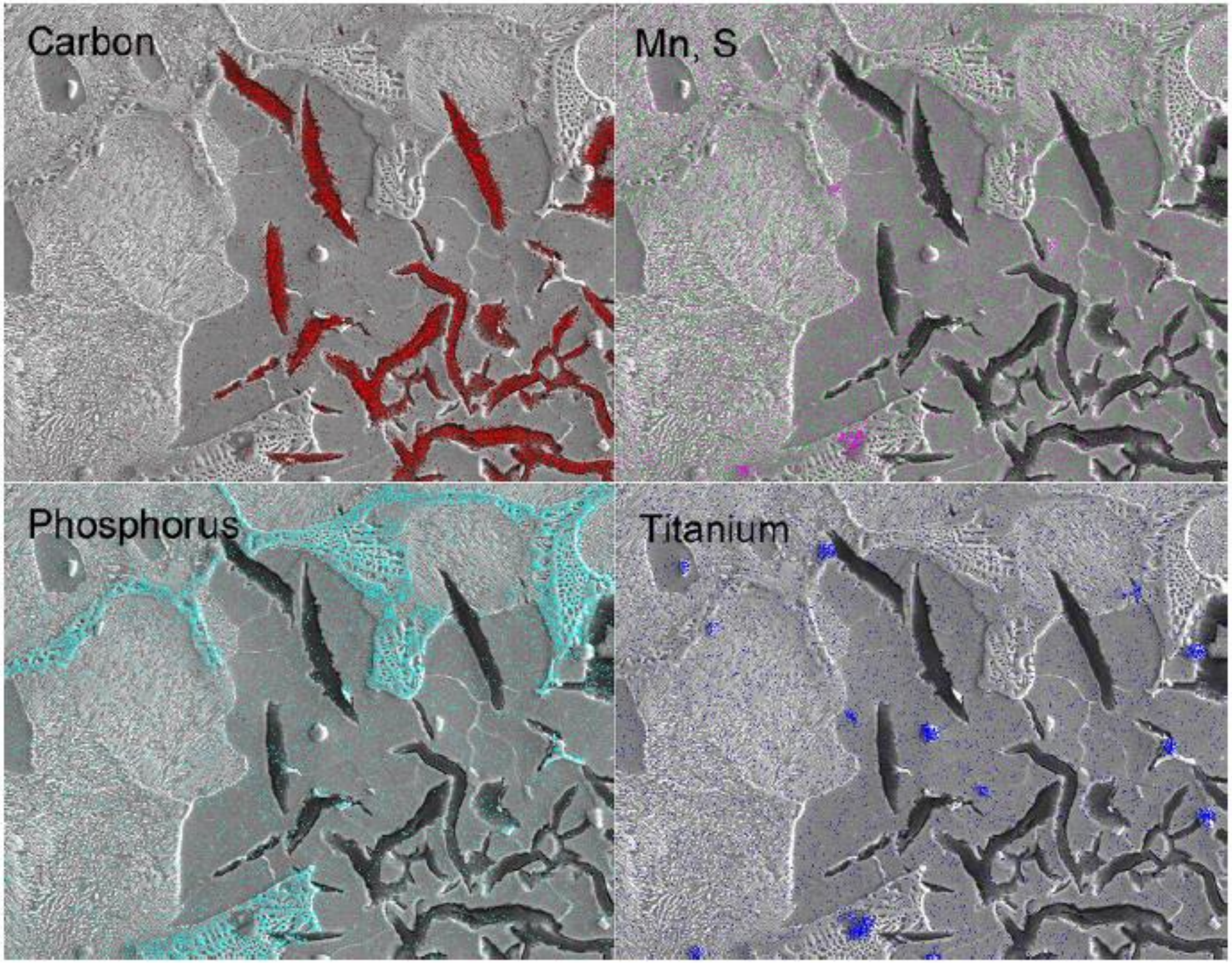
Backscattered electron image (compositional image)



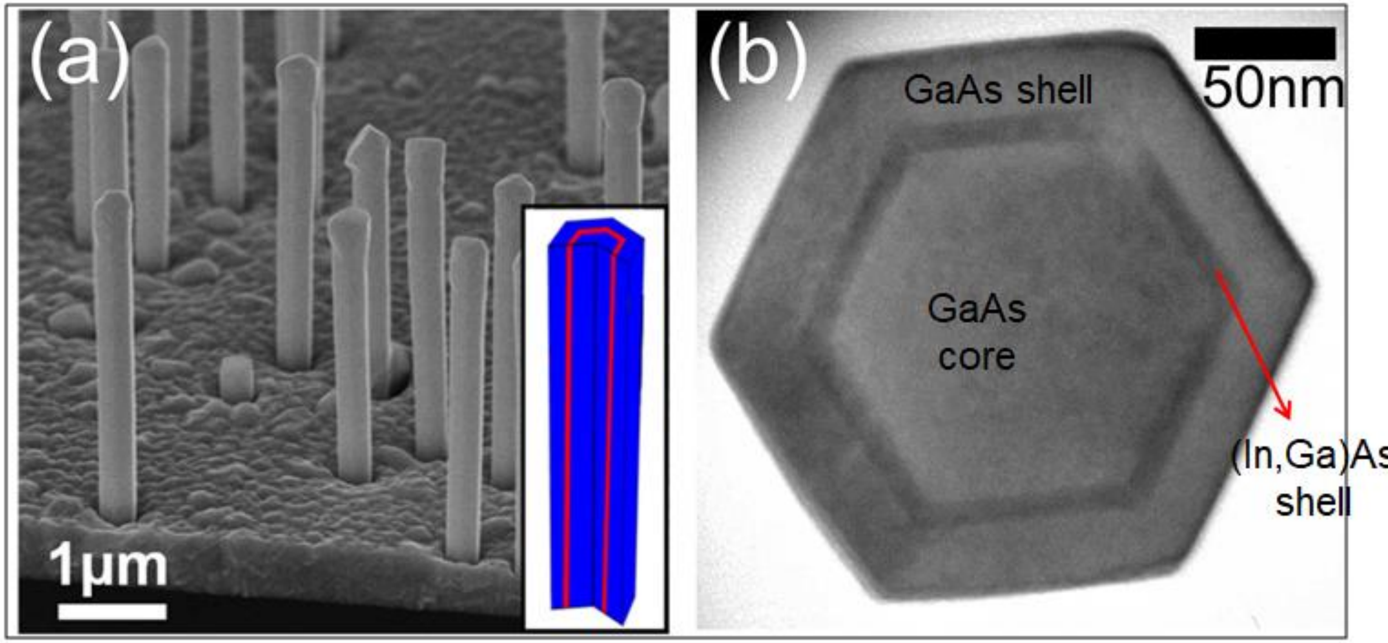


EDX spectrum

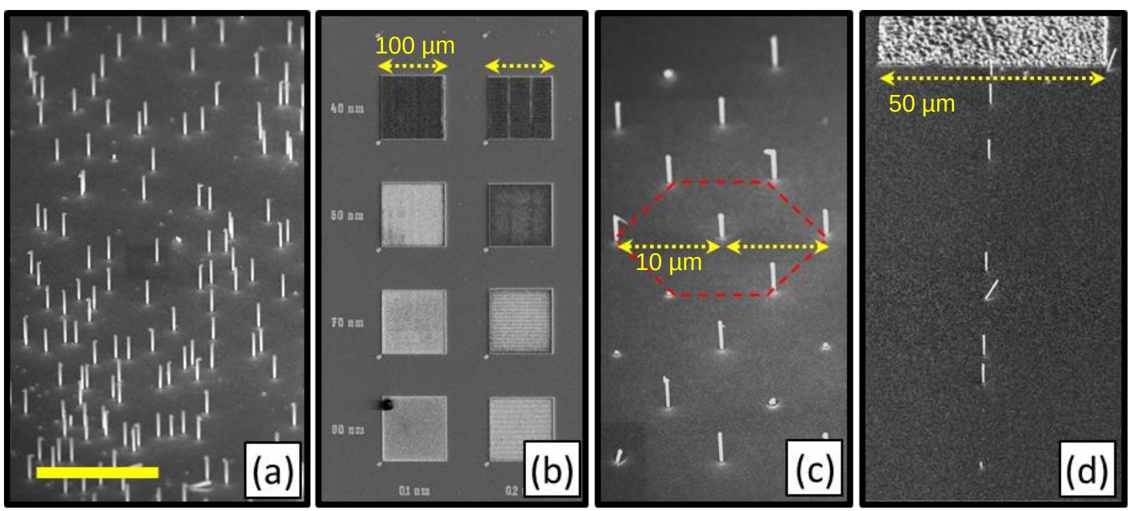
Compositional maps (or X-ray images)



SEM of semiconductor core-shell nanowires



Compositional map of core shell NW



(a) SEM picttures of randomly grwon NWs

(b) NW arrays

(c) NW distribution within one array

(d) NW along a single line

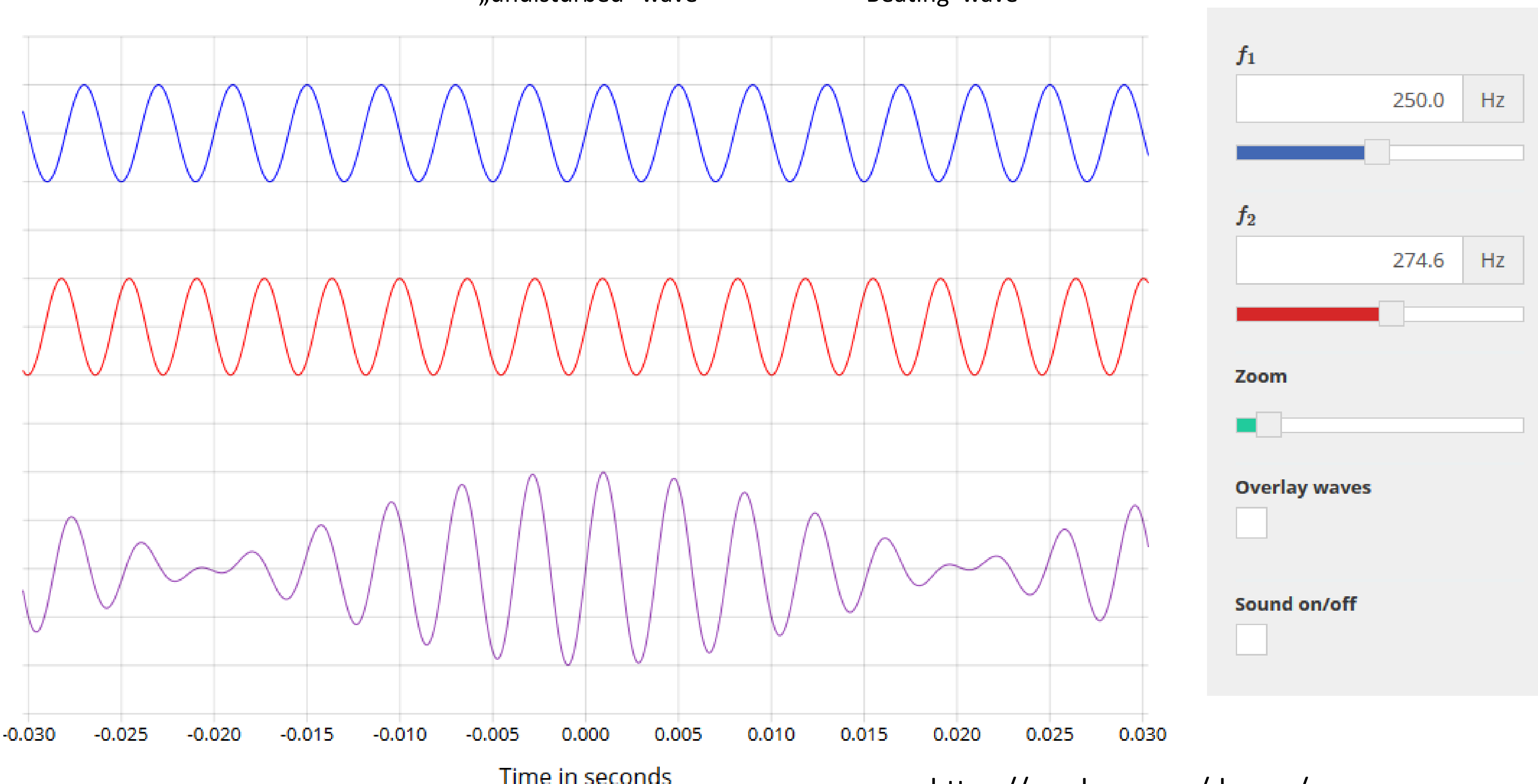
Dr. Ali AlHassan Uni Siegen

Encoding „Information“: Beating of frequency

$$E_1 + E_2 = E_0[\sin 2\pi(\nu_1 t - \frac{x}{\lambda_1}) + \sin 2\pi(\nu_2 t - \frac{x}{\lambda_2})]$$

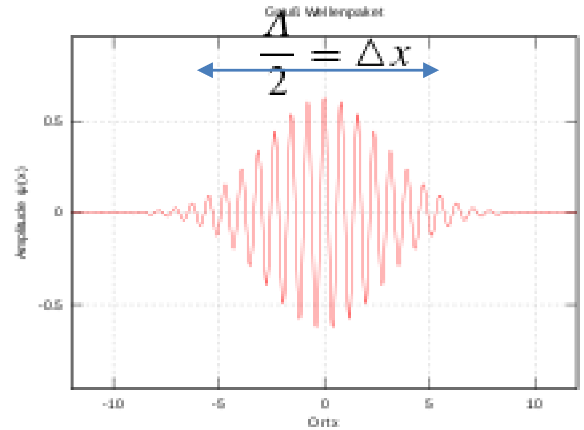
$$E_1 + E_2 = 2E_0 \sin 2\pi[\frac{(\nu_1+\nu_2)}{2}t - \frac{1}{2}(\frac{1}{\lambda_1}+\frac{1}{\lambda_2})x]\boxed{\cos 2\pi[\frac{(\nu_1-\nu_2)}{2}t - \frac{1}{2}(\frac{1}{\lambda_1}-\frac{1}{\lambda_2})x]}$$

„undisturbed“ wave | Beating wave



https://academo.org/demos/wave-interference-beat-frequency/

Shape of a wave group



$$\Delta x = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} = \frac{\frac{h}{mv_1}\frac{h}{mv_2}}{\frac{h}{mv_2} - \frac{h}{mv_1}} \approx \frac{\frac{h^2}{m^2 v^2}}{\frac{h}{v}\frac{v_1 - v_2}{v_1 v_2}} \approx \frac{h}{\Delta(mv)} = \frac{h}{\Delta p}$$

$$\Lambda = \frac{2\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \approx \frac{2\lambda^2}{\Delta\lambda}$$

The smaller Δx the smaller Δλ

$$\Delta x \cdot \Delta p_x \geq \hbar$$
$$\Delta y \cdot \Delta p_y \geq \hbar \qquad \text{(Heisenberg's uncertainty principle).}$$
$$\Delta z \cdot \Delta p_z \geq \hbar$$

Momentum and position can not be determined precisely simultaneous

Heisenberg´s uncertainty principle

$$\Delta x = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} = \frac{\frac{h}{mv_1}\frac{h}{mv_2}}{\frac{h}{mv_2} - \frac{h}{mv_1}} \approx \frac{\frac{h^2}{m^2 v^2}}{\frac{h}{v}\frac{v_1 - v_2}{v_1 v_2}} \approx \frac{h}{\Delta(mv)} = \frac{h}{\Delta p}$$

$$\Delta x \cdot \Delta p_x \geq \hbar$$
$$\Delta y \cdot \Delta p_y \geq \hbar \qquad \text{(Heisenberg's uncertainty principle).}$$
$$\Delta z \cdot \Delta p_z \geq \hbar$$

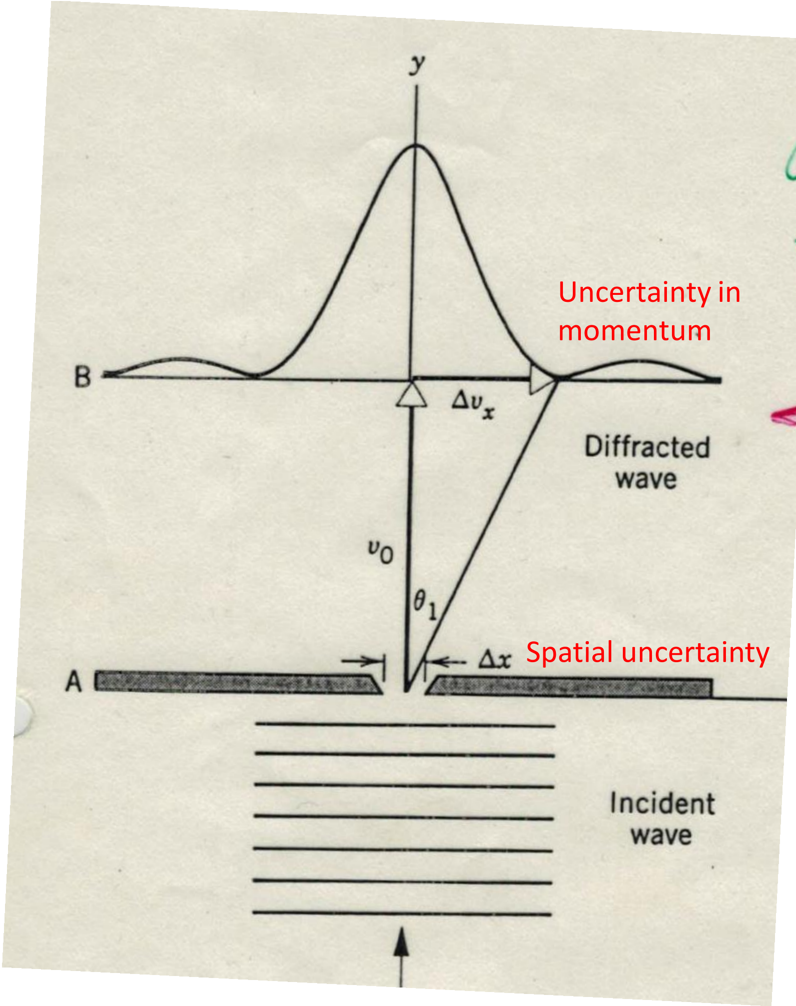
Proof for diffraction

1. minimum $\sin\theta \approx \theta = \frac{\lambda}{\Delta x}$

momentum $\theta \approx \frac{\Delta v_x}{v_0}$

$$\Delta v_x \cdot \Delta x = \lambda \cdot v_0 = \frac{h}{mv_0} v_0$$

$$m \Delta v_x \cdot \Delta x = \Delta p \cdot \Delta x = h$$



Uncertainty principle: position and momentum

Assume that an electron is moving along an x axis and that you measure its speed to be 2.05×10^6 m/s, which can be known with a precision of 0.50%. What is the minimum uncertainty (as allowed by the uncertainty principle in quantum theory) with which you can simultaneously measure the position of the electron along the x axis?

KEY IDEA

The minimum uncertainty allowed by quantum theory is given by Heisenberg's uncertainty principle in Eq. 38-20. We need only consider components along the x axis because we have motion only along that axis and want the uncertainty Δx in location along that axis. Since we want the minimum allowed uncertainty, we use the equality instead of the inequality in the x-axis part of Eq. 38-20, writing $\Delta x \cdot \Delta p_x = \hbar$.

Calculations: To evaluate the uncertainty Δp_x in the momentum, we must first evaluate the momentum component p_x. Because the electron's speed v_x is much less than the speed of light c, we can evaluate p_x with the classical expression for momentum instead of using a relativistic expression. We find

$$p_x = mv_x = (9.11 \times 10^{-31}\ \text{kg})(2.05 \times 10^6\ \text{m/s}) = 1.87 \times 10^{-24}\ \text{kg}\cdot\text{m/s}.$$

The uncertainty in the speed is given as 0.50% of the measured speed. Because p_x depends directly on speed, the uncertainty Δp_x in the momentum must be 0.50% of the momentum:

$$\begin{aligned}\Delta p_x &= (0.0050)p_x \\ &= (0.0050)(1.87 \times 10^{-24}\ \text{kg}\cdot\text{m/s}) \\ &= 9.35 \times 10^{-27}\ \text{kg}\cdot\text{m/s}.\end{aligned}$$

Then the uncertainty principle gives us

$$\Delta x = \frac{\hbar}{\Delta p_x} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})/2\pi}{9.35 \times 10^{-27}\ \text{kg}\cdot\text{m/s}}$$

$$= 1.13 \times 10^{-8}\ \text{m} \approx 11\ \text{nm}, \quad \text{(Answer)}$$

which is about 100 atomic diameters. Given your measurement of the electron's speed, it makes no sense to try to pin down the electron's position to any greater precision.

Schrödinger equation

Light and small particles can be described by a **matter wave = probability wave**

$$\Psi(x, y, z, t) = \psi(x, y, z)\, e^{-i\omega t},$$

Propability that a particle will be found at specific position x (e.g. at detector) in a specific time intervall is proportional to $|\psi|^2$, whereas ψ is often complex , $|\psi|^2$ is always real = **probability density,** how on find ψ ? **It must satisfy**

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U(x)]\psi = 0 \quad \text{(Schrödinger's equation, one-dimensional motion),}$$

E is total energy of the particle, U(x) is potential energy ,
→ **Schrödinger equation is a basic principle of physics, it cannot be derived !!!**

In case of U=0 → free particle function, $E=E_{kin}$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(\frac{mv^2}{2}\right)\psi = 0, \qquad \frac{d^2\psi}{dx^2} + k^2\psi = 0 \qquad \psi(x) = Ae^{ikx} + Be^{-ikx},$$

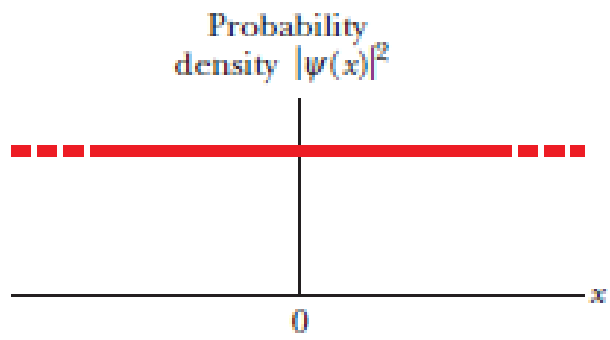
Probability density

$$\psi(x) = \psi_0 \, e^{ikx}.$$

$$|\psi|^2 = |\psi_0 \, e^{ikx}|^2 = (\psi_0^2)|e^{ikx}|^2.$$

$$|e^{ikx}|^2 = (e^{ikx})(e^{ikx})^* = e^{ikx} \, e^{-ikx} = e^{ikx-ikx} = e^0 = 1,$$

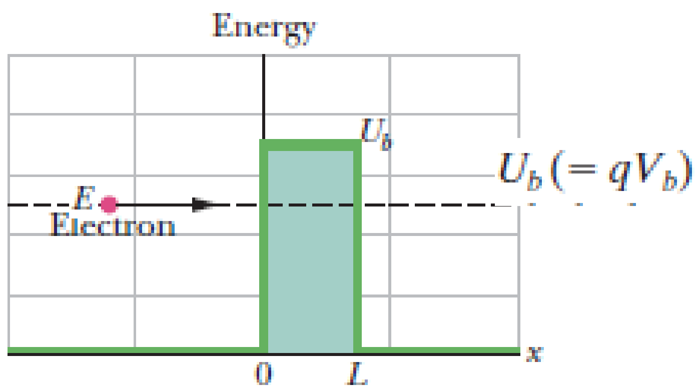
$$|\psi|^2 = (\psi_0^2)(1)^2 = \psi_0^2 \qquad \text{(a constant).}$$

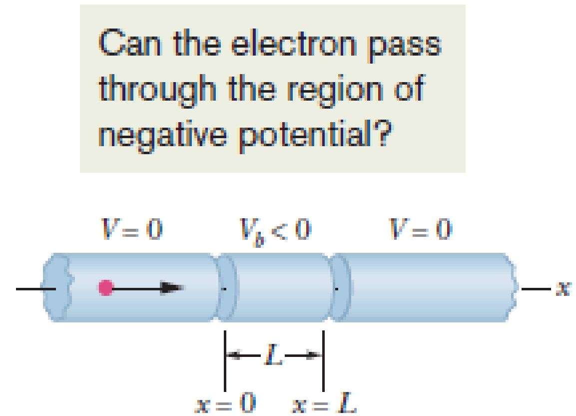


Probability density for a particle moving along x direction is constant for all x

Barrier tunneling

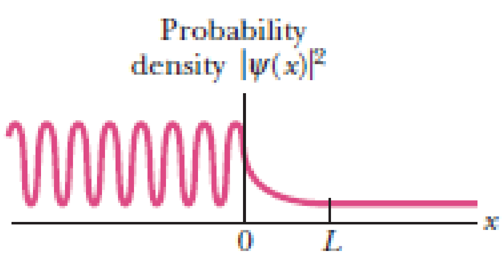
Find wave function ψ(x) for this problem





Incoming wave, mainly reflected

Transmitting wave decays exponentially



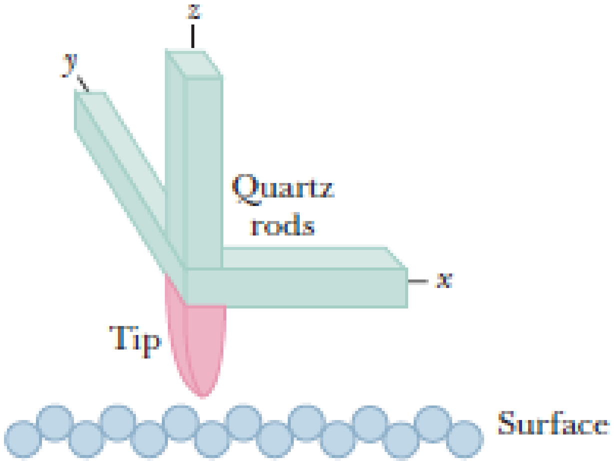
Transmission coefficient T

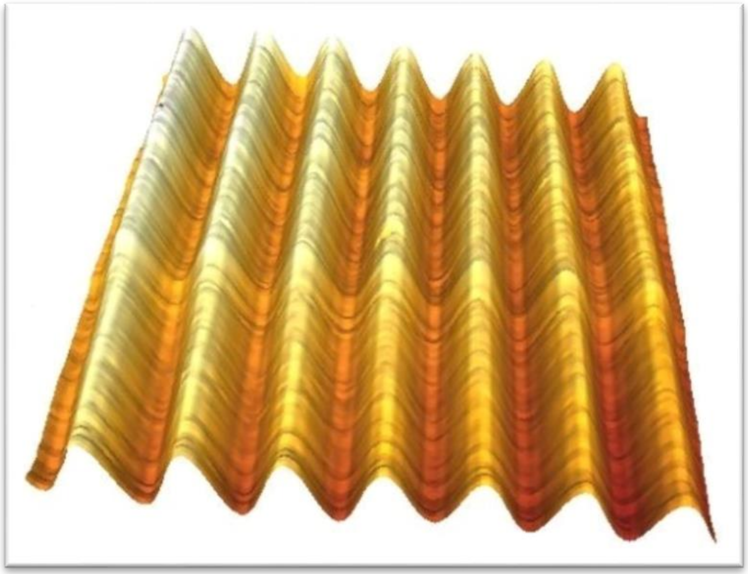
$$T \approx e^{-2bL},$$

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}},$$

Scanning tunnel microscopy

Tunnel current between tip and surface changes exponentially with distance L,
Sensitive feedback value





Polymer surface relief grating

300nm 1µm

P.Veer et. al. **J. Appl. Phys**. **106,** 014909, (2009)
P.Veer et. al. **Mol. Cryst. Liq. Cryst**., **486,** 1114, (2008)

Barrier tunneling by matter wave

Suppose that the electron in Fig. 38-15, having a total energy E of 5.1 eV, approaches a barrier of height $U_b = 6.8$ eV and thickness $L = 750$ pm.

(a) What is the approximate probability that the electron will be transmitted through the barrier, to appear (and be detectable) on the other side of the barrier?

KEY IDEA

The probability we seek is the transmission coefficient T as given by Eq. 38-21 ($T \approx e^{-2bL}$), where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

Calculations: The numerator of the fraction under the square-root sign is

$$(8\pi^2)(9.11 \times 10^{-31}\ \text{kg})(6.8\ \text{eV} - 5.1\ \text{eV}) \times (1.60 \times 10^{-19}\ \text{J/eV}) = 1.956 \times 10^{-47}\ \text{J}\cdot\text{kg}.$$

Thus, $$b = \sqrt{\frac{1.956 \times 10^{-47}\ \text{J}\cdot\text{kg}}{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})^2}} = 6.67 \times 10^9\ \text{m}^{-1}.$$

The (dimensionless) quantity $2bL$ is then

$$2bL = (2)(6.67 \times 10^9\ \text{m}^{-1})(750 \times 10^{-12}\ \text{m}) = 10.0$$

and, from Eq. 38-21, the transmission coefficient is

$$T \approx e^{-2bL} = e^{-10.0} = 45 \times 10^{-6}. \qquad \text{(Answer)}$$

Thus, of every million electrons that strike the barrier, about 45 will tunnel through it, each appearing on the other side with its original total energy of 5.1 eV. (The transmission through the barrier does not alter an electron's energy or any other property.)

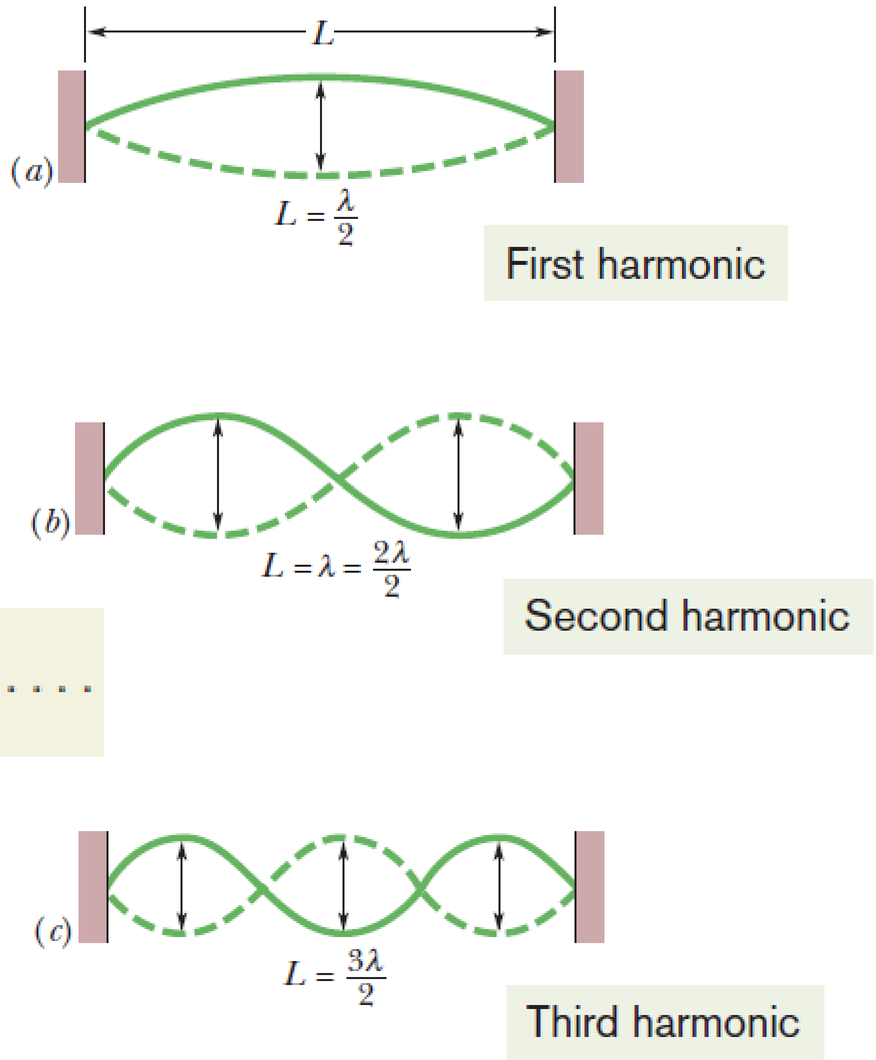
(b) What is the approximate probability that a proton with the same total energy of 5.1 eV will be transmitted through the barrier, to appear (and be detectable) on the other side of the barrier?

Reasoning: The transmission coefficient T (and thus the probability of transmission) depends on the mass of the particle. Indeed, because mass m is one of the factors in the exponent of e in the equation for T, the probability of transmission is very sensitive to the mass of the particle. This time, the mass is that of a proton (1.67×10^{-27} kg), which is significantly greater than that of the electron in (a). By substituting the proton's mass for the mass in (a) and then continuing as we did there, we find that $T \approx 10^{-186}$. Thus, although the probability that the proton will be transmitted is not exactly zero, it is barely more than zero. For even more massive particles with the same total energy of 5.1 eV, the probability of transmission is exponentially lower.

Harmonics

$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

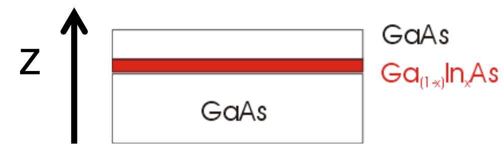
$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots .$$



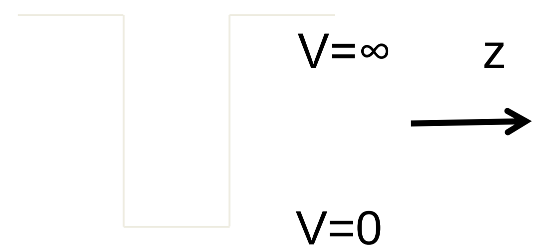
Electron in a infinite potential wall

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(x,y,z)+V(x,y,z)\psi(x,y,z)=E\psi(x,y,z)$$

x ,y → ∞ , stacking || z



$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\psi(z)+V(z)\psi(z)=E\psi(z)$$



Within QW V=0

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\psi(z)=E\psi(z)$$

Ansatz

$$\psi(z)=A\sin(kz)+B\cos(kz)$$

Boundary condition

$$\psi(0)=\psi(L)=0 \longrightarrow \text{B=0} \qquad \psi(z)=A\sin(kz)$$

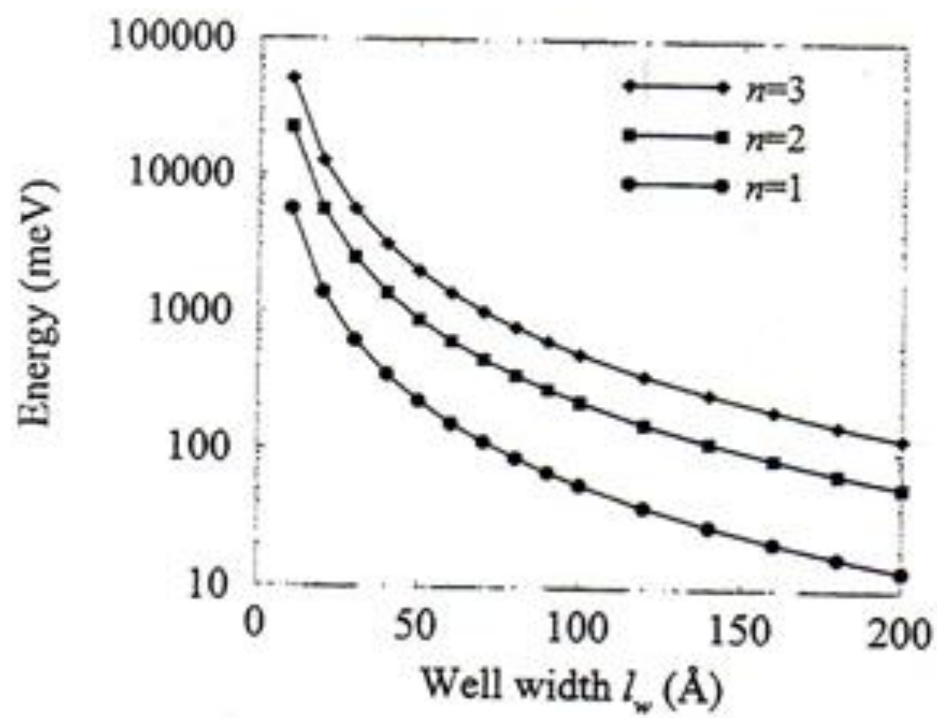
$$E_n=\frac{\hbar^2\pi^2n^2}{2m^*L^2} \qquad \psi(z)=\sqrt{\frac{2}{L}}\sin\left(\frac{\pi n z}{L}\right)$$

Quantization, infinite potential wall

$E_z = E_n$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2m^* L^2}$$

Electron energy is quantized



Harrison book

$$E = E_z + E_{x,y} = E_n + \frac{\hbar^2 |k_{x,y}|^2}{2m^*}$$

Question time about Physics 1

5.2.20 9:00 -11:00am

Mechanics – Electrodynamics – wave optics

Written exam 11.2.2020

Elegible to write the exam are those students which have submitted examination sheets and have received **50%** of possible points. !!!

You have the chance to complete your submission up to 29.1. 2020,
Next chance for exam at 24.3. 2020 (2nd exam)