

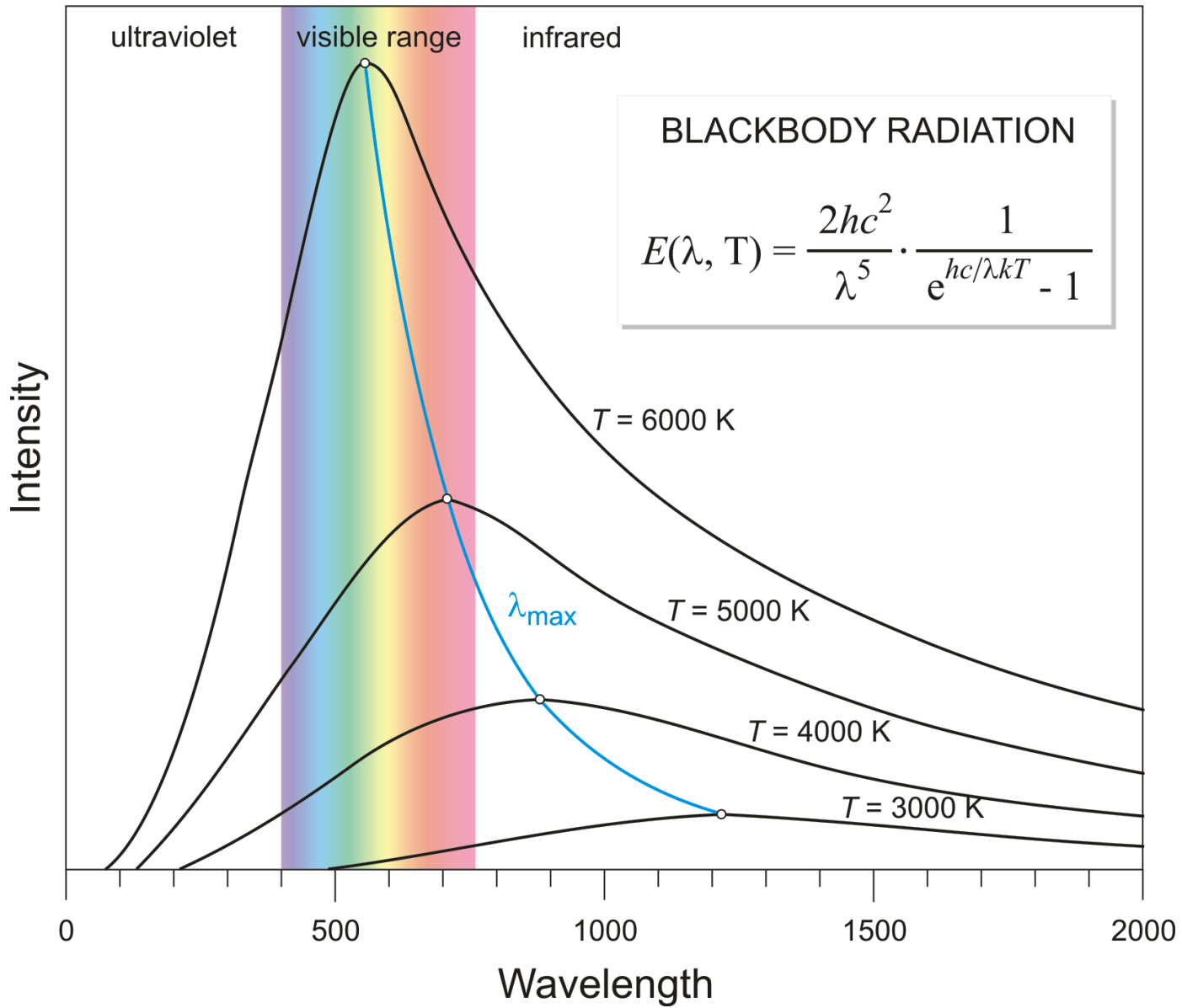
# Physics 1



## Lecture 11: Wave to Particle dualism

Prof. Dr. U. Pietsch

# Black body radiation



# Light is quantized

Classical theory ?

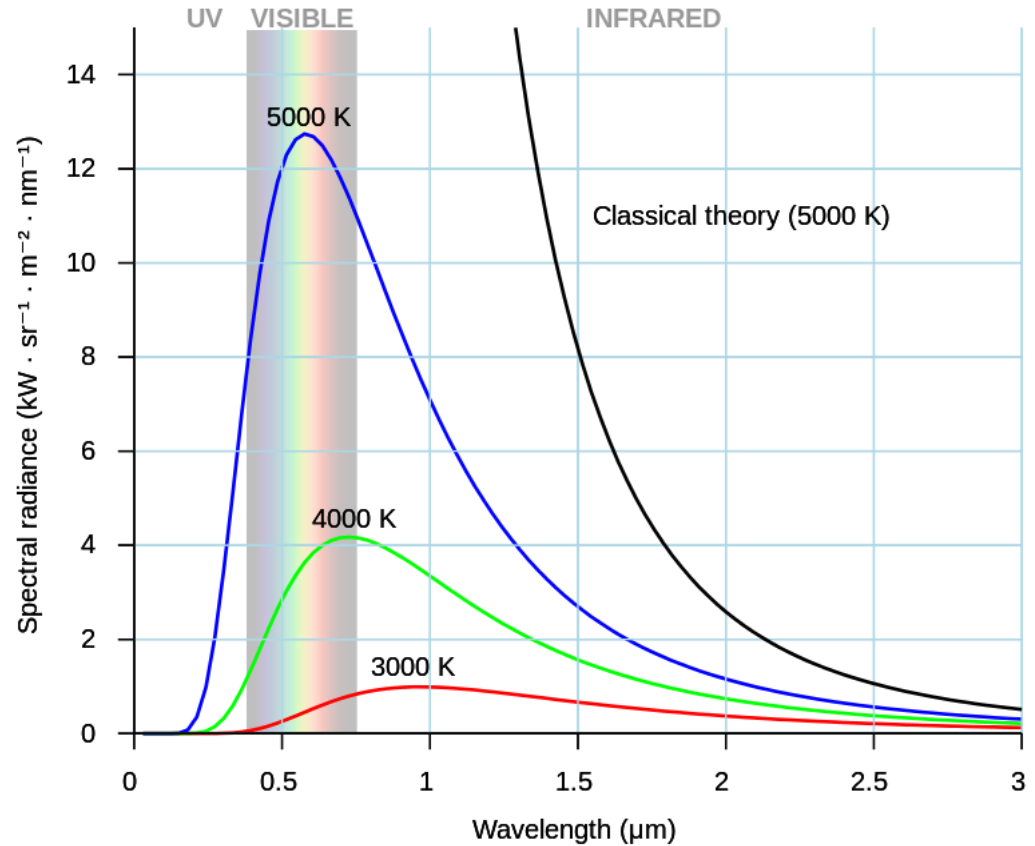
Planck's law states that

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Radiation is quantized

Energy quantum of a photon

$$E = h\nu$$



$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$

## Emission and absorption of light as photons

A sodium vapor lamp is placed at the center of a large sphere that absorbs all the light reaching it. The rate at which the lamp emits energy is 100 W; assume that the emission is entirely at a wavelength of 590 nm. At what rate are photons absorbed by the sphere?

### KEY IDEAS

The light is emitted and absorbed as photons. We assume that all the light emitted by the lamp reaches (and thus is absorbed by) the sphere. So, the rate  $R$  at which photons are absorbed by the sphere is equal to the rate  $R_{\text{emit}}$  at which photons are emitted by the lamp.

**Calculations:** That rate is

$$R_{\text{emit}} = \frac{\text{rate of energy emission}}{\text{energy per emitted photon}} = \frac{P_{\text{emit}}}{E}$$

Into this we can substitute from Eq. 38-2 ( $E = hf$ ), Einstein's proposal about the energy  $E$  of each quantum of light (which we here call a photon in modern language). We can then write the absorption rate as

$$R = R_{\text{emit}} = \frac{P_{\text{emit}}}{hf}$$

Using Eq. 38-1 ( $f = c/\lambda$ ) to substitute for  $f$  and then entering known data, we obtain

$$\begin{aligned} R &= \frac{P_{\text{emit}}\lambda}{hc} \\ &= \frac{(100 \text{ W})(590 \times 10^{-9} \text{ m})}{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(2.998 \times 10^8 \text{ m/s})} \\ &= 2.97 \times 10^{20} \text{ photons/s.} \end{aligned} \quad (\text{Answer})$$

# Photoelectric effect

Photons of frequency  $f$  is shining on target and ejects photo electrons

Stopping potential  $V_{\text{stop}}$  measures work function,  $K$ , of photoelectrons

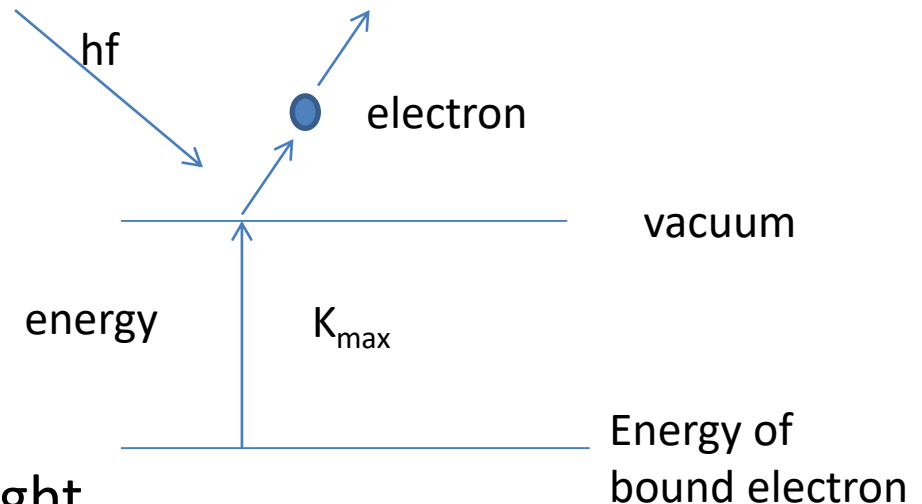
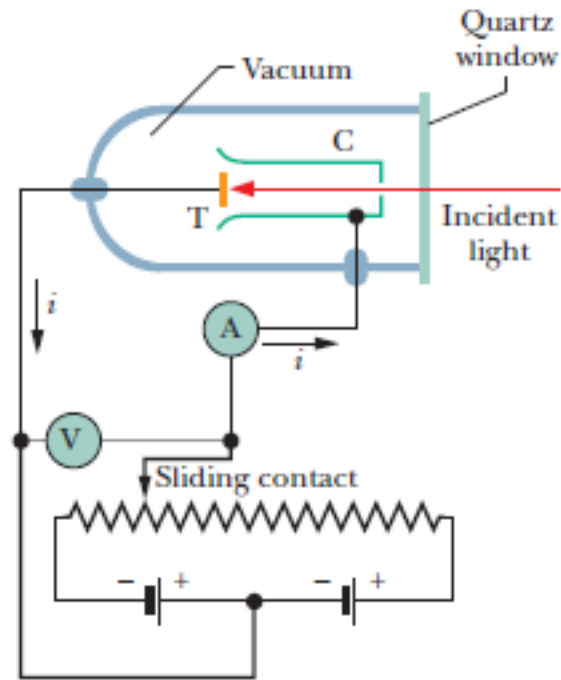
$$K_{\text{max}} = eV_{\text{stop}}$$

$K$  does not depend on intensity of light but from its energy

$$hf = K_{\text{max}} + \Phi \quad (\text{photoelectric equation}).$$

$\Phi$  kinetic energy of photo electrons

Proof of particle hypothesis of light



# Measuring $\Phi$

Electrons can escape only if the light frequency exceeds a certain value.

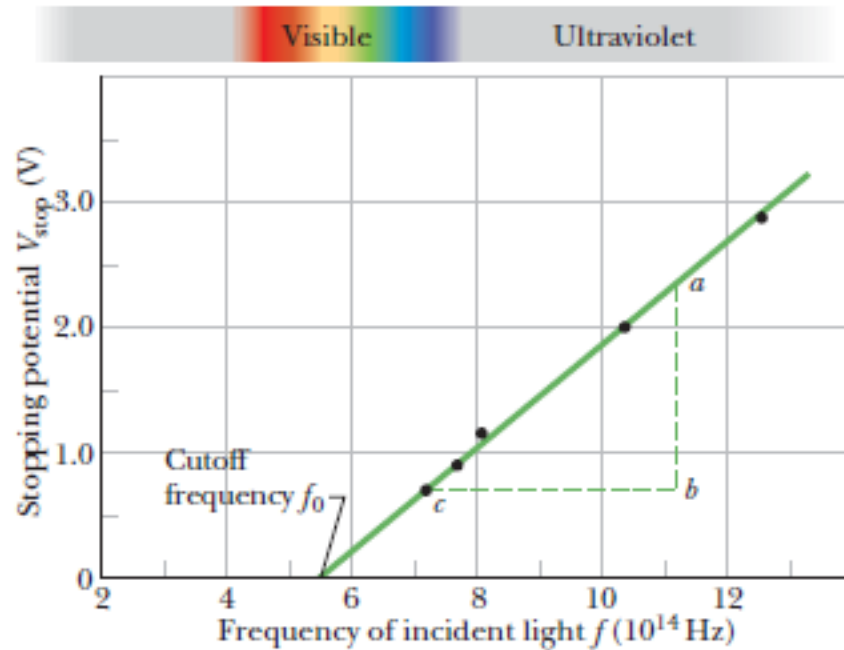
The escaping electron's kinetic energy is greater for a greater light frequency.

$$V_{\text{stop}} = \left( \frac{h}{e} \right) f - \frac{\Phi}{e}$$

$$\frac{h}{e} = \frac{ab}{bc} = \frac{2.35 \text{ V} - 0.72 \text{ V}}{(11.2 \times 10^{14} - 7.2 \times 10^{14}) \text{ Hz}}$$

$$= 4.1 \times 10^{-15} \text{ V} \cdot \text{s}$$

$$h = (4.1 \times 10^{-15} \text{ V} \cdot \text{s})(1.6 \times 10^{-19} \text{ C}) = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}$$



stopping potential  $V_{\text{stop}}$  as a function of the frequency  $f$  of the incident light for a sodium target T in the apparatus of Fig. 38-1. (Data reported by R. A. Millikan in 1916.)

Hint for the „particle“ nature of light

## Photoelectric effect and work function

Find the work function  $\Phi$  of sodium from Fig. 38-2.

### KEY IDEAS

We can find the work function  $\Phi$  from the cutoff frequency  $f_0$  (which we can measure on the plot). The reasoning is this: At the cutoff frequency, the kinetic energy  $K_{\max}$  in Eq. 38-5 is zero. Thus, all the energy  $hf$  that is transferred from a photon to an electron goes into the electron's escape, which requires an energy of  $\Phi$ .

**Calculations:** From that last idea, Eq. 38-5 then gives us, with  $f = f_0$ ,

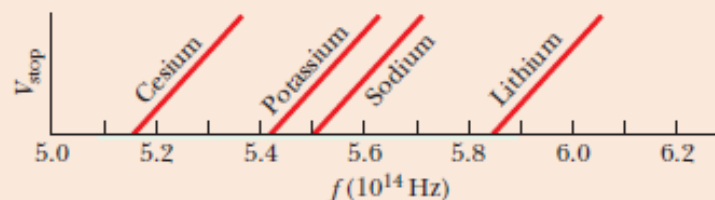
$$hf_0 = 0 + \Phi = \Phi.$$

In Fig. 38-2, the cutoff frequency  $f_0$  is the frequency at which the plotted line intercepts the horizontal frequency axis, about  $5.5 \times 10^{14}$  Hz. We then have

$$\begin{aligned}\Phi &= hf_0 = (6.63 \times 10^{-34} \text{ J}\cdot\text{s})(5.5 \times 10^{14} \text{ Hz}) \\ &= 3.6 \times 10^{-19} \text{ J} = 2.3 \text{ eV.} \quad (\text{Answer})\end{aligned}$$

### CHECKPOINT 2

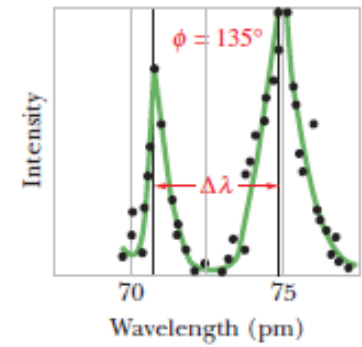
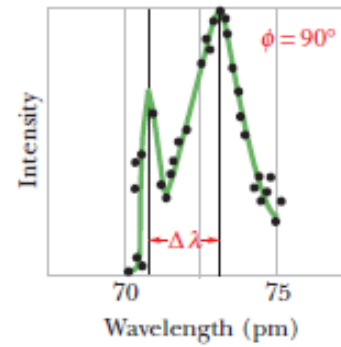
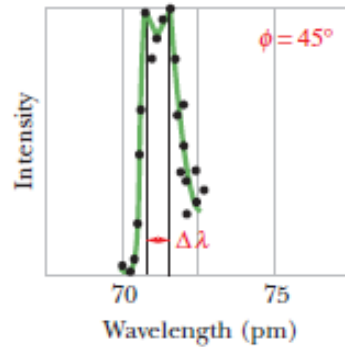
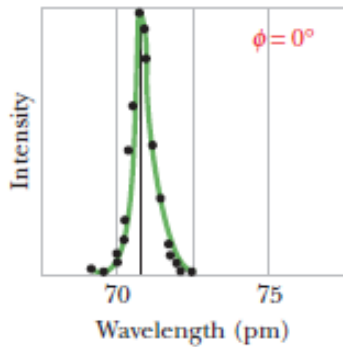
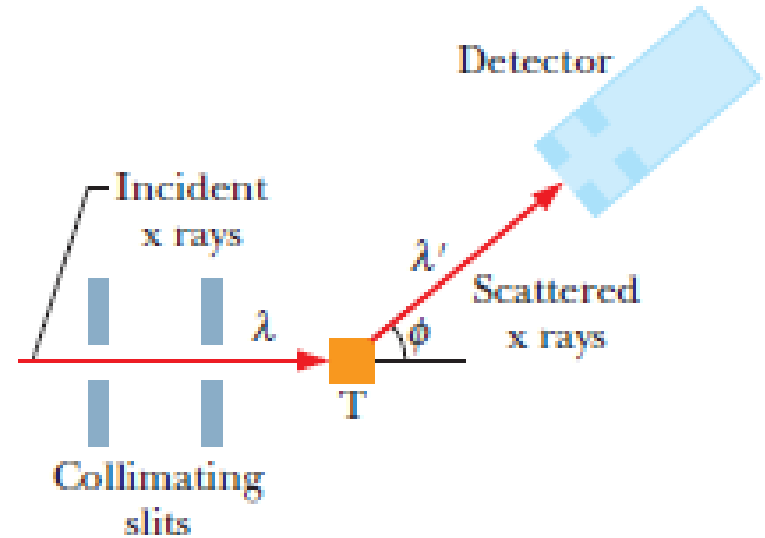
The figure shows data like those of Fig. 38-2 for targets of cesium, potassium, sodium, and lithium. The plots are parallel. (a) Rank the targets according to their work functions, greatest first. (b) Rank the plots according to the value of  $h$  they yield, greatest first.



# Compton effect

$$E = h\nu = hc/\lambda \rightarrow p = m c = E/c = h/\lambda$$

$$p = \frac{hf}{c} = \frac{h}{\lambda} \quad (\text{photon momentum}),$$



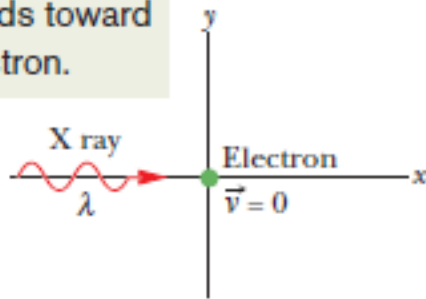
Wave length shift increases with increasing scattering angle

Photons have a momentum

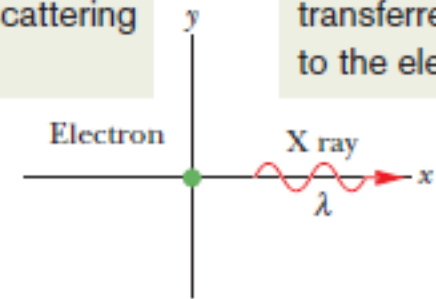


# Possible scenarios of photon-electron scattering

An x ray heads toward a target electron.

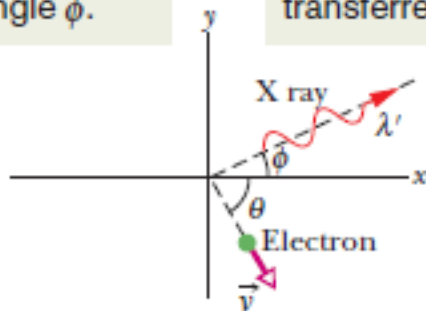


The x ray can bypass the electron at scattering angle  $\phi = 0$ .



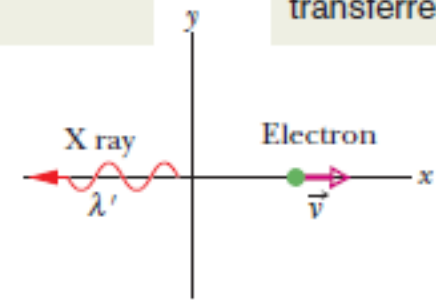
No energy is transferred to the electron.

Or it can scatter at some intermediate angle  $\phi$ .



Intermediate energy is transferred.

Or it can backscatter at the maximum angle  $\phi = 180^\circ$ .



Maximum energy is transferred.

$$E_{kin} = h\nu - h\nu'$$

Conservation of momentum

$$mv = \frac{h\nu}{c} \sin \frac{\theta}{2} + \frac{h\nu'}{c} \sin \frac{\theta}{2}$$

$$v \approx v' \quad \frac{1}{2}mv = \frac{h\nu}{c} \sin \frac{\theta}{2}$$

Conservation of energy

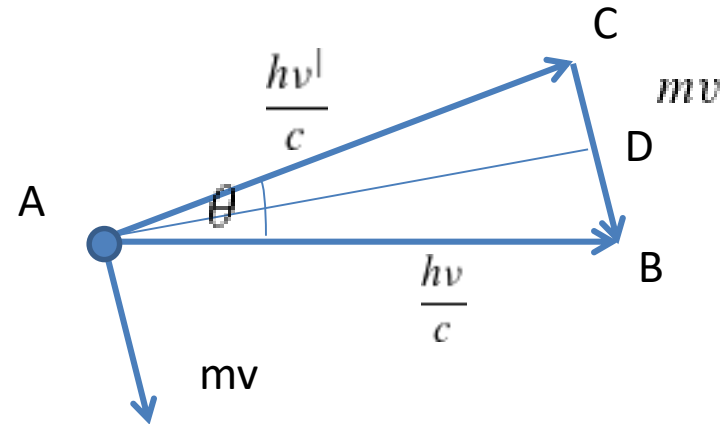
$$\frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{4h^2\nu^2 \sin^2 \frac{\theta}{2}}{2mc^2} = h\nu - h\nu'$$

\*  $1/(h\nu^2)$  and using  $v' \approx v$

$$\frac{2h}{mc^2} \sin^2 \frac{\theta}{2} = \frac{\nu - \nu'}{\nu^2} \approx \frac{1}{\nu} - \frac{1}{\nu'}$$

$$\lambda = \frac{c}{\nu}$$

$$\boxed{\lambda - \lambda' = \frac{2h}{mc} \sin^2 \frac{\theta}{2}}$$



Proof of the „particle“ character of light

Compton wave length

$$\lambda_c = \frac{2h}{mc} = 0.0243 \text{ \AA}$$

$$\Delta\lambda = \frac{h}{mc} (1 - \cos \theta)$$

## Compton scattering of light by electrons

X rays of wavelength  $\lambda = 22$  pm (photon energy = 56 keV) are scattered from a carbon target, and the scattered rays are detected at  $85^\circ$  to the incident beam.

(a) What is the Compton shift of the scattered rays?

### KEY IDEA

The Compton shift is the wavelength change of the x rays due to scattering from loosely bound electrons in a target. Further, that shift depends on the angle at which the scattered x rays are detected, according to Eq. 38-11. The shift is zero for forward scattering at angle  $\phi = 0^\circ$ , and it is maximum for back scattering at angle  $\phi = 180^\circ$ . Here we have an intermediate situation at angle  $\phi = 85^\circ$ .

**Calculation:** Substituting  $85^\circ$  for that angle and  $9.11 \times 10^{-31}$  kg for the electron mass (because the scattering is from electrons) in Eq. 38-11 gives us

$$\begin{aligned}\Delta\lambda &= \frac{h}{mc} (1 - \cos \phi) \\ &= \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(1 - \cos 85^\circ)}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} \\ &= 2.21 \times 10^{-12} \text{ m} \approx 2.2 \text{ pm.} \quad (\text{Answer})\end{aligned}$$

(b) What percentage of the initial x-ray photon energy is transferred to an electron in such scattering?

### KEY IDEA

We need to find the *fractional energy loss* (let us call it *frac*) for photons that scatter from the electrons:

$$\text{frac} = \frac{\text{energy loss}}{\text{initial energy}} = \frac{E - E'}{E}.$$

**Calculations:** From Eq. 38-2 ( $E = hf$ ), we can substitute for the initial energy  $E$  and the detected energy  $E'$  of the x rays in terms of frequencies. Then, from Eq. 38-1 ( $f = c/\lambda$ ), we can substitute for those frequencies in terms of the wavelengths. We find

$$\begin{aligned}\text{frac} &= \frac{hf - hf'}{hf} = \frac{c/\lambda - c/\lambda'}{c/\lambda} = \frac{\lambda' - \lambda}{\lambda} \\ &= \frac{\Delta\lambda}{\lambda + \Delta\lambda}.\end{aligned} \quad (38-12)$$

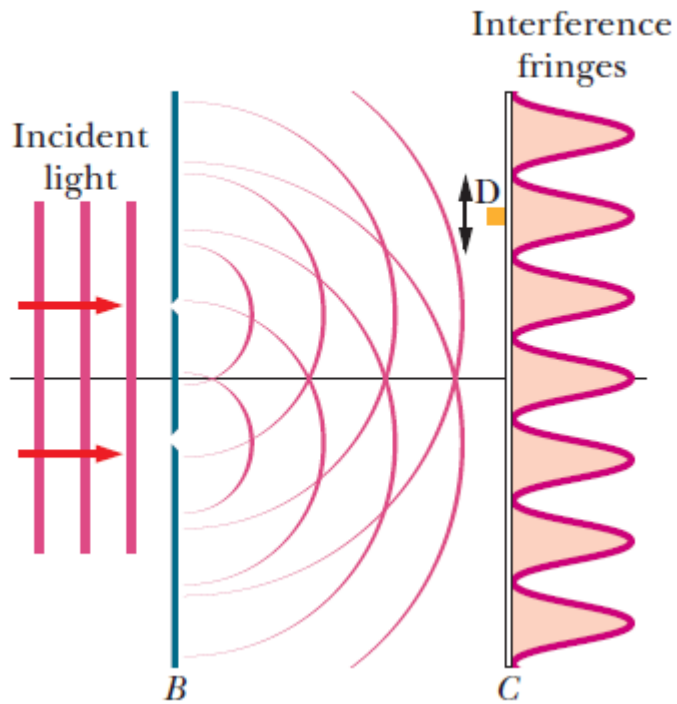
Substitution of data yields

$$\text{frac} = \frac{2.21 \text{ pm}}{22 \text{ pm} + 2.21 \text{ pm}} = 0.091, \text{ or } 9.1\%. \quad (\text{Answer})$$

Although the Compton shift  $\Delta\lambda$  is independent of the wavelength  $\lambda$  of the incident x rays (see Eq. 38-11), the *fractional* photon energy loss of the x rays does depend on  $\lambda$ , increasing as the wavelength of the incident radiation decreases, as indicated by Eq. 38-12.

# Light as a probability wave

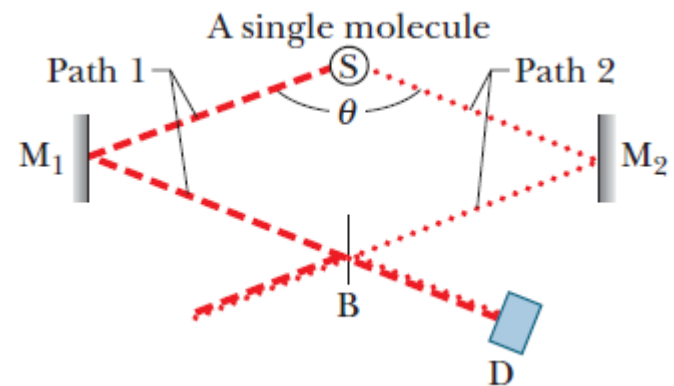
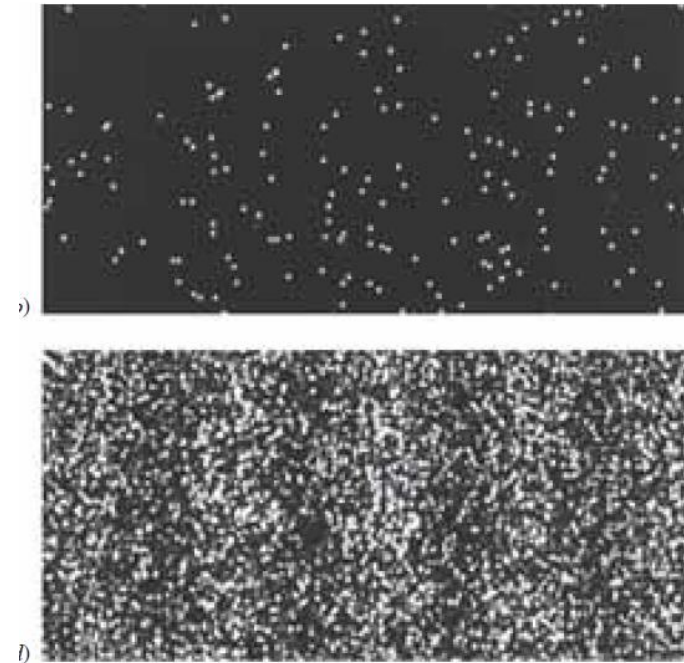
Measure by screen



$$\lambda = \frac{h}{p} \quad (\text{de Broglie wavelength}),$$

Proof of wave to particle character of light

Measure by photon counter



# Neutron scattering

deBroglie relation

$$E = h\nu = hc/\lambda \rightarrow p = m c = E/c = h/\lambda$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m_n E}}$$

$$\lambda = \frac{h}{p} \quad (\text{de Broglie wavelength}),$$

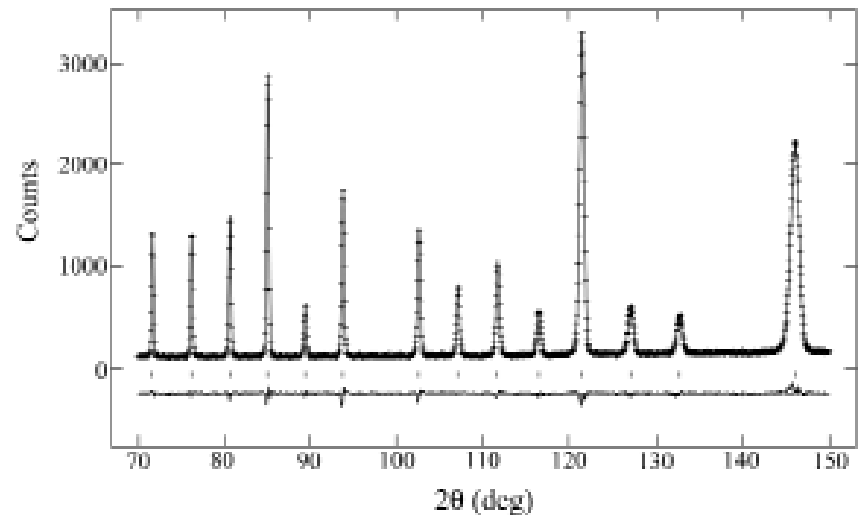
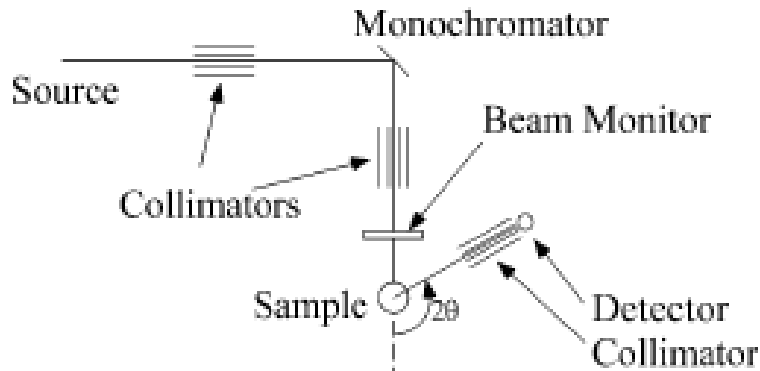
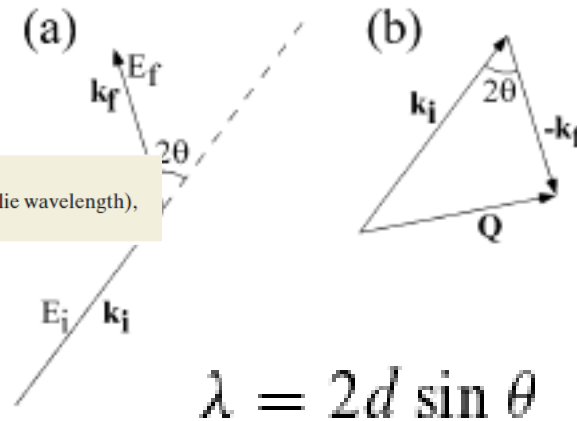


Fig. 12. The high angle part of the neutron powder diffraction pattern of cubic  $\text{LaBa}_2\text{Fe}_3\text{O}_{9-\delta}$ , measured at 295 K. The incident wavelength was 1.54 Å,

Proof of „wave“ nature of particles

# Properties of the neutron

- Mass

Neutron	$m_n = 1.675 \times 10^{-24} \text{g}$
Electron	$m_e = 9.10 \times 10^{-28} \text{g}$
Photon	0

- Dispersion

$$E = \frac{\hbar^2 k^2}{2m} = 2.07214 \text{ meV} \text{ \AA}^2 \times k^2$$

$$\lambda = \frac{2\pi}{k} = \frac{9.044605 \text{ \AA}}{\sqrt{E[\text{meV}]}}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2m_n E}}$$

$$1 \text{ meV} = 1.6 \times 10^{-9} \text{ erg} = 11.6 \text{ K} = 0.24 \text{ THz}$$

$$E = U_e$$

$$E = \kappa T$$

$$E = h\nu$$

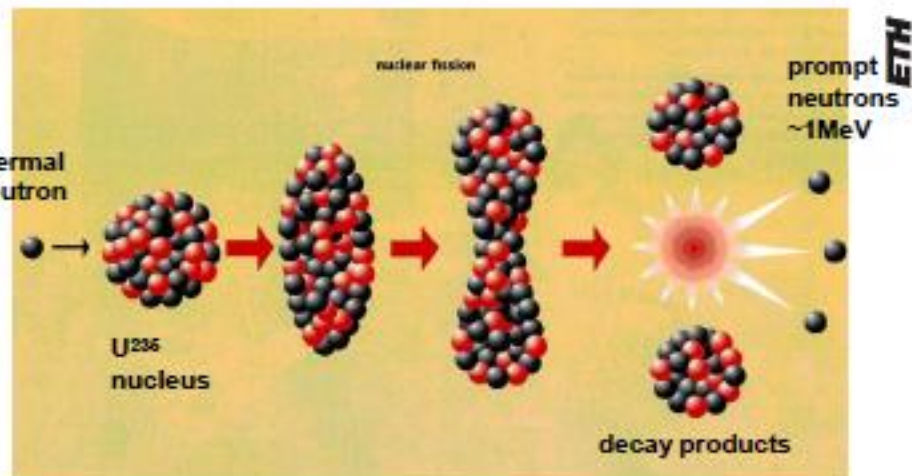
- Energy at  $\lambda = 5 \text{ \AA}$

Neutron	$E = 3.3 \text{ meV} \sim 40 \text{ K}$
Electron	$E \sim 6 \text{ eV} \sim 70,000 \text{ K}$
Photon	$E = hc/\lambda \sim 2.5 \text{ keV} \sim 3 \text{ MK}$

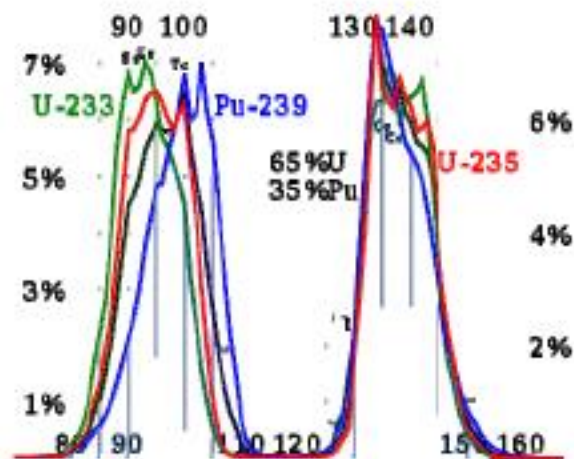
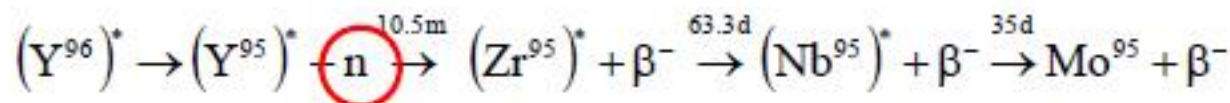
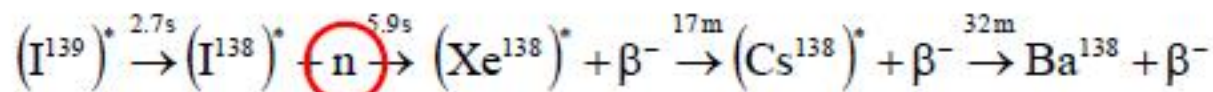
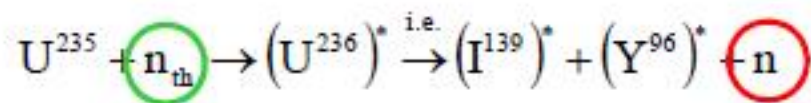
• Neutron energy and wavelengths are perfectly matched to condensed matter physics

# Fission

- Neutron-induced nuclear decay
- Average yield:
  - thermal fission of  $U^{235}$ : 2.5 n
  - fast fission of  $U^{238}$ : 2.6 n
  - spontaneous decay of  $U^{238}$ : 2.4 n



- $U^{235}$  example:

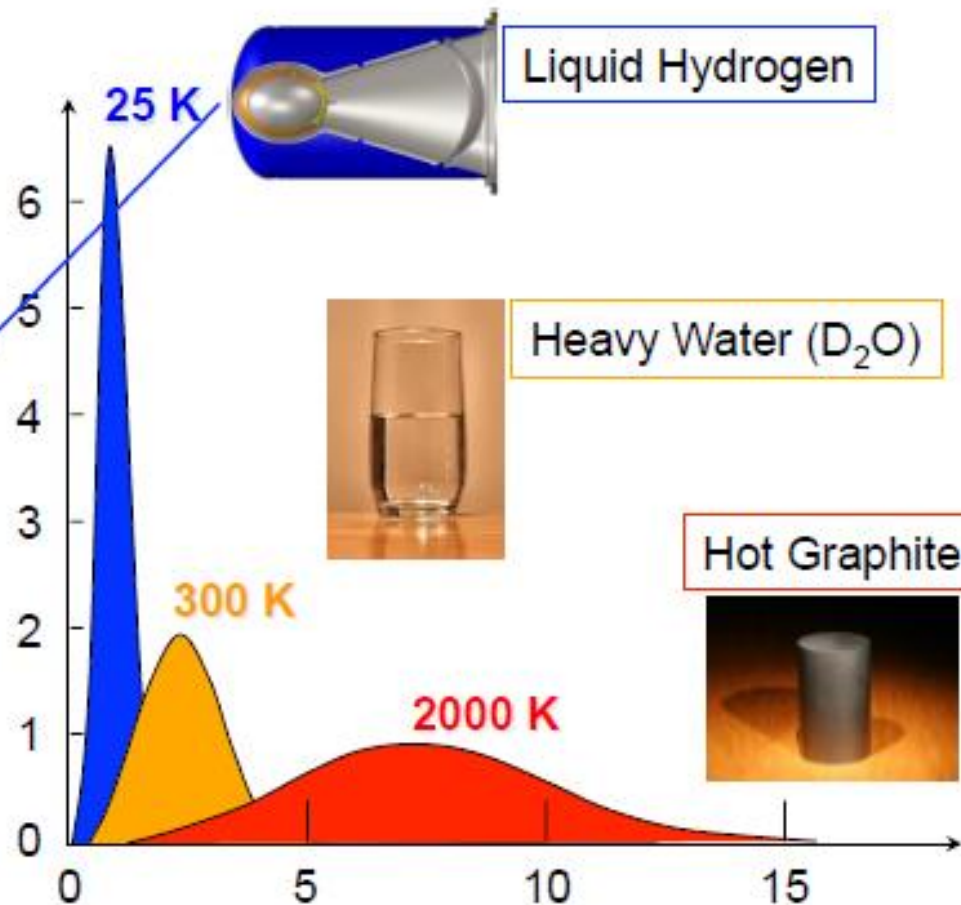
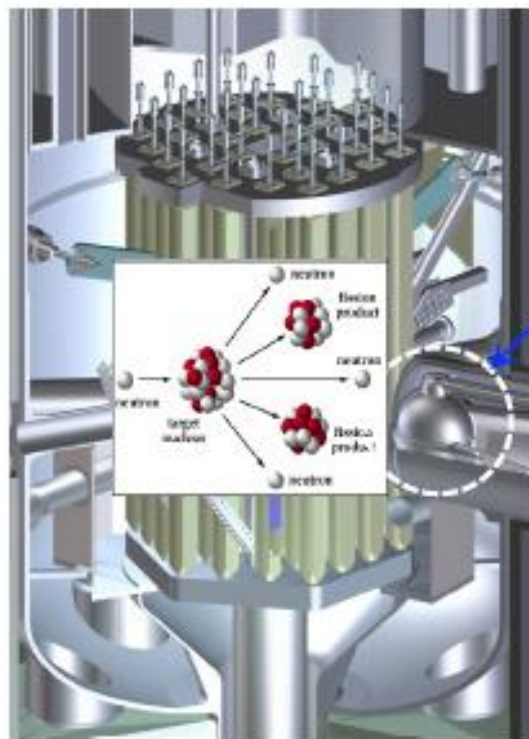


- Moderation will sustain chain reaction

# Neutron Moderation

Maxwellian Distribution

$$\Phi \sim v^3 e^{(-mv^2/2k_B T)}$$

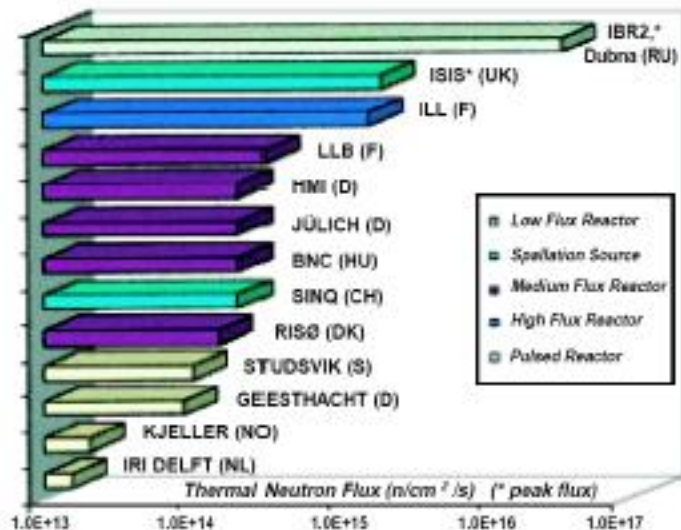
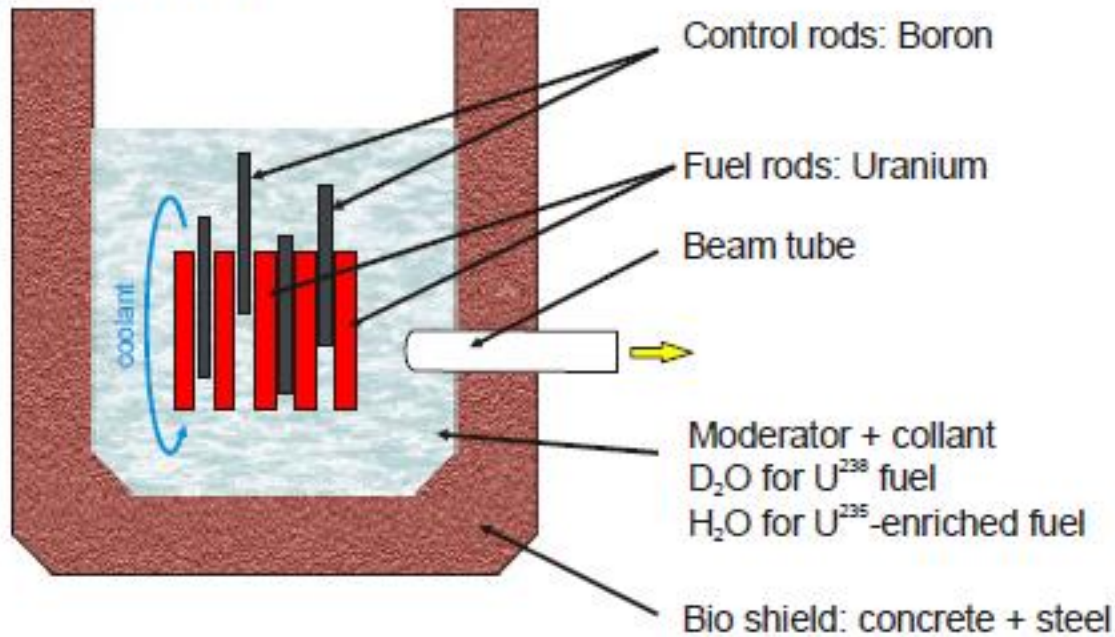


“Fast” neutrons:  $v = 20,000$  km/sec

Neutron velocity  $v$  (km/sec)



# Reactors



# Reactor neutrons

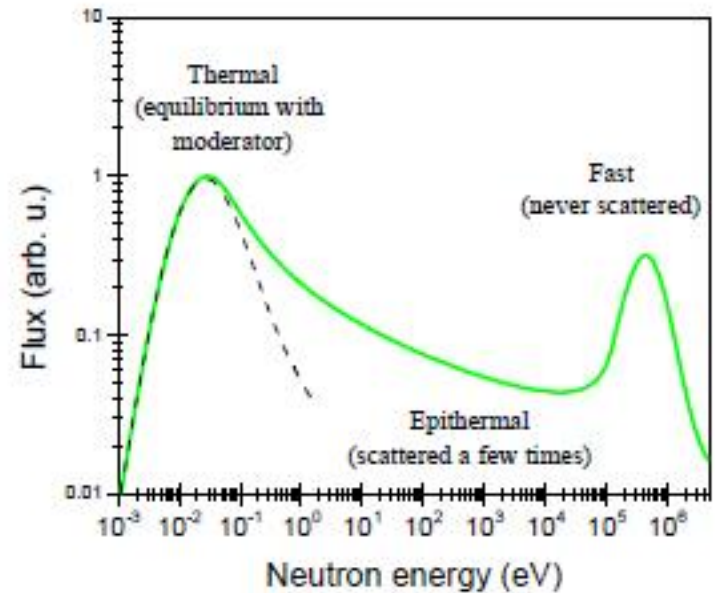
- Spectrum

$$n_v \propto v^2 \exp\left(-\frac{mv^2}{2\kappa T}\right)$$

Moderator temperature

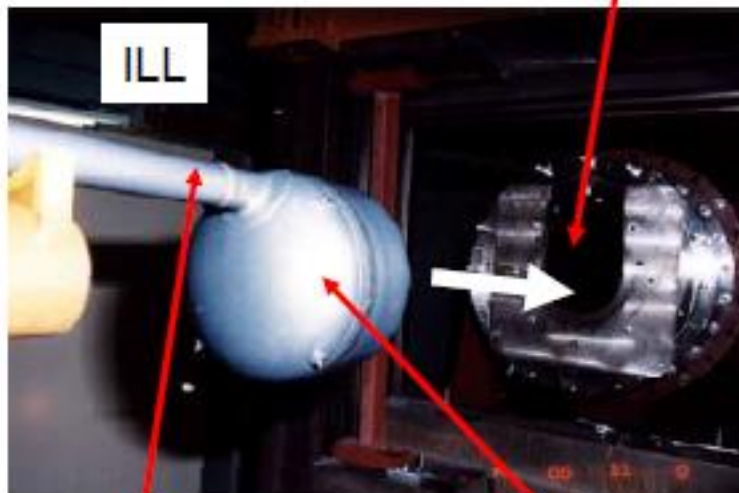
$$E_{\max} = \kappa T$$

$$\bar{E} = \frac{\kappa T}{2}$$



- Cold source:  $E/\kappa \sim 20\text{K}$

Beam tube



ILL

Cryogen circulation

Liquid hydrogen

- Hot source:  $E/\kappa \sim 2000\text{K}$

