Physics 1

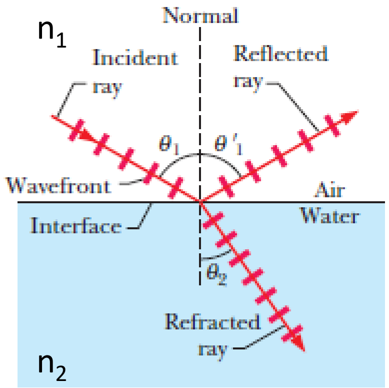
UNIVERSITÄT SIEGEN

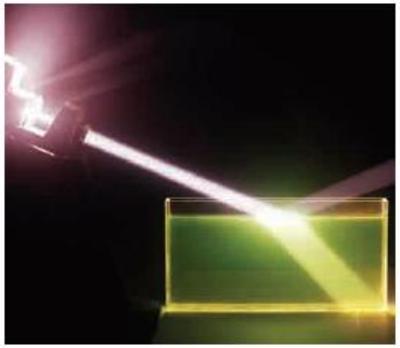
FESTKÖRPERPHYSIK

Lecture 10: Wave optics : interference and diffraction

Prof. Dr. U. Pietsch

Reflection and refraction





Law of reflection

$$\theta'_1 = \theta_1 \qquad \text{(reflection).}$$

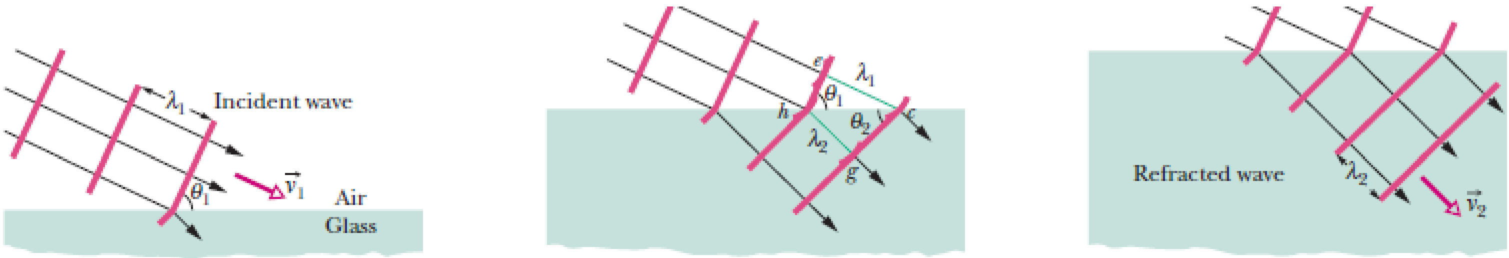
Law of refraction

$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \qquad \text{(refraction).}$$

Some Indexes of Refraction[a]

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

Law of refraction – revised



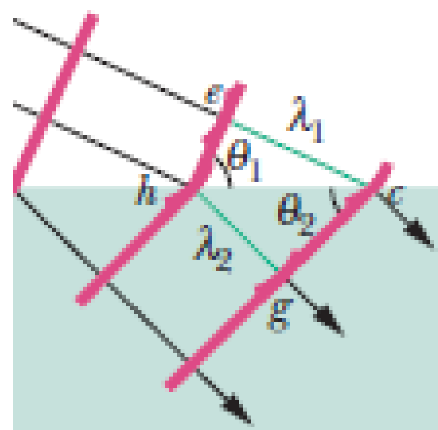
(a) (b) (c)

c = λ f

Since c is constant

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2},$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}.$$



$$\sin\theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle } hce\text{)}$$

$$\sin\theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle } hcg\text{).}$$

$$n = \frac{c}{v} \quad \text{(index of refraction).}$$

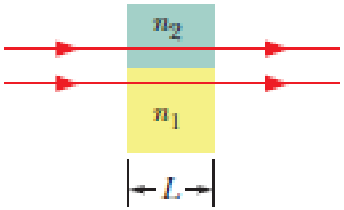
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

Wavelength , λ_n, and velocity of light, v_n, are changing in medium but frequence, f, stays unchanged

Phase difference

$$\lambda_n = \lambda \frac{v}{c}. \qquad \lambda_n = \frac{\lambda}{n}.$$

The difference in indexes causes a phase shift between the rays.

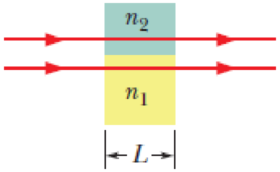


Number of wave lengths within length L

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda}. \qquad N_2 = \frac{L}{\lambda_{n2}} = \frac{Ln_2}{\lambda}.$$

Phase difference $$N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_1}{\lambda} = \frac{L}{\lambda}(n_2 - n_1).$$

The phase difference between two light waves can change if the waves travel through different materials having different indexes of refraction.



In Fig. 35-4, the two light waves that are represented by the rays have wavelength 550.0 nm before entering media 1 and 2. They also have equal amplitudes and are in phase. Medium 1 is now just air, and medium 2 is a transparent plastic layer of index of refraction 1.600 and thickness 2.600 μm.

(a) What is the phase difference of the emerging waves in wavelengths, radians, and degrees? What is their effective phase difference (in wavelengths)?

KEY IDEA

The phase difference of two light waves can change if they travel through different media, with different indexes of refraction. The reason is that their wavelengths are different in the different media. We can calculate the change in phase difference by counting the number of wavelengths that fits into each medium and then subtracting those numbers.

Calculations: When the path lengths of the waves in the two media are identical, Eq. 35-11 gives the result of the subtraction. Here we have $n_1 = 1.000$ (for the air), $n_2 = 1.600$, $L = 2.600\ \mu$m, and $\lambda = 550.0$ nm. Thus, Eq. 35-11 yields

$$\begin{aligned} N_2 - N_1 &= \frac{L}{\lambda}(n_2 - n_1) \\ &= \frac{2.600 \times 10^{-6}\text{ m}}{5.500 \times 10^{-7}\text{ m}}(1.600 - 1.000) \\ &= 2.84. \end{aligned} \qquad \text{(Answer)}$$

Thus, the phase difference of the emerging waves is 2.84 wavelengths. Because 1.0 wavelength is equivalent to 2π rad and 360°, you can show that this phase difference is equivalent to

$$\text{phase difference} = 17.8\text{ rad} \approx 1020°. \qquad \text{(Answer)}$$

The effective phase difference is the decimal part of the actual phase difference *expressed in wavelengths.* Thus, we have

$$\text{effective phase difference} = 0.84\text{ wavelength.} \qquad \text{(Answer)}$$

You can show that this is equivalent to 5.3 rad and about 300°. *Caution:* We do *not* find the effective phase difference by taking the decimal part of the actual phase difference as expressed in radians or degrees. For example, we do *not* take 0.8 rad from the actual phase difference of 17.8 rad.

(b) If the waves reached the same point on a distant screen, what type of interference would they produce?

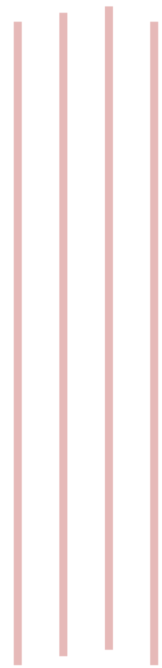
Reasoning: We need to compare the effective phase difference of the waves with the phase differences that give the extreme types of interference. Here the effective phase difference of 0.84 wavelength is between 0.5 wavelength (for fully destructive interference, or the darkest possible result) and 1.0 wavelength (for fully constructive interference, or the brightest possible result), but closer to 1.0 wavelength. Thus, the waves would produce intermediate interference that is closer to fully constructive interference—they would produce a relatively bright spot.

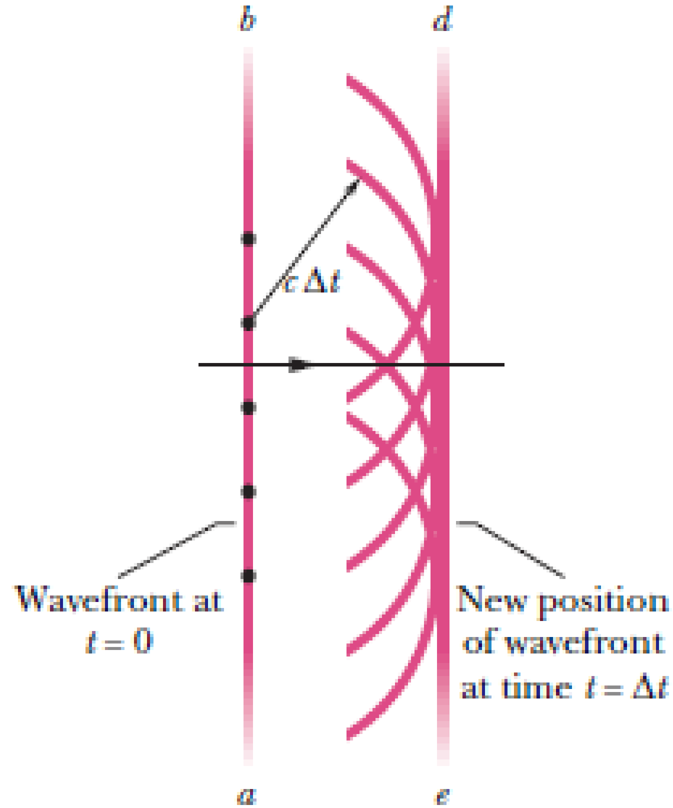
Huygen´s Principle

Christiaan Huygens
1629 - 1695

Incoming
wavefront

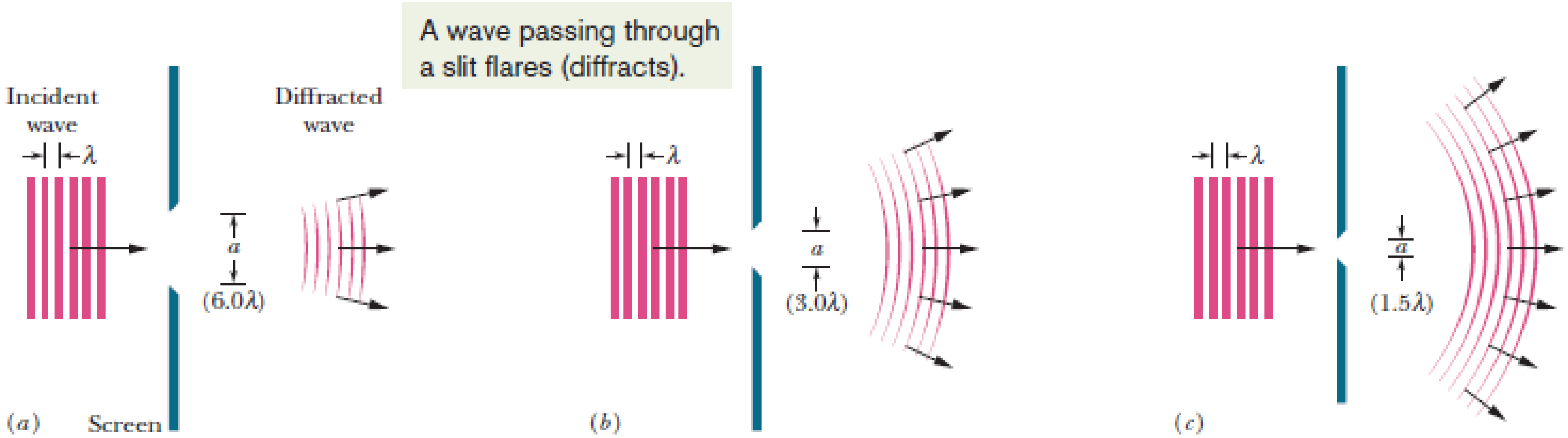
Often a
plane wave





All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

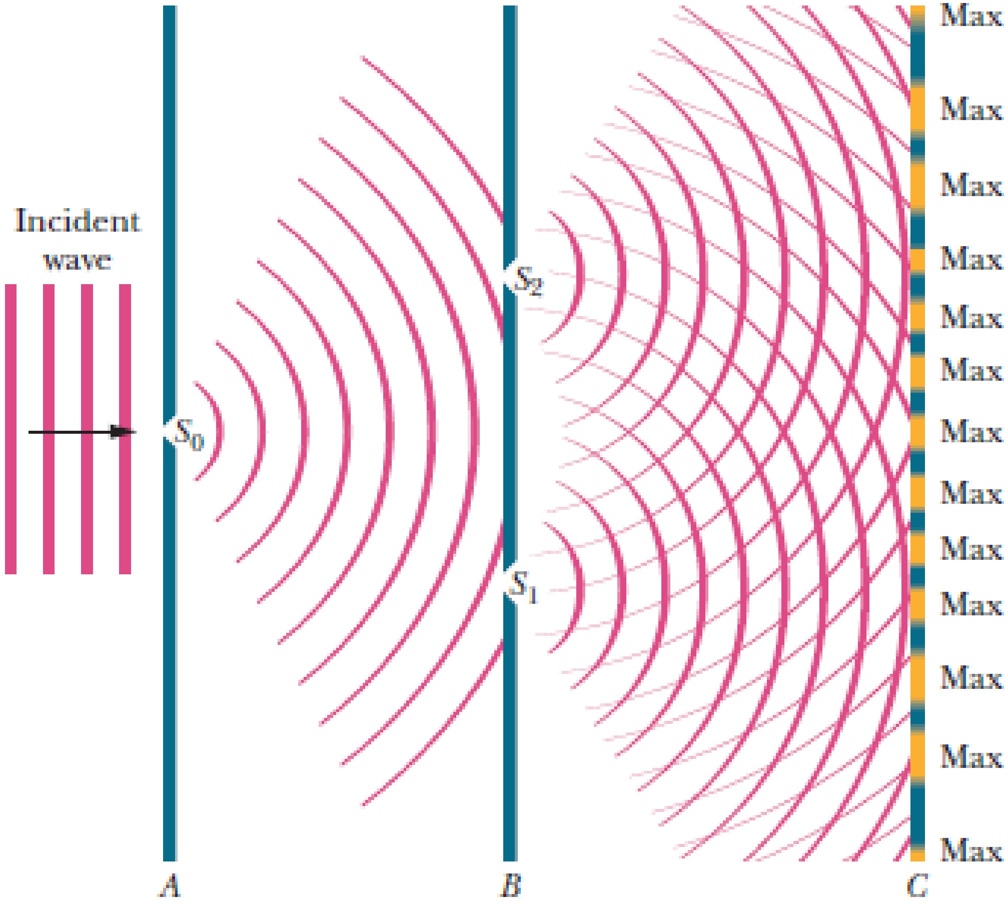
Diffraction

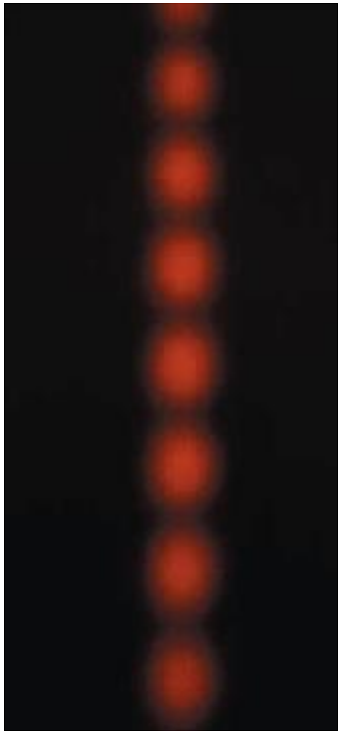


The smaller the slit the smaller is bending radius of the created spherical wave

Young´s double slit experiment

Bright and dark fringes

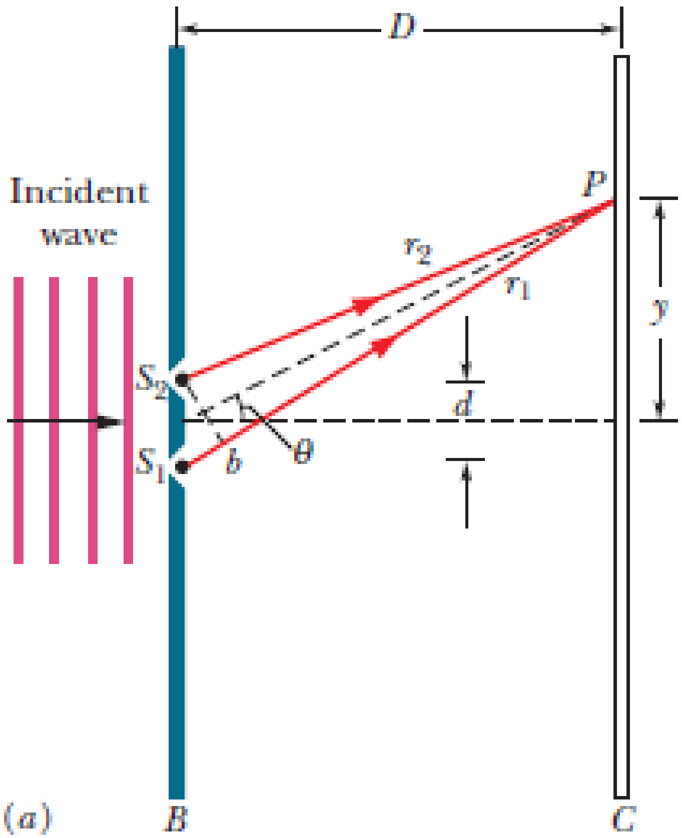




The waves emerging from the two slits overlap and form an interference pattern.

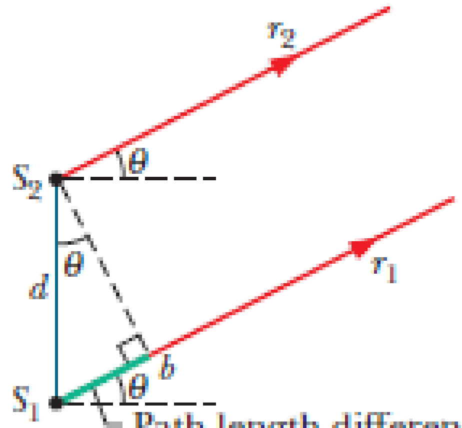
Path length difference

The phase difference between two waves can change if the waves travel paths of different lengths.



(a)

$\sin\theta = \Delta L/d$



The ΔL shifts one wave from the other, which determines the interference.

$\sin\theta = \Delta L/d$

For bright fringes

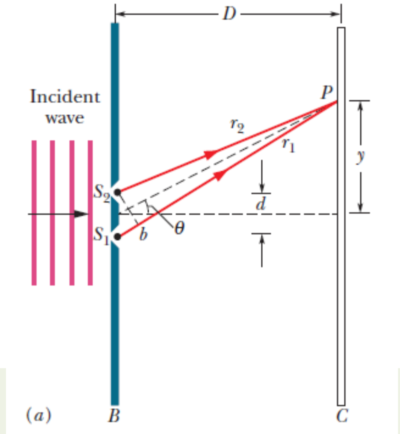
$$\Delta L = d\sin\theta = (\text{integer})(\lambda),$$

$$d\sin\theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright fringes).}$$

For dark fringes

$$\Delta L = d\sin\theta = (\text{odd number})(\tfrac{1}{2}\lambda),$$

$$d\sin\theta = (m + \tfrac{1}{2})\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark fringes).}$$



Double-slit interference pattern

What is the distance on screen C in Fig. 35-10a between adjacent maxima near the center of the interference pattern? The wavelength λ of the light is 546 nm, the slit separation d is 0.12 mm, and the slit–screen separation D is 55 cm. Assume that θ in Fig. 35-10 is small enough to permit use of the approximations $\sin\theta \approx \tan\theta \approx \theta$, in which θ is expressed in radian measure.

KEY IDEAS

(1) First, let us pick a maximum with a low value of m to ensure that it is near the center of the pattern. Then, from the geometry of Fig. 35-10a, the maximum's vertical distance y_m from the center of the pattern is related to its angle θ from the central axis by

$$\tan\theta \approx \theta = \frac{y_m}{D}.$$

(2) From Eq. 35-14, this angle θ for the mth maximum is given by

$$\sin\theta \approx \theta = \frac{m\lambda}{d}.$$

Calculations: If we equate our two expressions for angle θ and then solve for y_m, we find

$$y_m = \frac{m\lambda D}{d}. \qquad (35\text{-}17)$$

For the next maximum as we move away from the pattern's center, we have

$$y_{m+1} = \frac{(m+1)\lambda D}{d}. \qquad (35\text{-}18)$$

We find the distance between these adjacent maxima by subtracting Eq. 35-17 from Eq. 35-18:

$$\Delta y = y_{m+1} - y_m = \frac{\lambda D}{d}$$

$$= \frac{(546 \times 10^{-9}\text{ m})(55 \times 10^{-2}\text{ m})}{0.12 \times 10^{-3}\text{ m}}$$

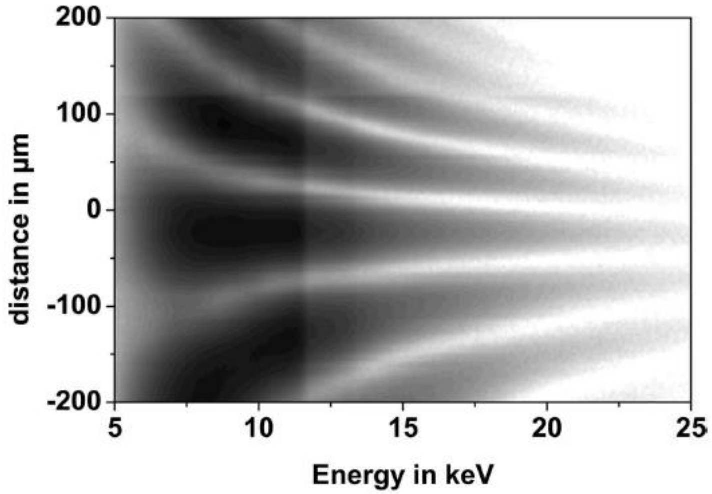
$$= 2.50 \times 10^{-3}\text{ m} \approx 2.5\text{ mm}. \qquad \text{(Answer)}$$

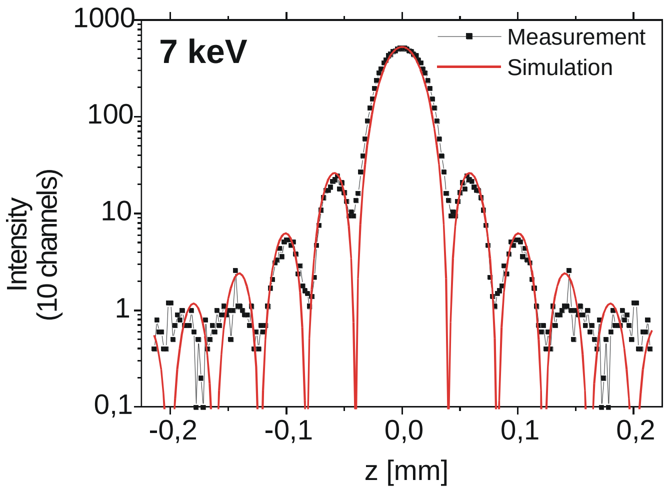
As long as d and θ in Fig. 35-10a are small, the separation of the interference fringes is independent of m; that is, the fringes are evenly spaced.

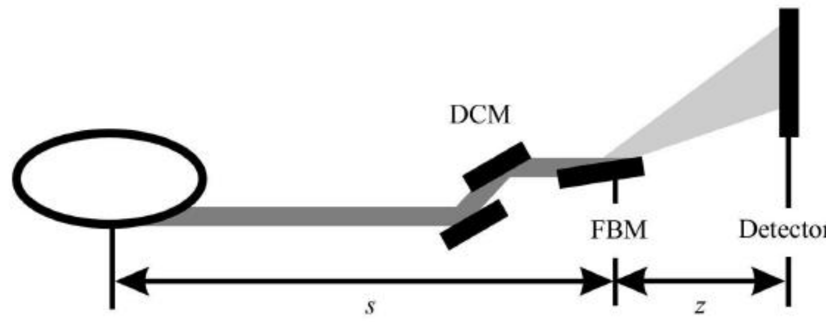
Fresnel bi-layer experiment with x-rays

A monolithic Fresnel bimirror for hard X-rays and its application for coherence measurements

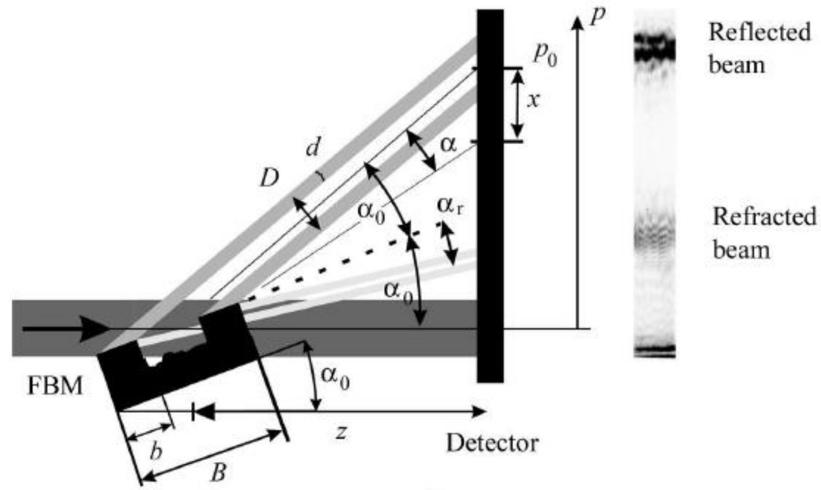
Wolfram Leitenberger[a]* and Ullrich Pietsch[b]





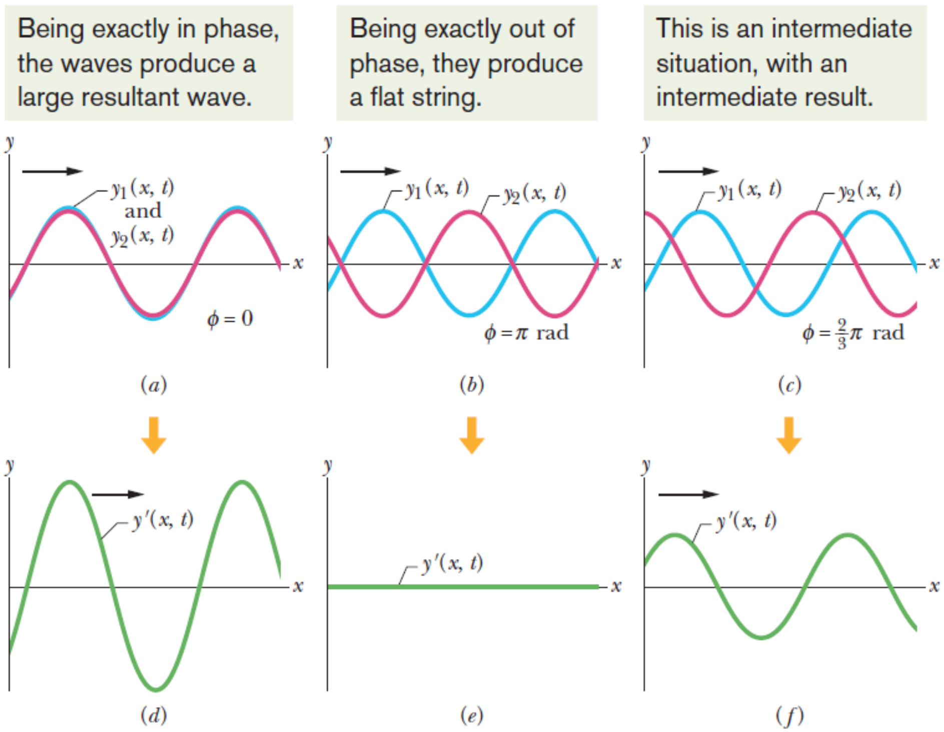


(a)



(b)

Intensity of double slit interference



$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin(\omega t + \phi),$$

$$I = (E_1 + E_2)^2, \quad I_0 = E_0^2$$

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$$

$$\phi = \frac{2\pi d}{\lambda} \sin \theta.$$

Intensity maxima at

$$\tfrac{1}{2}\phi = m\pi, \qquad \text{for } m = 0, 1, 2, \ldots.$$

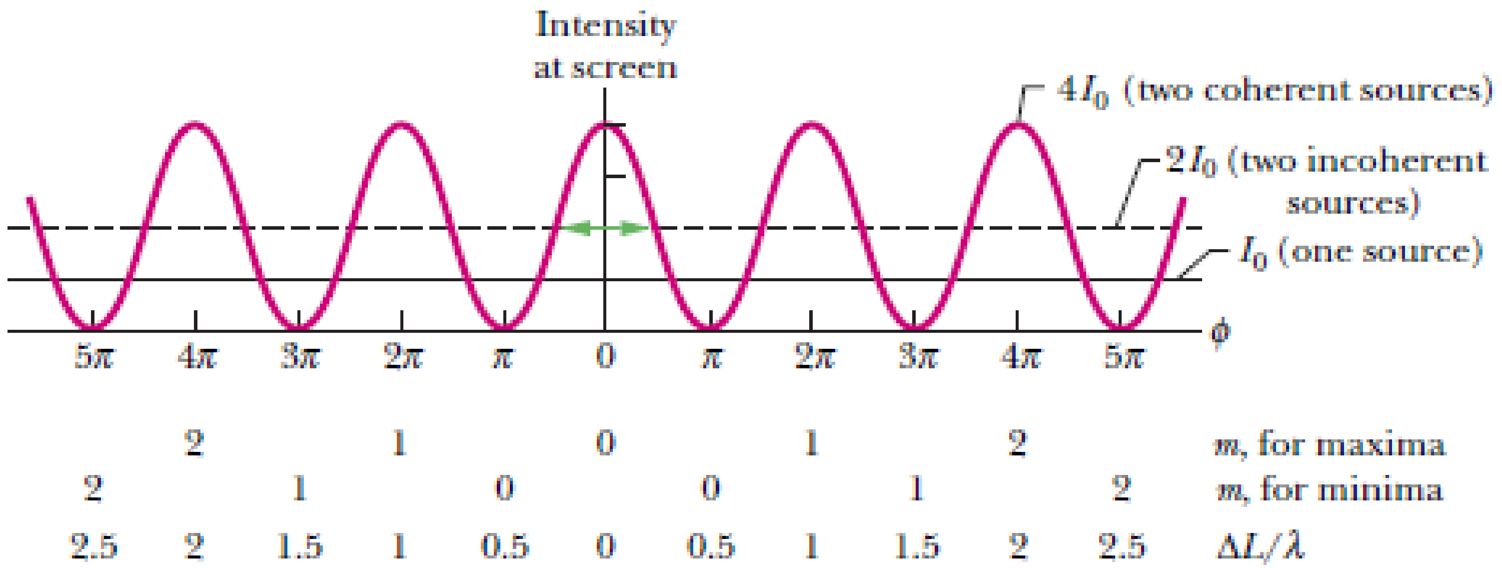
$$2m\pi = \frac{2\pi d}{\lambda} \sin \theta, \qquad \text{for } m = 0, 1, 2, \ldots$$

$$d \sin \theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \qquad \text{(maxima)},$$

Intensity minima at

$$\tfrac{1}{2}\phi = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 0, 1, 2, \ldots.$$

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \qquad \text{(minima)},$$



Coherence : the two interfering wave must be able to interfere, i.e. wave fronts must have same wave length and same shape.

Temporal coherence length $\Lambda = \lambda^2/2\Delta\lambda$

Spatial coherence length $\Lambda = \lambda/2\Delta\alpha$

$$\Lambda_{raum} = \frac{\lambda R}{d} = \frac{\lambda}{\alpha}$$

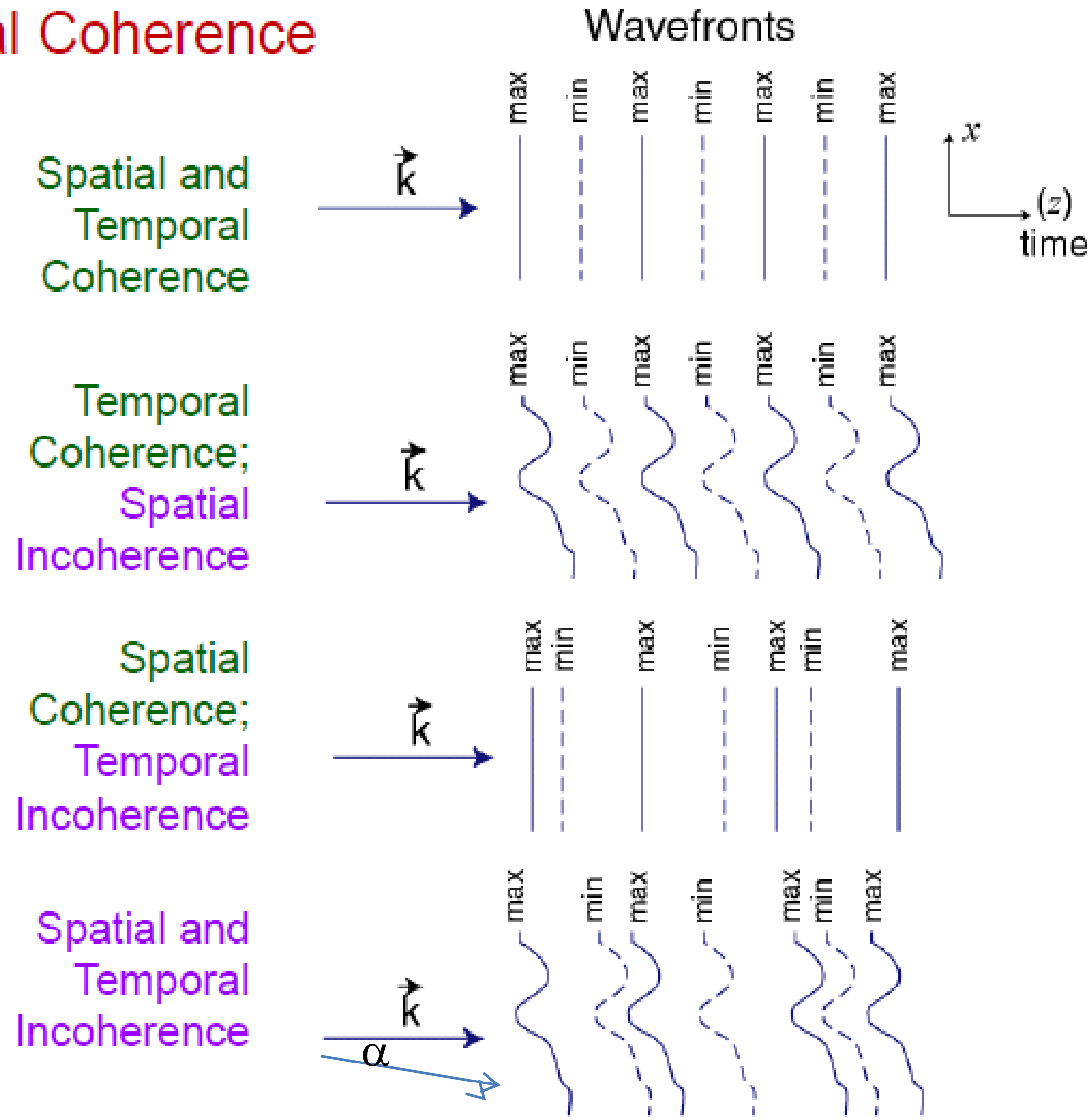
Spatial and Temporal Coherence

Temporal coherence length

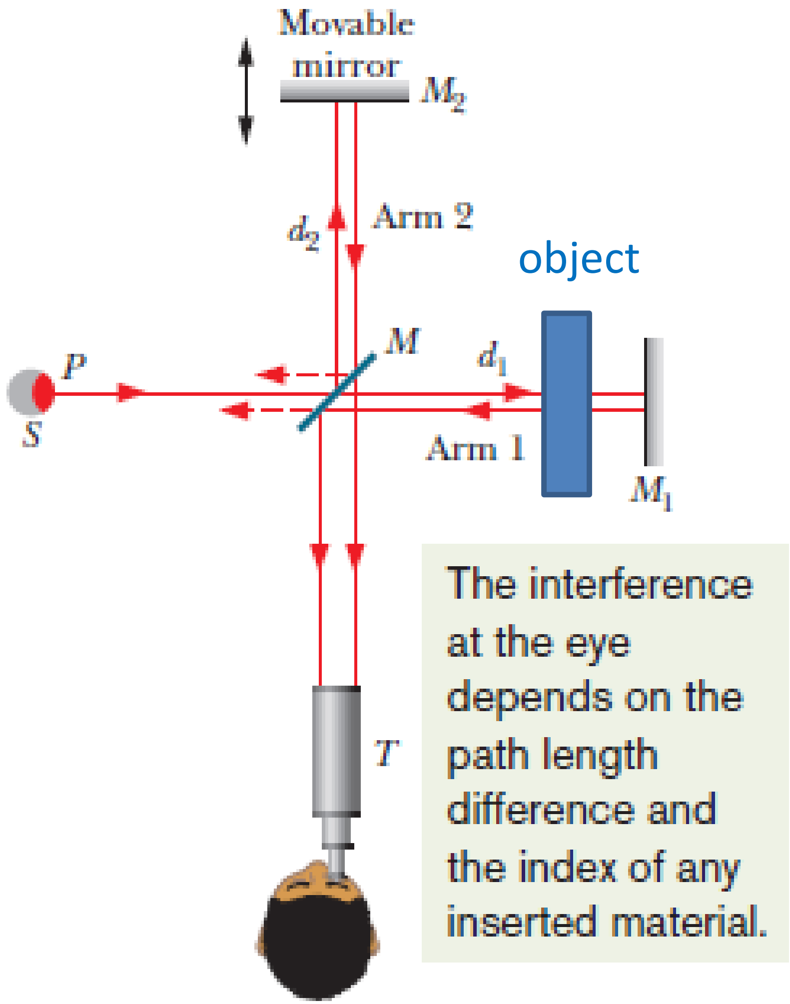
$$\Lambda = \lambda^2/(2\Delta\lambda)$$

Spatial coherence length

$$\Lambda = \lambda/(2\Delta\alpha)$$



Michelson interferometer



Object thickness L, index n

Number of fringes induced by the object

$$N_m = \frac{2L}{\lambda_n} = \frac{2Ln}{\lambda}.$$

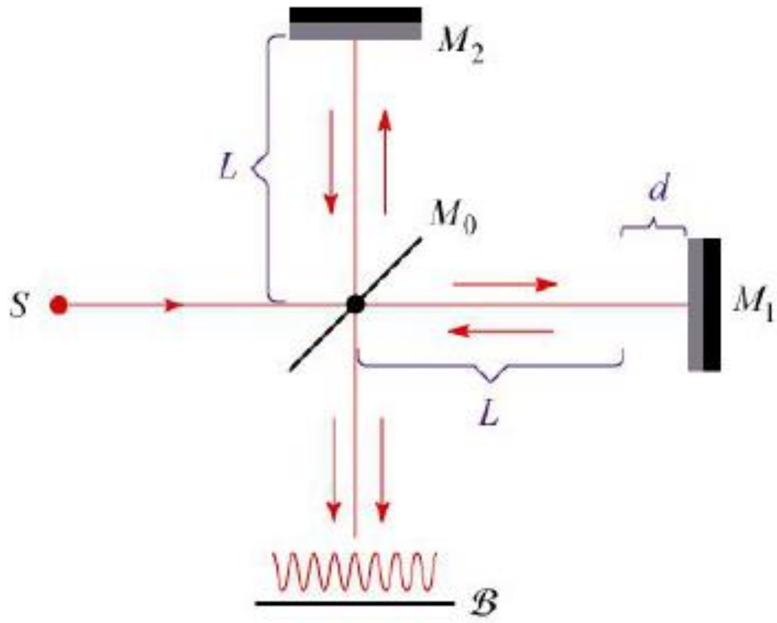
Number of fringes without object

$$N_a = \frac{2L}{\lambda}.$$

To measure L

$$N_m - N_a = \frac{2Ln}{\lambda} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n-1).$$

Temporal Coherence



Separate two subsequent Maxima

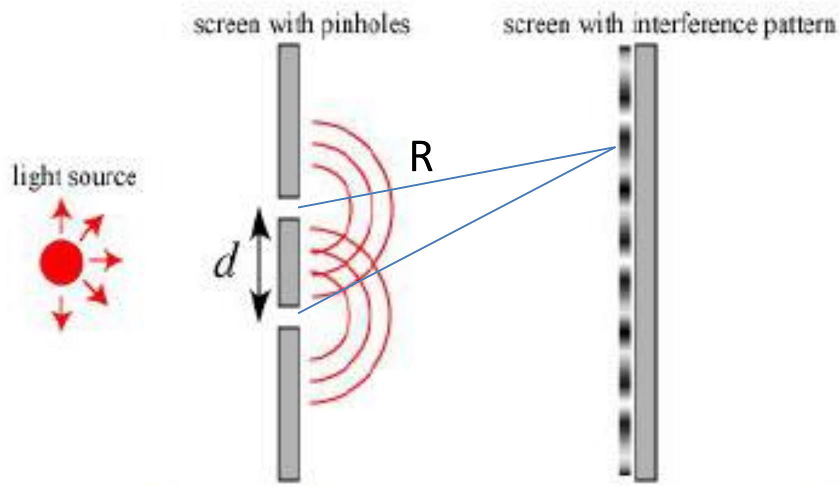
$$(N_{max} - \frac{1}{2})(\lambda + \frac{1}{2}\Delta\lambda) = (N_{max} + \frac{1}{2})(\lambda - \frac{1}{2}\Delta\lambda)$$

$$N_{max} = \frac{\lambda}{\Delta\lambda)}$$

$$L_{temporal} = N_{max}\lambda = \frac{\lambda^2}{\Delta\lambda}$$

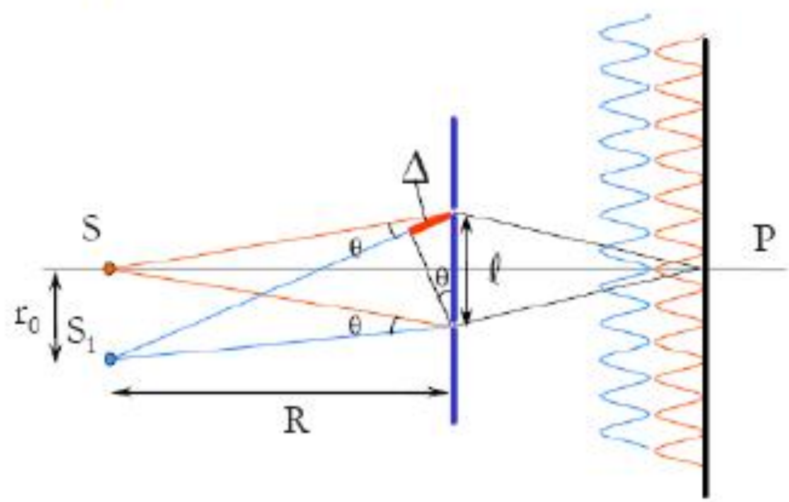
Temporal Coherence Time, τ_c Temporal Coherence Length, $\ell_c = 2d = c\,\tau_c$

Spatial Coherence



$$L_{spatial} = \frac{\lambda R}{d} = \frac{\lambda}{\theta}$$

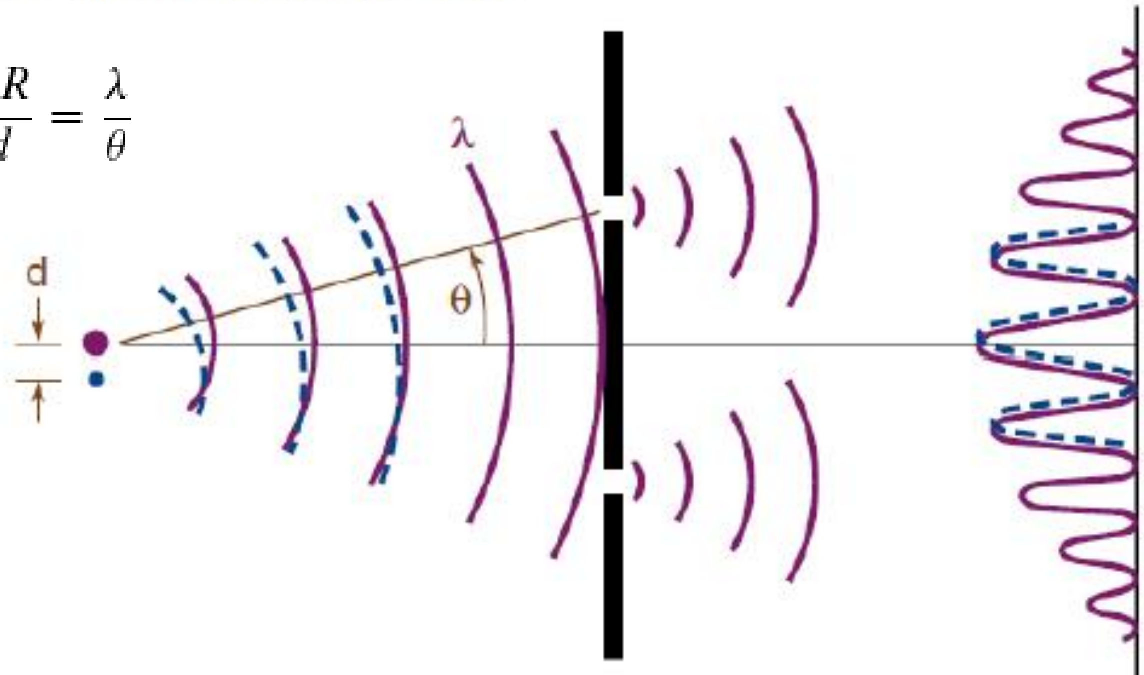
Spatial Coherence Area, $A_c = \pi d^2$

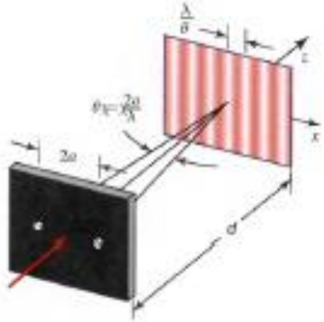


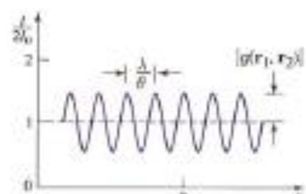
Young's Experiment to Demonstrate Spatial Coherence

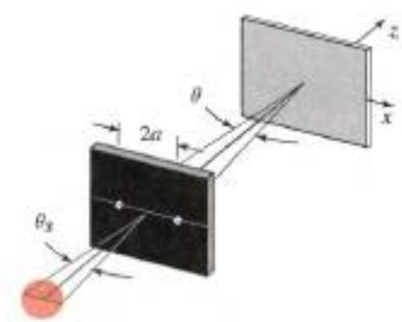
Persistence of fringes as the source grows
from a point source to finite size.

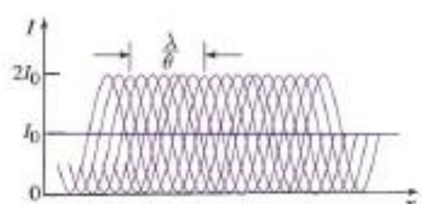
$$L_{spatial} = \frac{\lambda R}{d} = \frac{\lambda}{\theta}$$







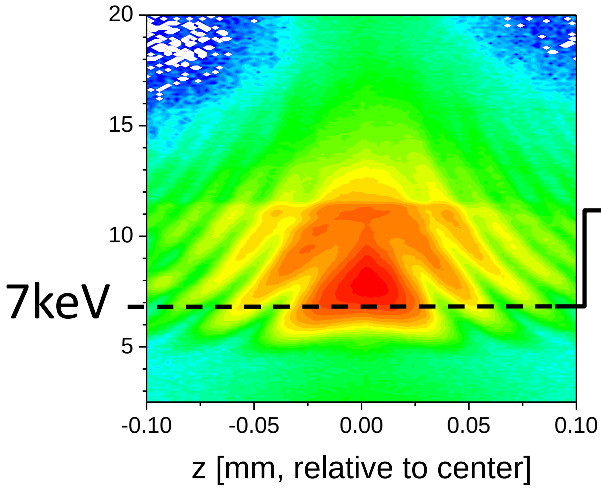


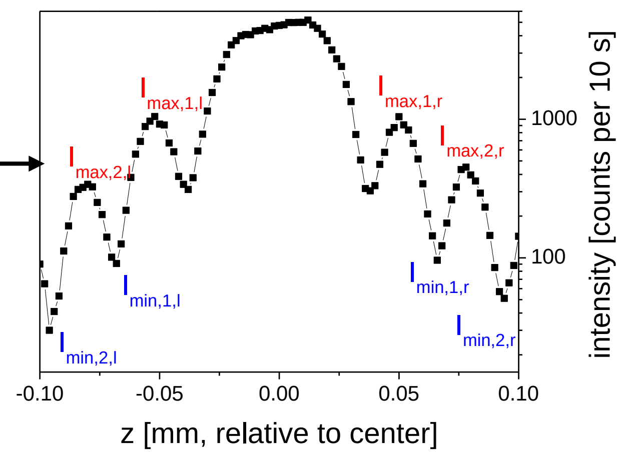


Calculation of coherence properties from the Fraunhofer X-ray Diffraction at Circular Aperture

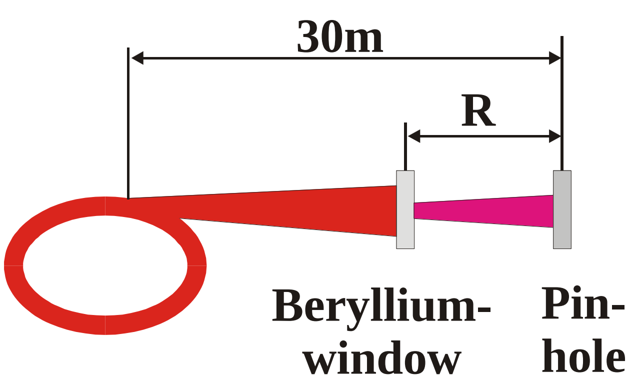
Coherence experiments at the white-beam beamline of BESSYII

Tobias Panzner [a], Gudrun Gleber [a], Tushar Sant [a], Wolfram Leitenberger [b], Ullrich Pietsch [a,*]



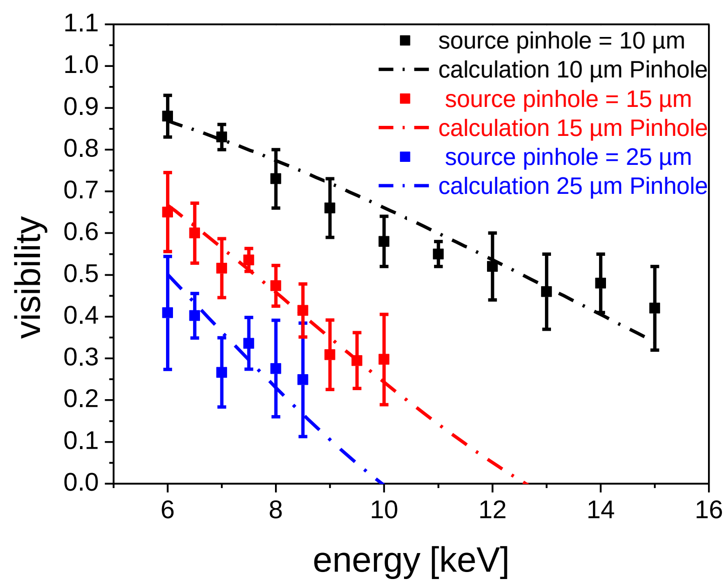


$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\sin\left(\frac{\pi \cdot s \cdot d \cdot E}{12{,}398 \cdot R}\right)}{\frac{\pi \cdot s \cdot d \cdot E}{12{,}398 \cdot R}}$$



averaged vertical source size s:
(46 +/- 6)µm

(for R=7,43m , distance of a Be-window in the beam path (looks like a virtual source))



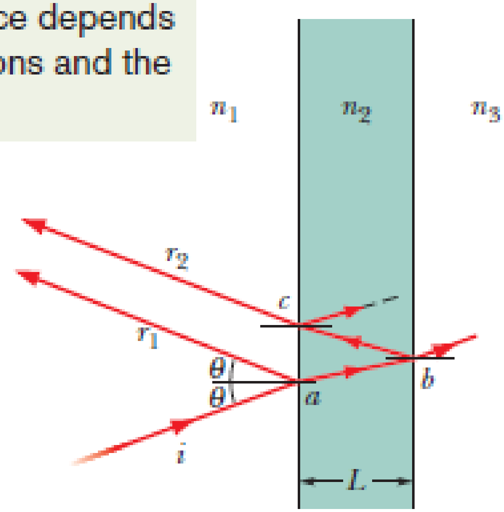
Interference at thin films

Constructive interference of r_1 (phase shift 0.5) and r_2 (no phase shift)

$$2L = \frac{\text{odd number}}{2} \times \lambda_{n2} \quad \text{(in-phase waves).}$$

The interference depends on the reflections and the path lengths.

$$\lambda_{n2} = \frac{\lambda}{n_2},$$

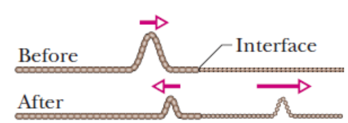


$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

Destructive interference of r_1 (phase shift 0.5) and r_2 (no phase shift)

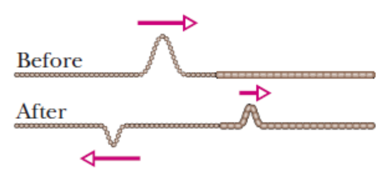
Reflection	Reflection phase shift	
Off lower index	0	n_1/n_2 interface
Off higher index	0.5 wavelength	n_2/n_3 interface

$$2L = \text{integer} \times \lambda_{n2} \quad \text{(out-of-phase waves).}$$



at b

$$2L = m\frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$



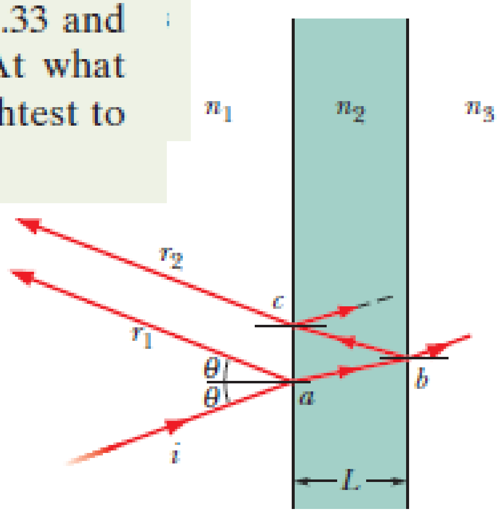
(b)

at a, c

$n_1 = n_3 = 1$

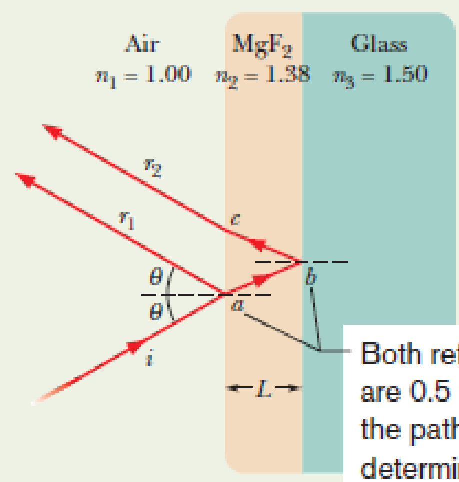
examples

White light, with a uniform intensity across the visible wavelength range of 400 to 690 nm, is perpendicularly incident on a water film, of index of refraction $n_2 = 1.33$ and thickness $L = 320$ nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?



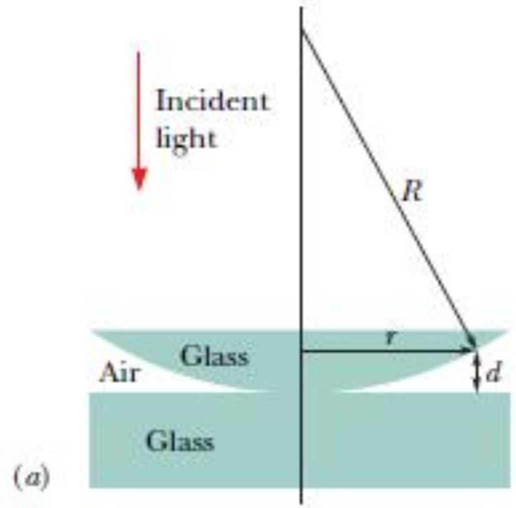
$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}.$$

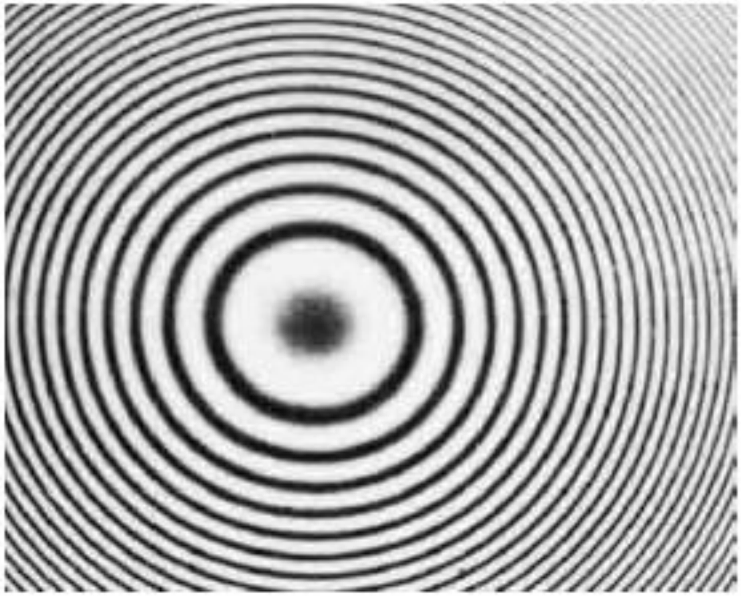
In Fig. 35-19, a glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface. The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550$ nm)? Assume that the light is approximately perpendicular to the lens surface.



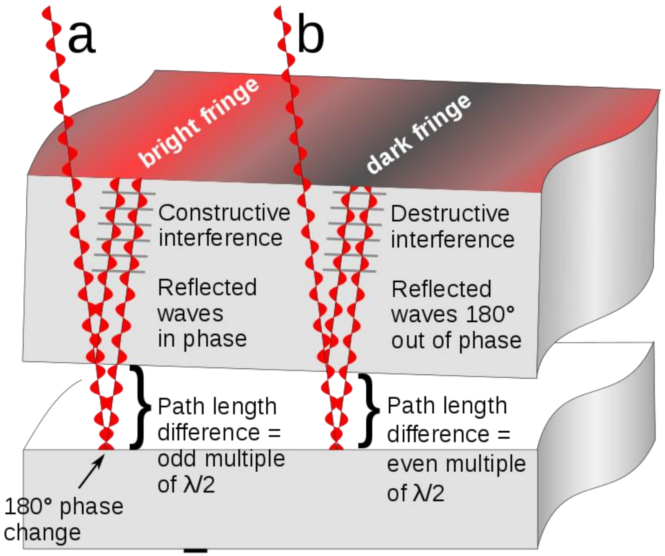
Both reflection phase shifts are 0.5 wavelength. So, only the path length difference determines the interference.

Newton´s rings





b)



https://en.wikipedia.org/wiki/Newton%27s_rings#/media/File:Optical_flat_interference.svg

Radius of the N^{th} ring is given by

$$r_N = \left[\lambda R\left(N - \frac{1}{2}\right)\right]^{\frac{1}{2}}$$

Refraction index for X-rays

$$n = 1 - \frac{N_A}{2\pi} r_0 \lambda^2 \frac{Z}{A} \rho = 1 - \delta - i\beta \quad < 1$$

$$\delta = \frac{N_A}{2\pi} r_0 \lambda^2 \sum_k \frac{\rho_k}{A_k} (f_k + f'_k) \approx 10^{-4}..10^{-5}$$

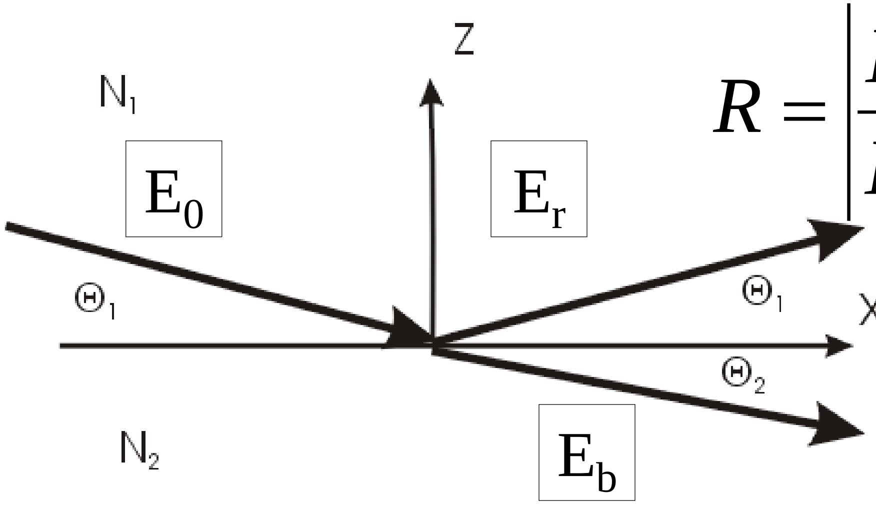
$$\beta = \frac{N_A}{2\pi} r_0 \lambda^2 \sum_k \frac{\rho_k}{A_k} f''_k \approx 10^{-6}..10^{-8}$$

Reflection and Refraction for X-rays

Using grazing angle θ

- Snellius Law

$$\frac{\cos\Theta_1}{\cos\Theta_2} = \frac{N_2}{N_1}$$



$$R = \left|\frac{E_r}{E_0}\right|^2$$

- Fresnel formulas

$$\frac{E_r}{E_0} = \frac{\sin(\Theta_1 - \Theta_2)}{\sin(\Theta_1 + \Theta_2)} \approx \frac{\Theta_1 - \Theta_2}{\Theta_1 + \Theta_2}$$

$$T = \left|\frac{E_b}{E_0}\right|^2$$

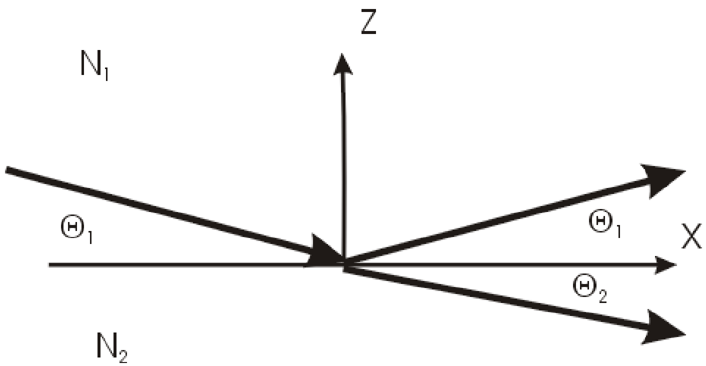
$$\frac{E_b}{E_0} = \frac{2\sin(\Theta_1)\cos(\Theta_2)}{\sin(\Theta_1 + \Theta_2)} \approx \frac{2\Theta_1}{\Theta_1 + \Theta_2}$$

Grazing incidence, varying Θ

$$\cos\Theta_1 n_1 = \cos\Theta_2 n_2$$

$$(1-\frac{1}{2}\Theta_1^2) \approx (1-\frac{1}{2}\Theta_2^2)(1-\delta)$$

$$\Theta_1^2 = \Theta_2^2 + 2\delta$$



$$\longrightarrow \Theta_{1c} = \sqrt{2\delta} \approx 0.15° \ldots 0.4°$$

critical angle

$$\Theta_2 = \left\{ \begin{matrix} i\sqrt{2\delta - \Theta_1^2} & \Theta_1 < \Theta_{1c} \\ \sqrt{\Theta_1^2 - 2\delta} & \Theta_1 > \Theta_{1c} \end{matrix} \right\}$$

Total external reflection
→Fresnel-Reflectivity

Fresnel equations: helpful approximations

$$t = \frac{2\Theta_1}{\Theta_1 + \sqrt{\Theta_1^2 - 2\delta}} \approx \frac{2\Theta_1}{\Theta_1 + \Theta_1(1 + \frac{1}{2}\frac{2\delta}{\Theta_1^2})}$$

$$t \approx \frac{2\Theta_1}{2\Theta_1 + \frac{\delta}{\Theta_1}} \approx \frac{\Theta_1}{\delta} \; for\_\Theta_1 < 2\delta$$

$$t \approx \frac{2\Theta_1}{2\Theta_1} = 1 \; for\_\Theta_1 >> 2\delta$$

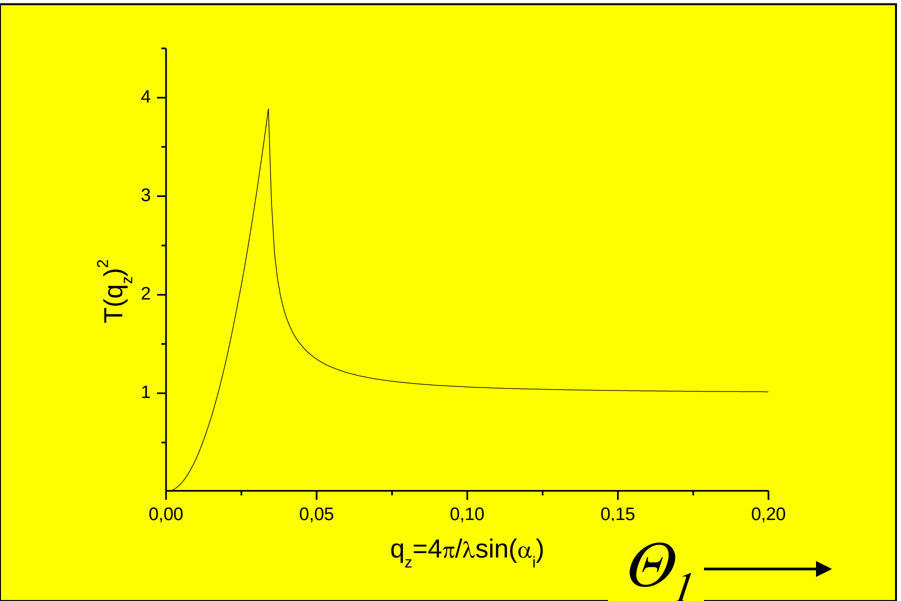
$T=t^2$

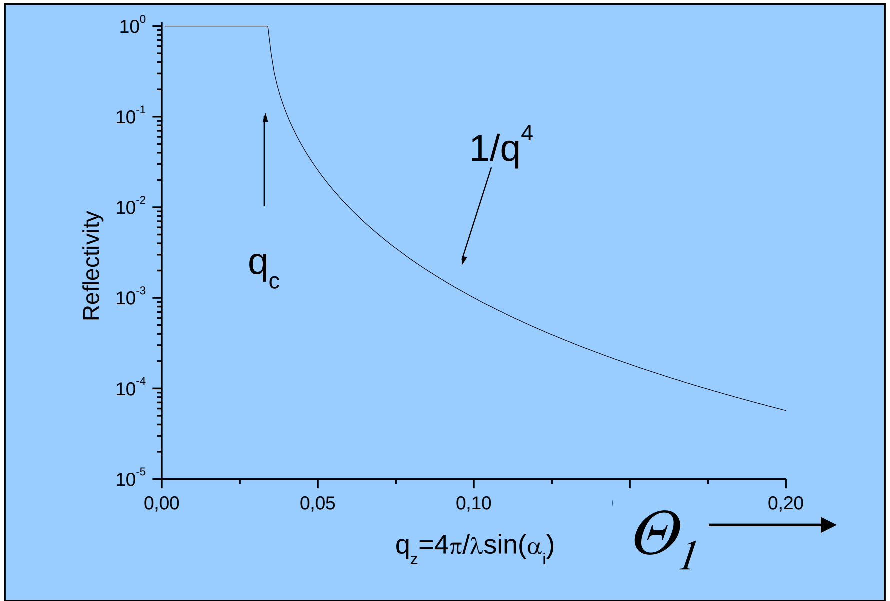
$$r = \frac{\Theta_1 - \sqrt{\Theta_1^2 - 2\delta}}{\Theta_1 + \sqrt{\Theta_1^2 - 2\delta}} \approx \frac{\Theta_1 - \Theta_1(1 + \frac{1}{2}\frac{2\delta}{\Theta_1})}{\Theta_1 + \Theta_1(1 + \frac{1}{2}\frac{2\delta}{\Theta_1^2})}$$

$$r = 1 \; for\_\Theta_1 << 2\delta$$

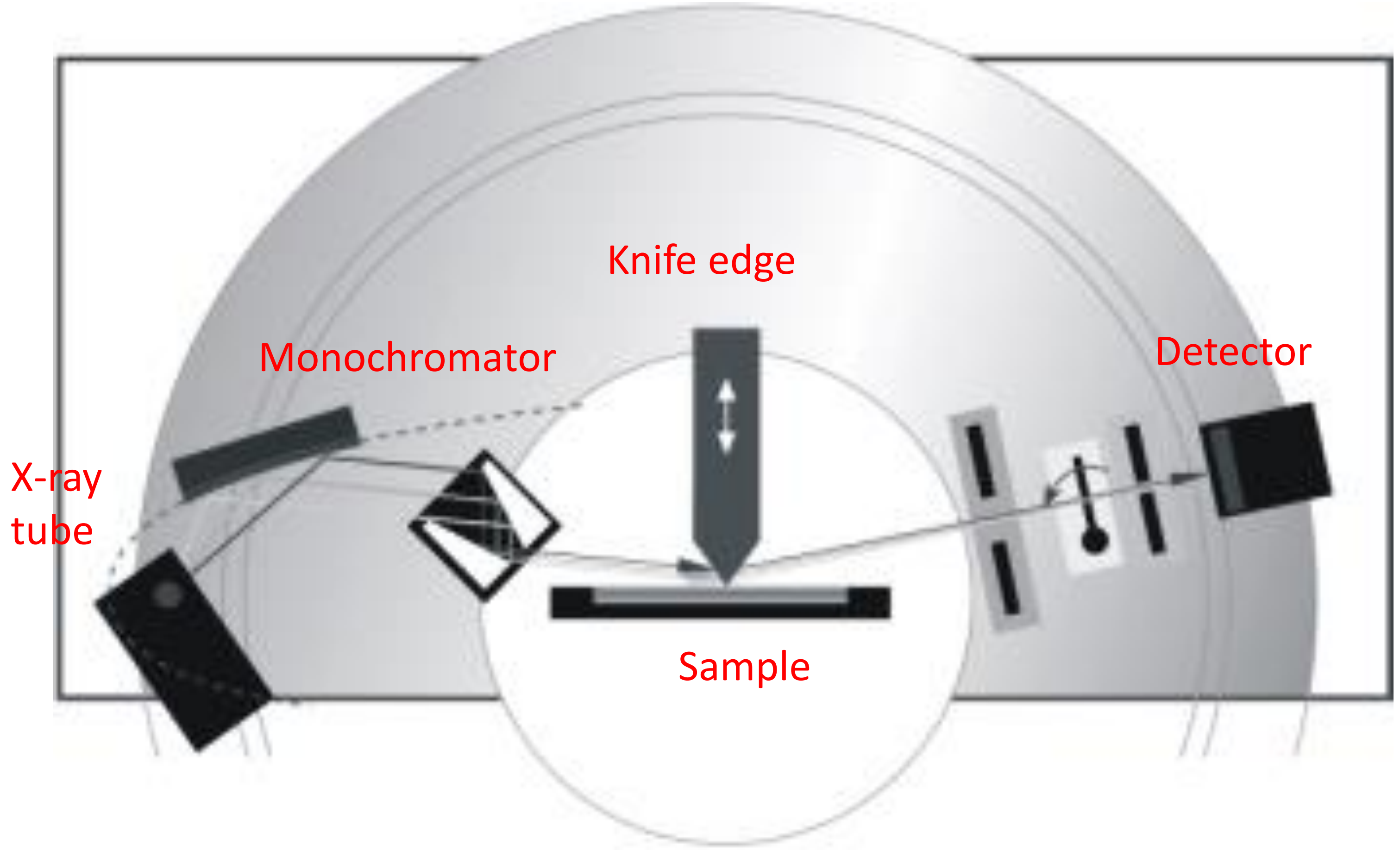
$$r = -\frac{\delta}{2\Theta_1^2} \; for\_\Theta_1 >> 2\delta$$

$R=r^2\approx\theta^{-4}$





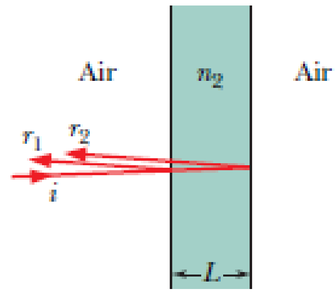
Experimental set-up

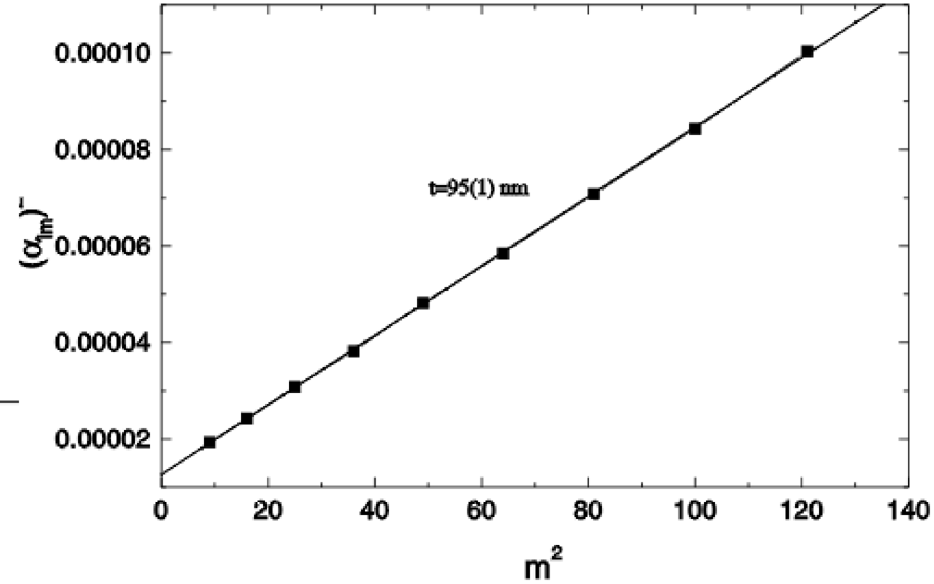


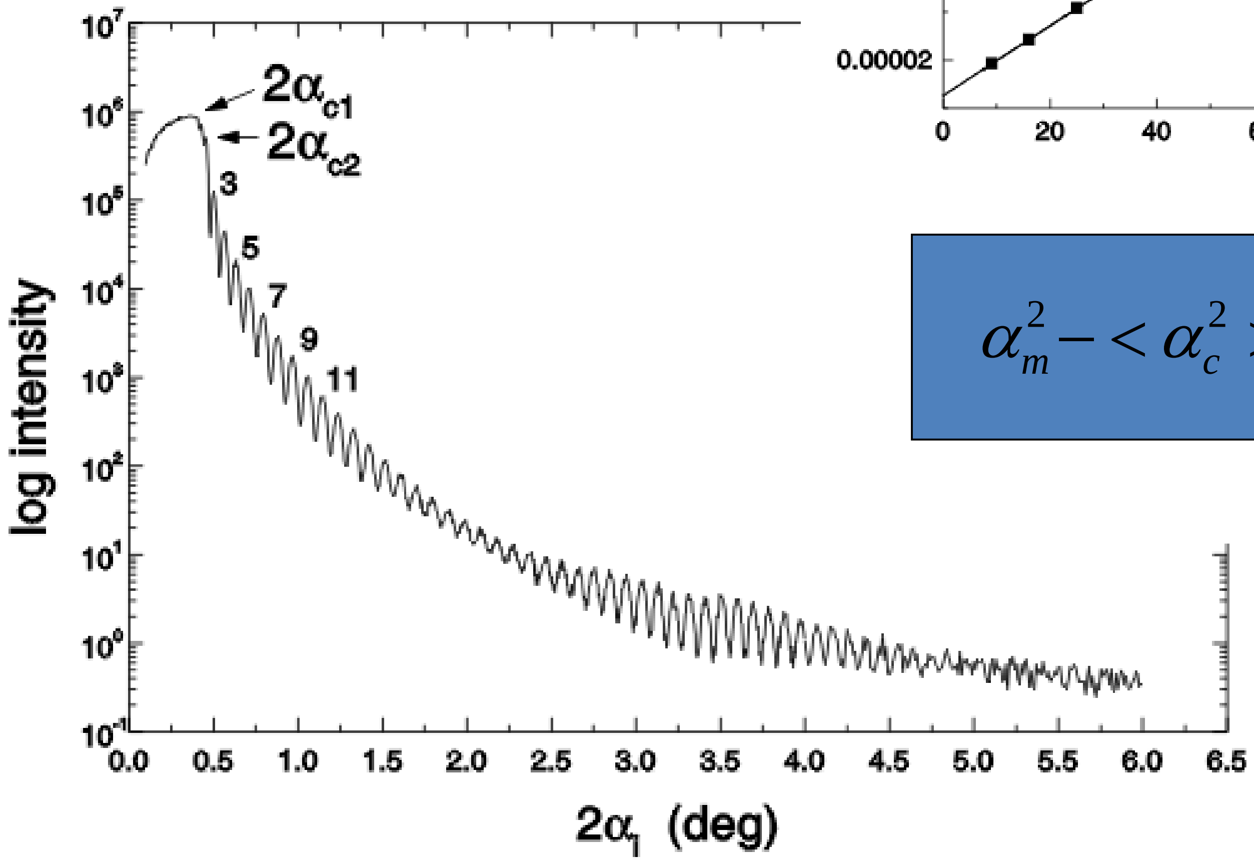
Home equipment

Layer thickness

BN film on Silicon





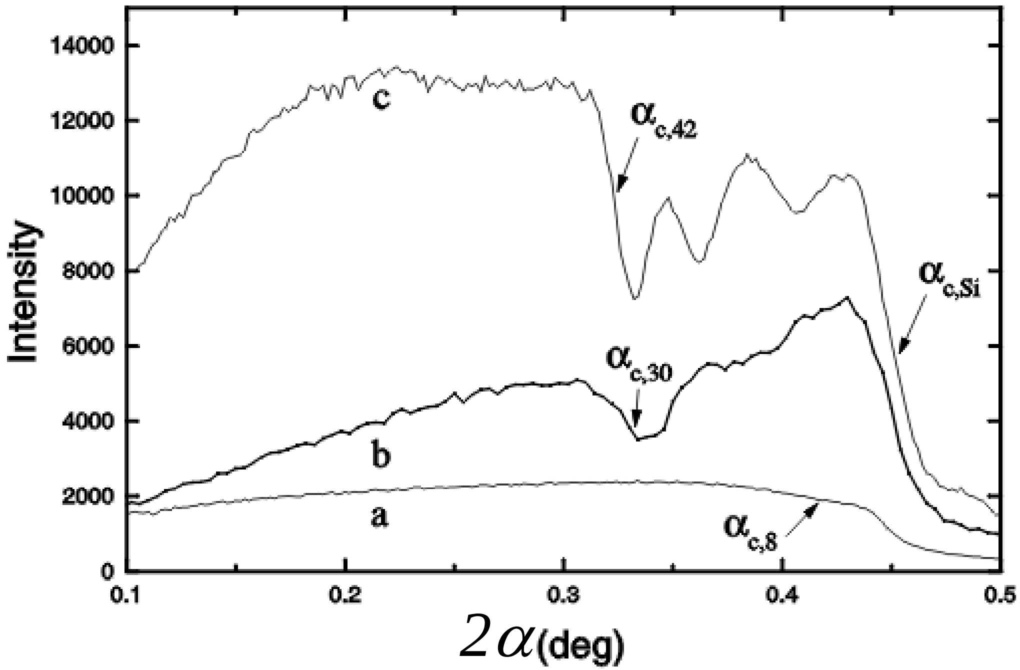


$$\alpha_m^2 - <\alpha_c^2> = m^2\left(\frac{\lambda}{2d}\right)^2$$

$$\Delta t/t = \Delta\alpha_i/\alpha_i$$

Determination of density and mass

Organic film on silicon



$$\alpha_c = \sqrt{2\delta}$$

$$2\delta = \frac{\lambda^2}{\pi} r_{el} \rho_{el}$$

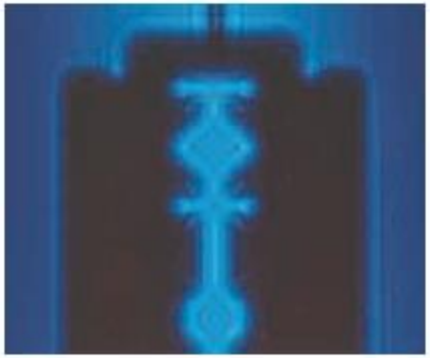
$$\rho_{el} = \alpha_c^2 \frac{\pi}{\lambda^2 r_{el}}$$

$$\rho_{mass} = \rho_{el} \frac{A}{N_A Z}$$

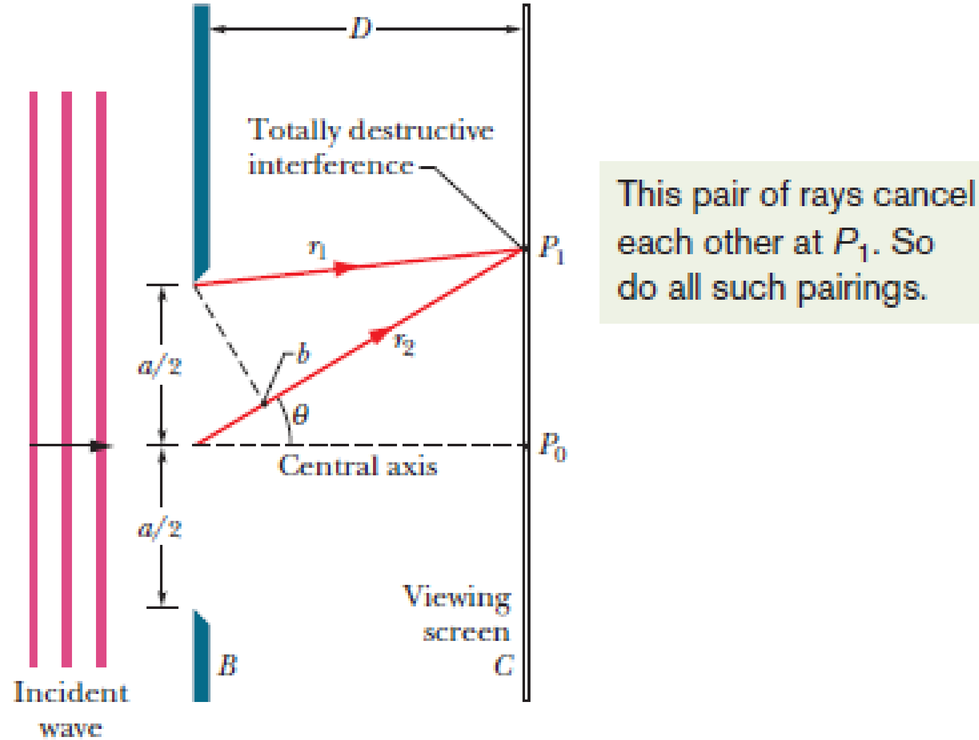
ρ_{si}=7 $10^{23}$$cm^{-3}$, ρ_m =2.32gcm^{-3}
ρ_{LB30}=4.6 $10^{23}$$cm^{-3}$, ρ_m =1.54gcm^{-3}

$\Delta\rho/\rho=2\delta\alpha_i/\alpha_c$

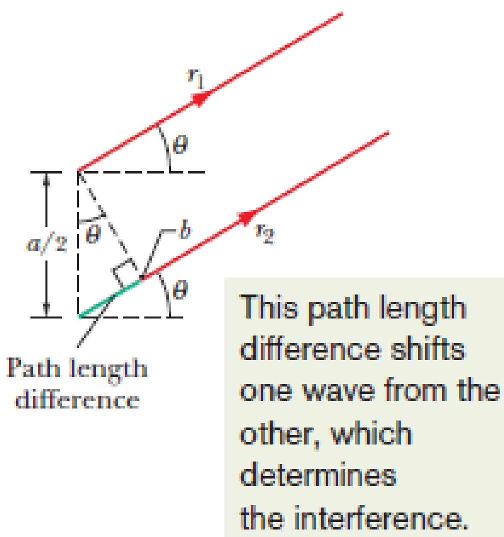
Diffraction



Diffraction at a single slit



b = a/2 sin θ



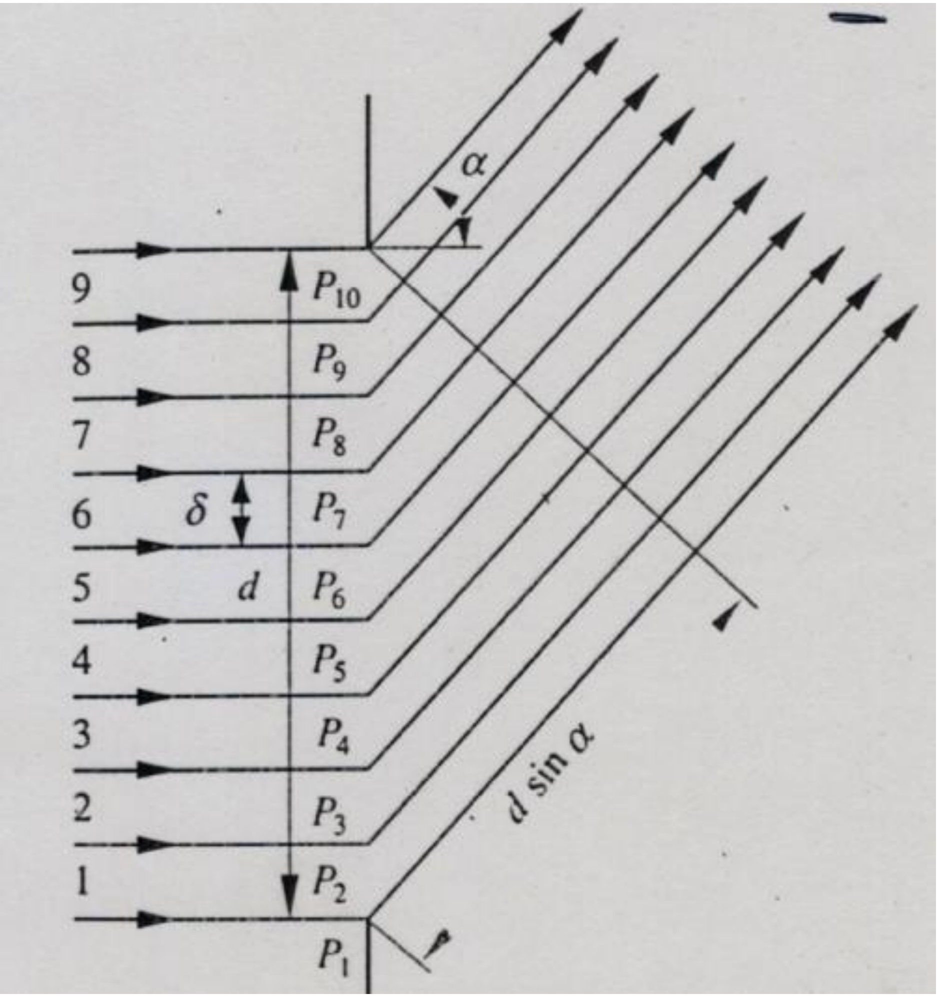
Condition for cancellation (minimum)

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$$

Generell:

$$a \sin\theta = m\,\lambda, \qquad m=1,2,3$$

Single slit interference - quantitative

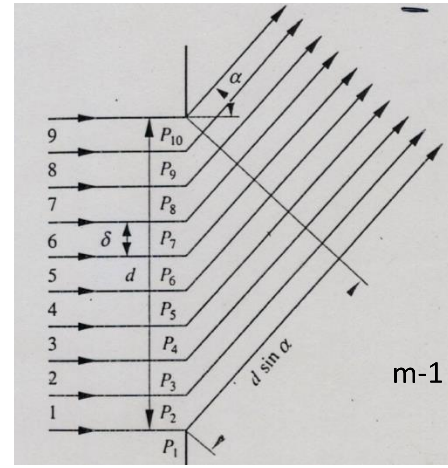


Divide slit width, d, into m-1 virtual slits

Single slit interference - quantitative

$$E = \frac{E_0 e^{i(\omega t - kr)}}{(m-1)r}[1 + e^{i\Delta\varphi} + e^{i2\Delta\varphi} + e^{i3\Delta\varphi} + \cdots + e^{i(m-1)\Delta\varphi}]$$

$$\Delta\varphi = \frac{2\pi}{\lambda} d \frac{\sin\alpha}{m-1}$$

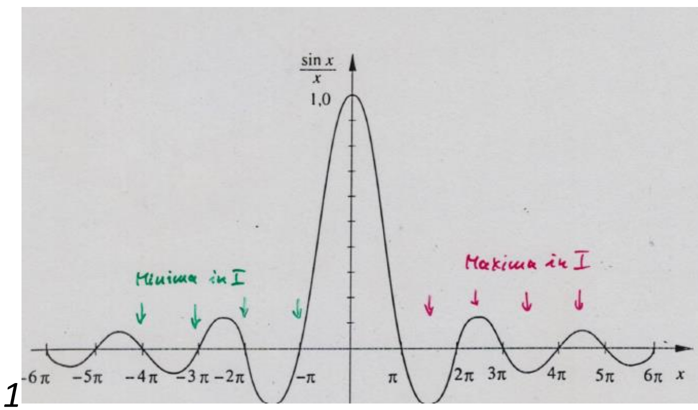


m-1 virtual slits

$$\sum_{j=0}^{m-1} e^{im_j\Delta\varphi} = \frac{e^{im\Delta\varphi} - 1}{e^{i\Delta\varphi} - 1}$$

$$\sum_{j=0}^{m-1} e^{im_j\Delta\varphi} = \frac{e^{im\Delta\varphi} - 1}{e^{i\Delta\varphi} - 1} = e^{i\frac{1}{2}\Delta\varphi(m-1)} \frac{\sin(m\frac{1}{2}\Delta\varphi)}{\sin(\frac{1}{2}\Delta\varphi)}$$

$$I = EE^{\cdot} = \frac{E_0^2}{r^2} \lim_{n\to\infty} \frac{\sin^2(m\frac{1}{2}\Delta\varphi)}{\sin^2(\frac{1}{2}\Delta\varphi)(m-1)^2} = \frac{E_0^2}{r^2} \frac{\sin^2(\frac{m}{m-1}\frac{\pi d \sin\alpha}{\lambda})}{(m-1)^2 \sin^2(\frac{1}{m-1}\frac{\pi d \sin\alpha}{\lambda})}$$



$$(m-1)^2 \sin^2(\frac{1}{m-1}\frac{\pi d \sin\alpha}{\lambda}) \approx (\frac{(\pi d \sin\alpha))^2}{(m-1)^2\lambda^2}(m-1)^2$$

and : m/(m-1)≈ 1

$$I = I_0 \frac{\sin^2(\frac{\pi d}{\lambda}\sin\alpha)}{(\frac{\pi d}{\lambda}\sin\alpha)^2}$$

$$n.minimum \longmapsto \frac{\pi d \sin\alpha}{\lambda} = \pi, 2\pi, 3\pi \qquad \sin\alpha = n\frac{\lambda}{d}$$

$$n.maximum \longmapsto \frac{\pi d \sin\alpha}{\lambda} = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi \qquad \sin\alpha = \frac{2n+1}{2}\frac{\lambda}{d}$$

Intensity of single slit diffraction

Phase difference of two interfering waves

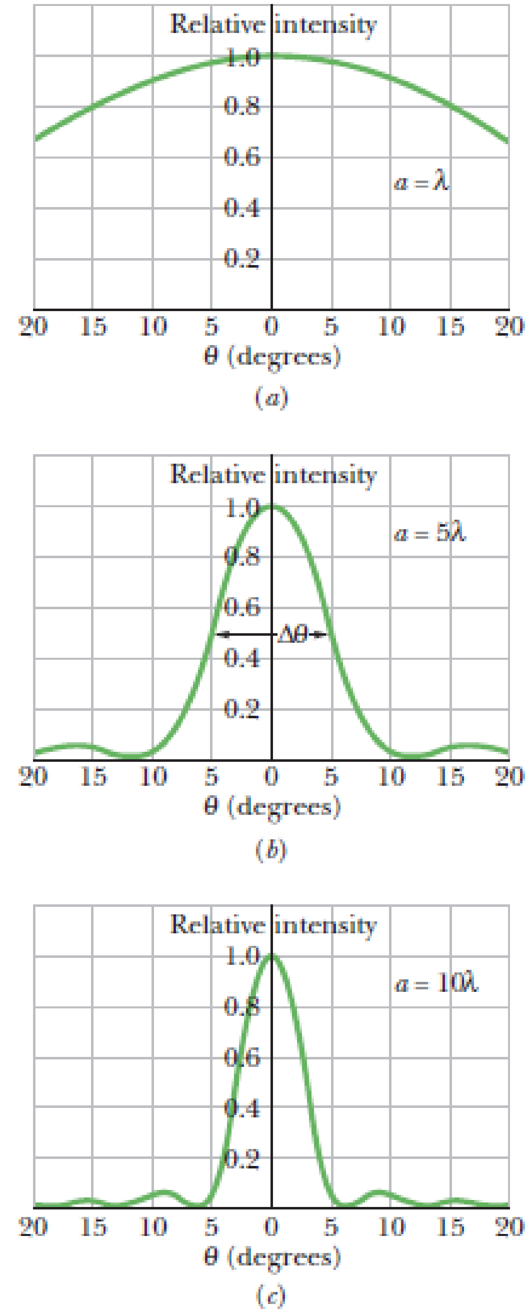
$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2,$$

$$\alpha = \tfrac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta.$$

Minima at $\alpha = m\pi, \qquad \text{for } m = 1, 2, 3, \ldots.$

$$m\pi = \frac{\pi a}{\lambda} \sin \theta,$$

1.Min at $\sin \theta = \lambda/a$ = 90° for $\lambda/a = 1$
= 11.5 for $\lambda/a = 0.2$
= 5.7 for $\lambda/a = 0.1$



Intensities of the maxima in a single-slit interference pattern

Find the intensities of the first three secondary maxima (side maxima) in the single-slit diffraction pattern of Fig. 36-1, measured as a percentage of the intensity of the central maximum.

KEY IDEAS

The secondary maxima lie approximately halfway between the minima, whose angular locations are given by Eq. 36-7 ($\alpha = m\pi$). The locations of the secondary maxima are then given (approximately) by

$$a = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 1, 2, 3, \ldots,$$

with α in radian measure. We can relate the intensity I at any point in the diffraction pattern to the intensity I_m of the central maximum via Eq. 36-5.

Calculations: Substituting the approximate values of α for the secondary maxima into Eq. 36-5 to obtain the relative intensities at those maxima, we get

$$\frac{I}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin(m + \frac{1}{2})\pi}{(m + \frac{1}{2})\pi}\right)^2, \quad \text{for } m = 1, 2, 3, \ldots.$$

The first of the secondary maxima occurs for $m = 1$, and its relative intensity is

$$\frac{I_1}{I_m} = \left(\frac{\sin(1 + \frac{1}{2})\pi}{(1 + \frac{1}{2})\pi}\right)^2 = \left(\frac{\sin 1.5\pi}{1.5\pi}\right)^2$$

$$= 4.50 \times 10^{-2} \approx 4.5\%. \qquad \text{(Answer)}$$

For $m = 2$ and $m = 3$ we find that

$$\frac{I_2}{I_m} = 1.6\% \quad \text{and} \quad \frac{I_3}{I_m} = 0.83\%. \qquad \text{(Answer)}$$

As you can see from these results, successive secondary maxima decrease rapidly in intensity. Figure 36-1 was deliberately overexposed to reveal them.

Single-slit diffraction pattern with white light

A slit of width a is illuminated by white light.

(a) For what value of a will the first minimum for red light of wavelength $\lambda = 650$ nm appear at $\theta = 15°$?

KEY IDEA

Diffraction occurs separately for each wavelength in the range of wavelengths passing through the slit, with the locations of the minima for each wavelength given by Eq. 36-3 ($a \sin \theta = m\lambda$).

Calculation: When we set $m = 1$ (for the first minimum) and substitute the given values of θ and λ, Eq. 36-3 yields

$$a = \frac{m\lambda}{\sin \theta} = \frac{(1)(650 \text{ nm})}{\sin 15°} = 2511 \text{ nm} \approx 2.5\ \mu\text{m}. \quad \text{(Answer)}$$

For the incident light to flare out that much ($\pm 15°$ to the first minima) the slit has to be very fine indeed—in this case, a mere four times the wavelength. For comparison, note that a fine human hair may be about 100 μm in diameter.

(b) What is the wavelength λ' of the light whose first side diffraction maximum is at 15°, thus coinciding with the first minimum for the red light?

KEY IDEA

The first side maximum for any wavelength is about halfway between the first and second minima for that wavelength.

Calculations: Those first and second minima can be located with Eq. 36-3 by setting $m = 1$ and $m = 2$, respectively. Thus, the first side maximum can be located *approximately* by setting $m = 1.5$. Then Eq. 36-3 becomes

$$a \sin \theta = 1.5\lambda'.$$

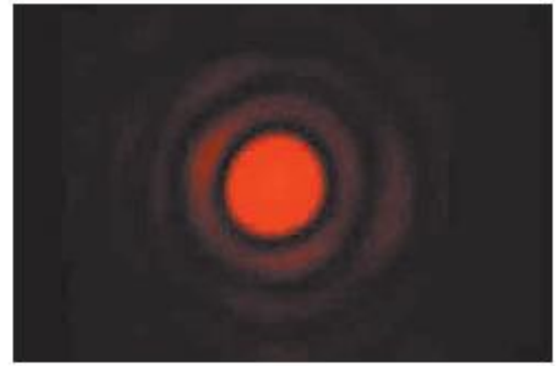
Solving for λ' and substituting known data yield

$$\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15°)}{1.5} = 430 \text{ nm}. \quad \text{(Answer)}$$

Light of this wavelength is violet (far blue, near the short-wavelength limit of the human range of visible light). From the two equations we used, can you see that the first side maximum for light of wavelength 430 nm will always coincide with the first minimum for light of wavelength 650 nm, no matter what the slit width is? However, the angle θ at which this overlap occurs does depend on slit width. If the slit is relatively narrow, the angle will be relatively large, and conversely.

Diffraction at circular aperture

$$\sin\theta = 1.22\,\frac{\lambda}{d} \qquad \text{(first minimum—circular aperture).}$$



compare

$$\sin\theta = \frac{\lambda}{a} \qquad \text{(first minimum—single slit),}$$

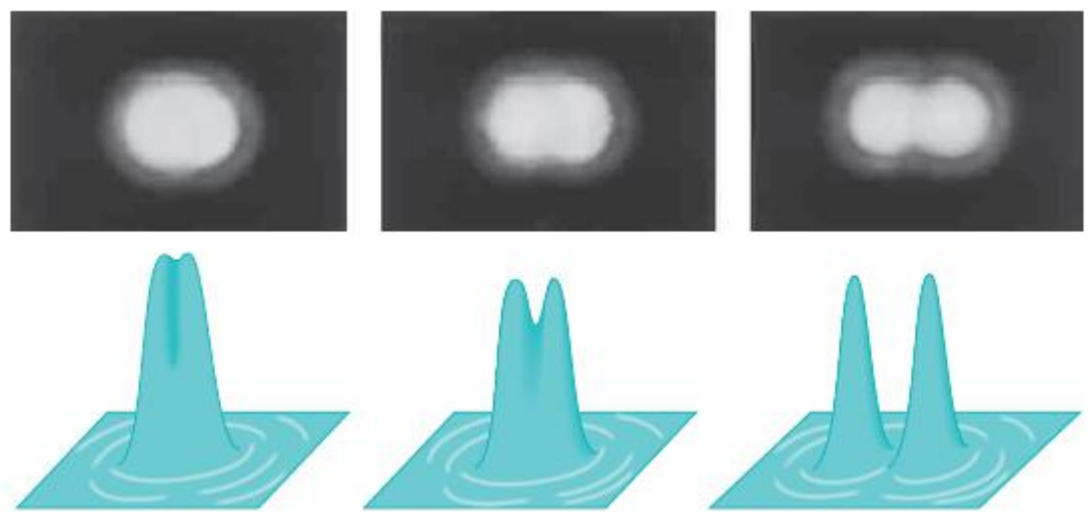
Resolvability of two neighbored apertures

Requested angular separation

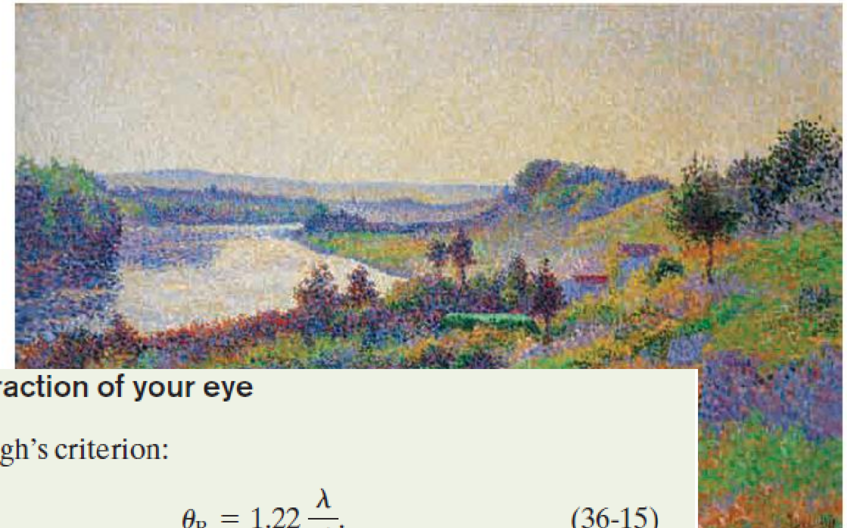
$$\theta_R = \sin^{-1}\frac{1.22\lambda}{d}.$$

Small angles

$$\theta_R = 1.22\,\frac{\lambda}{d} \qquad \text{(Rayleigh's criterion).}$$



(a) (b) (c)



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France. Photo

Pointillistic paintings use the diffraction of your eye

Figure 36-13*a* is a representation of the colored dots on a pointillistic painting. Assume that the average center-to-center separation of the dots is $D = 2.0$ mm. Also assume that the diameter of the pupil of your eye is $d = 1.5$ mm and that the least angular separation between dots you can resolve is set only by Rayleigh's criterion. What is the least viewing distance from which you cannot distinguish any dots on the painting?

KEY IDEA

Consider any two adjacent dots that you can distinguish when you are close to the painting. As you move away, you continue to distinguish the dots until their angular separation θ (in your view) has decreased to the angle given by Rayleigh's criterion:

$$\theta_R = 1.22\frac{\lambda}{d}. \qquad (36\text{-}15)$$

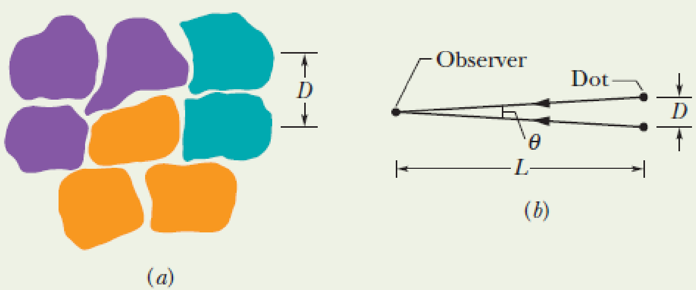


Fig. 36-13 (*a*) Representation of some dots on a pointillistic painting, showing an average center-to-center separation D. (*b*) The arrangement of separation D between two dots, their angular separation θ, and the viewing distance L.

Calculations: Figure 36-13*b* shows, from the side, the angular separation θ of the dots, their center-to-center separation D, and your distance L from them. Because D/L is small, angle θ is also small and we can make the approximation

$$\theta = \frac{D}{L}. \qquad (36\text{-}16)$$

Setting θ of Eq. 36-16 equal to θ_R of Eq. 36-15 and solving for L, we then have

$$L = \frac{Dd}{1.22\lambda}. \qquad (36\text{-}17)$$

Equation 36-17 tells us that L is larger for smaller λ. Thus, as you move away from the painting, adjacent red dots (long wavelengths) become indistinguishable before adjacent blue dots do. To find the least distance L at which *no* colored dots are distinguishable, we substitute $\lambda = 400$ nm (blue or violet light) into Eq. 36-17:

$$L = \frac{(2.0 \times 10^{-3}\text{ m})(1.5 \times 10^{-3}\text{ m})}{(1.22)(400 \times 10^{-9}\text{ m})} = 6.1\text{ m.} \quad \text{(Answer)}$$

At this or a greater distance, the color you perceive at any given spot on the painting is a blended color that may not actually exist there.

Rayleigh's criterion for resolving two distant objects

A circular converging lens, with diameter $d = 32$ mm and focal length $f = 24$ cm, forms images of distant point objects in the focal plane of the lens. The wavelength is $\lambda = 550$ nm.

(a) Considering diffraction by the lens, what angular separation must two distant point objects have to satisfy Rayleigh's criterion?

KEY IDEA

Figure 36-14 shows two distant point objects P_1 and P_2, the lens, and a viewing screen in the focal plane of the lens. It also shows, on the right, plots of light intensity I versus position on the screen for the central maxima of the images formed by the lens. Note that the angular separation θ_o of the objects equals the angular separation θ_i of the images. Thus, if the images are to satisfy Rayleigh's criterion for resolvability, the angular separations on both sides of the lens must be given by Eq. 36-14 (assuming small angles).

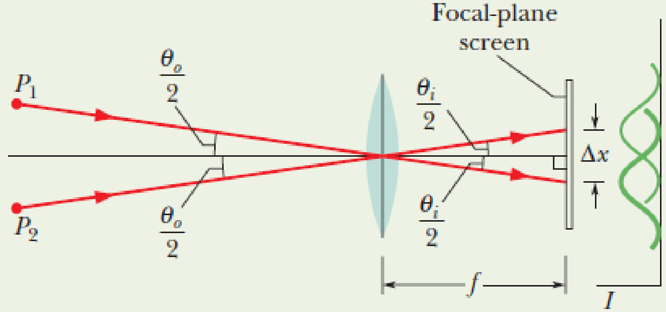


Fig. 36-14 Light from two distant point objects P_1 and P_2 passes through a converging lens and forms images on a viewing screen in the focal plane of the lens. Only one representative ray from each object is shown. The images are not points but diffraction patterns, with intensities approximately as plotted at the right. The angular separation of the objects is θ_o and that of the images is θ_i; the central maxima of the images have a separation Δx.

Calculations: From Eq. 36-14, we obtain

$$\theta_o = \theta_i = \theta_R = 1.22\frac{\lambda}{d}$$

$$= \frac{(1.22)(550 \times 10^{-9}\text{ m})}{32 \times 10^{-3}\text{ m}} = 2.1 \times 10^{-5}\text{ rad.} \quad \text{(Answer)}$$

At this angular separation, each central maximum in the two intensity curves of Fig. 36-14 is centered on the first minimum of the other curve.

(b) What is the separation Δx of the centers of the *images* in the focal plane? (That is, what is the separation of the *central* peaks in the two intensity-versus-position curves?)

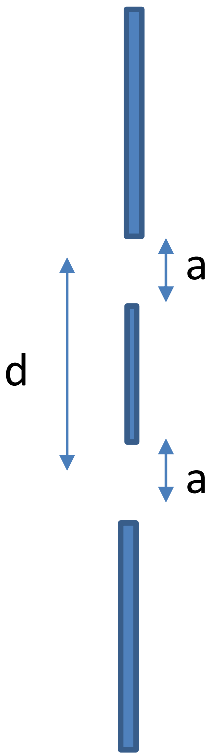
Calculations: From either triangle between the lens and the screen in Fig. 36-14, we see that $\tan \theta_i/2 = \Delta x/2f$. Rearranging this equation and making the approximation $\tan\theta \approx \theta$, we find

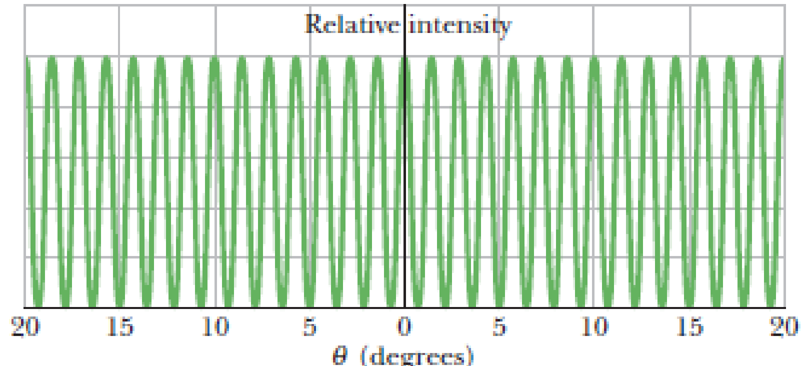
$$\Delta x = f\theta_i, \tag{36-18}$$

where θ_i is in radian measure. Substituting known data then yields

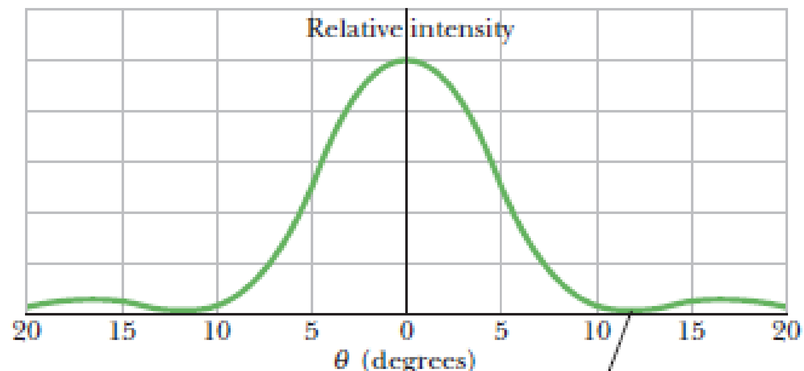
$$\Delta x = (0.24\text{ m})(2.1 \times 10^{-5}\text{ rad}) = 5.0\ \mu\text{m.} \quad \text{(Answer)}$$

Double slit experiment (again)

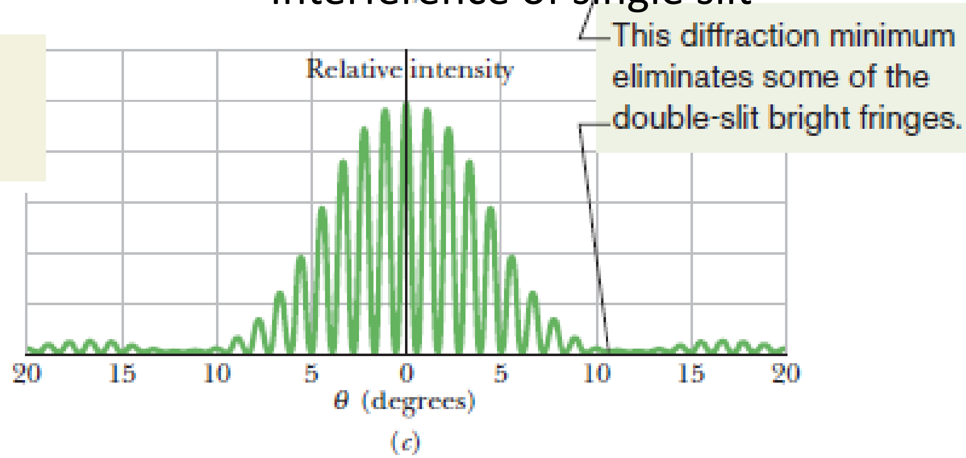




Interference for zero slit width



Interference of single slit



$$I(\theta) = I_m(\cos^2 \beta)\left(\frac{\sin \alpha}{\alpha}\right)^2 \qquad \text{(double slit),}$$

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

$$\alpha = \frac{\pi a}{\lambda} \sin \theta.$$

Double-slit experiment with diffraction of each slit included

In a double-slit experiment, the wavelength λ of the light source is 405 nm, the slit separation d is 19.44 μm, and the slit width a is 4.050 μm. Consider the interference of the light from the two slits and also the diffraction of the light through each slit.

(a) How many bright interference fringes are within the central peak of the diffraction envelope?

KEY IDEAS

We first analyze the two basic mechanisms responsible for the optical pattern produced in the experiment:

1. *Single-slit diffraction:* The limits of the central peak are the first minima in the diffraction pattern due to either slit individually. (See Fig. 36-15.) The angular locations of those minima are given by Eq. 36-3 ($a \sin \theta = m\lambda$). Here let us rewrite this equation as $a \sin \theta = m_1\lambda$, with the subscript 1 referring to the one-slit diffraction. For the first minima in the diffraction pattern, we substitute $m_1 = 1$, obtaining
$$a \sin \theta = \lambda. \tag{36-22}$$
2. *Double-slit interference:* The angular locations of the bright fringes of the double-slit interference pattern are given by Eq. 35-14, which we can write as
$$d \sin \theta = m_2\lambda, \qquad \text{for } m_2 = 0, 1, 2, \ldots. \tag{36-23}$$
Here the subscript 2 refers to the double-slit interference.

Calculations: We can locate the first diffraction minimum within the double-slit fringe pattern by dividing Eq. 36-23 by Eq. 36-22 and solving for m_2. By doing so and then substituting the given data, we obtain

$$m_2 = \frac{d}{a} = \frac{19.44\ \mu\text{m}}{4.050\ \mu\text{m}} = 4.8.$$

This tells us that the bright interference fringe for $m_2 = 4$ fits into the central peak of the one-slit diffraction pattern, but the fringe for $m_2 = 5$ does not fit. Within the central diffraction peak we have the central bright fringe ($m_2 = 0$), and four bright fringes (up to $m_2 = 4$) on each side of it. Thus, a total of nine bright fringes of the double-slit interference pattern are within the central peak of the diffraction envelope.

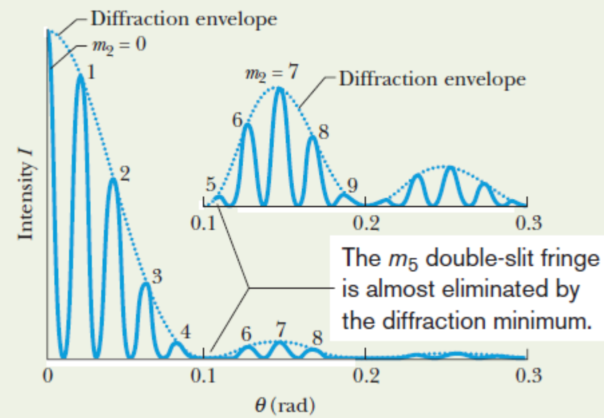


Fig. 36-17 One side of the intensity plot for a two-slit interference experiment. The inset shows (vertically expanded) the plot within the first and second side peaks of the diffraction envelope.

The bright fringes to one side of the central bright fringe are shown in Fig. 36-17.

(b) How many bright fringes are within either of the first side peaks of the diffraction envelope?

KEY IDEA

The outer limits of the first side diffraction peaks are the second diffraction minima, each of which is at the angle θ given by $a \sin \theta = m_1\lambda$ with $m_1 = 2$:

$$a \sin \theta = 2\lambda. \tag{36-24}$$

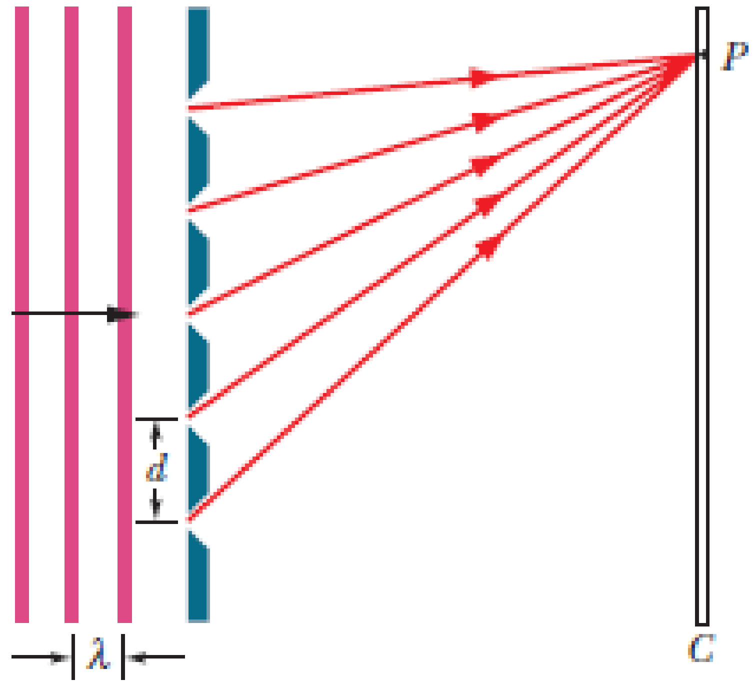
Calculation: Dividing Eq. 36-23 by Eq. 36-24, we find

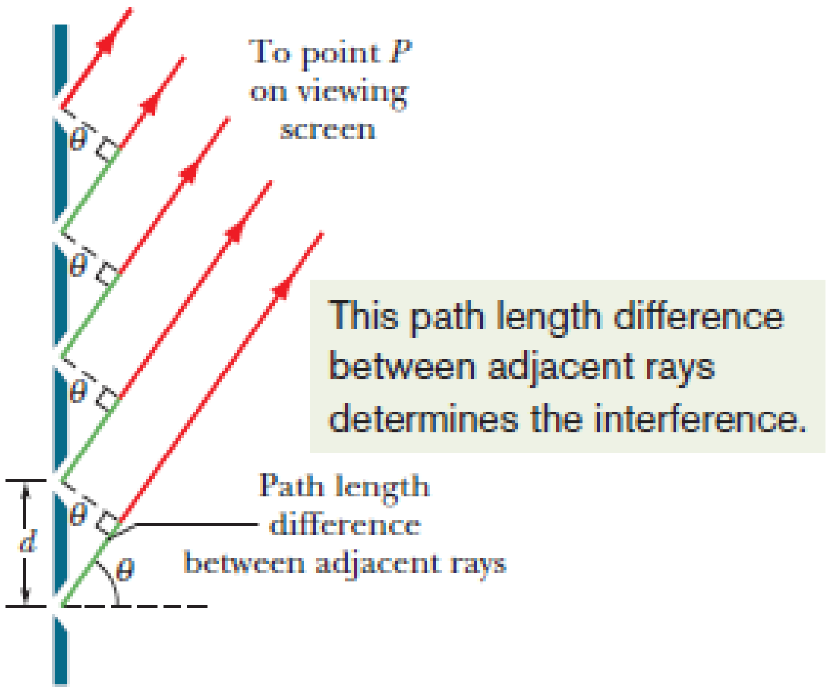
$$m_2 = \frac{2d}{a} = \frac{(2)(19.44\ \mu\text{m})}{4.050\ \mu\text{m}} = 9.6.$$

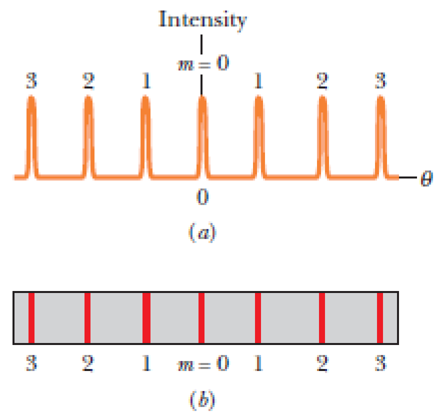
This tells us that the second diffraction minimum occurs just before the bright interference fringe for $m_2 = 10$ in Eq. 36-23. Within either first side diffraction peak we have the fringes from $m_2 = 5$ to $m_2 = 9$, for a total of five bright fringes of the double-slit interference pattern (shown in the inset of Fig. 36-17). However, if the $m_2 = 5$ bright fringe, which is almost eliminated by the first diffraction minimum, is considered too dim to count, then only four bright fringes are in the first side diffraction peak.

Diffraction gratings

$d \sin \theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad$ (maxima—lines),







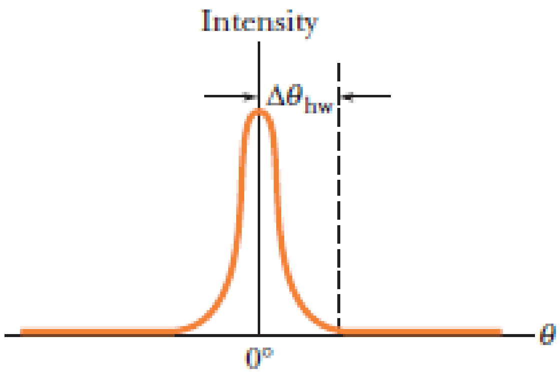
Width of the interference lines

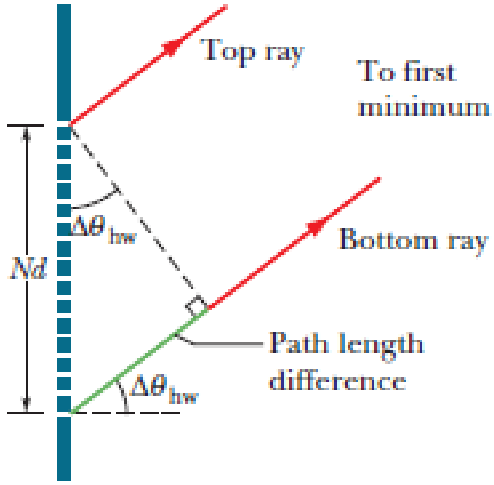
First minimum occurs, if N is number of slits

$$Nd \sin \Delta\theta_{hw} = \lambda.$$

$$\Delta\theta_{hw} = \frac{\lambda}{Nd} \qquad \text{(half-width of central line).}$$

$$\Delta\theta_{hw} = \frac{\lambda}{Nd \cos \theta} \qquad \text{(half-width of line at } \theta\text{).}$$





Grating spectrometer

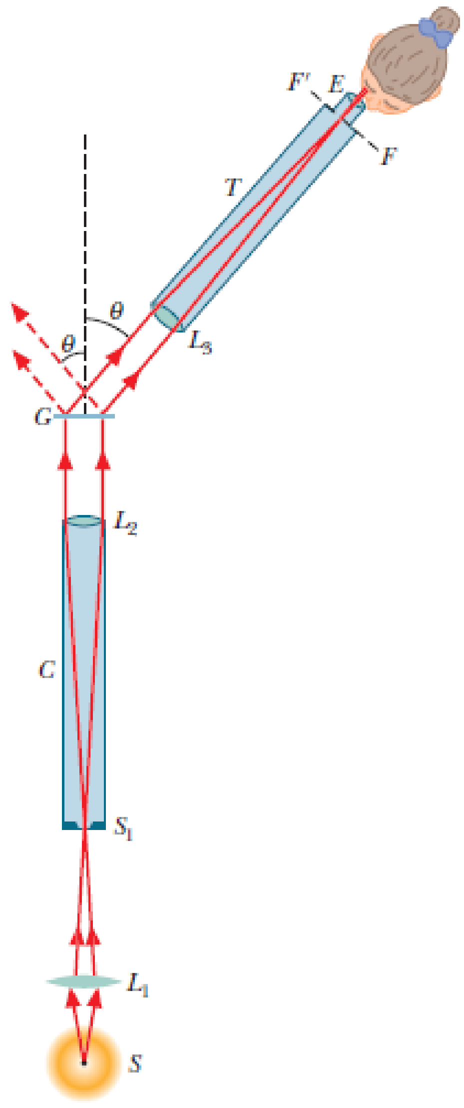
$$D = \frac{\Delta\theta}{\Delta\lambda} \quad \text{(dispersion defined).}$$

$$D = \frac{m}{d\cos\theta} \quad \text{(dispersion of a grating).}$$

$$R = \frac{\lambda_{avg}}{\Delta\lambda} \quad \text{(resolving power defined).}$$

$$R = Nm \quad \text{(resolving power of a grating).}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{Nm}$$



Dispersion and resolving power of a diffraction grating

A diffraction grating has 1.26×10^4 rulings uniformly spaced over width $w = 25.4$ mm. It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced emission lines (known as the sodium doublet) of wavelengths 589.00 nm and 589.59 nm.

(a) At what angle does the first-order maximum occur (on either side of the center of the diffraction pattern) for the wavelength of 589.00 nm?

KEY IDEA

The maxima produced by the diffraction grating can be determined with Eq. 36-25 ($d \sin\theta = m\lambda$).

Calculations: The grating spacing d is

$$d = \frac{w}{N} = \frac{25.4 \times 10^{-3}\text{ m}}{1.26 \times 10^4}$$
$$= 2.016 \times 10^{-6}\text{ m} = 2016\text{ nm}.$$

The first-order maximum corresponds to $m = 1$. Substituting these values for d and m into Eq. 36-25 leads to

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(589.00\text{ nm})}{2016\text{ nm}}$$
$$= 16.99° \approx 17.0°. \qquad \text{(Answer)}$$

(b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.

KEY IDEAS

(1) The angular separation $\Delta\theta$ between the two lines in the first order depends on their wavelength difference $\Delta\lambda$ and the dispersion D of the grating, according to Eq. 36-29 ($D = \Delta\theta/\Delta\lambda$). (2) The dispersion D depends on the angle θ at which it is to be evaluated.

Calculations: We can assume that, in the first order, the two sodium lines occur close enough to each other for us to evaluate D at the angle $\theta = 16.99°$ we found in part (a) for one of those lines. Then Eq. 36-30 gives the dispersion as

$$D = \frac{m}{d\cos\theta} = \frac{1}{(2016\text{ nm})(\cos 16.99°)}$$
$$= 5.187 \times 10^{-4}\text{ rad/nm}.$$

From Eq. 36-29 and with $\Delta\lambda$ in nanometers, we then have

$$\Delta\theta = D\,\Delta\lambda = (5.187 \times 10^{-4}\text{ rad/nm})(589.59 - 589.00)$$
$$= 3.06 \times 10^{-4}\text{ rad} = 0.0175°. \qquad \text{(Answer)}$$

You can show that this result depends on the grating spacing d but not on the number of rulings there are in the grating.

(c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?

KEY IDEAS

(1) The resolving power of a grating in any order m is physically set by the number of rulings N in the grating according to Eq. 36-32 ($R = Nm$). (2) The smallest wavelength difference $\Delta\lambda$ that can be resolved depends on the average wavelength involved and on the resolving power R of the grating, according to Eq. 36-31 ($R = \lambda_{avg}/\Delta\lambda$).

Calculation: For the sodium doublet to be barely resolved, $\Delta\lambda$ must be their wavelength separation of 0.59 nm, and λ_{avg} must be their average wavelength of 589.30 nm. Thus, we find that the smallest number of rulings for a grating to resolve the sodium doublet is

$$N = \frac{R}{m} = \frac{\lambda_{avg}}{m\,\Delta\lambda}$$
$$= \frac{589.30\text{ nm}}{(1)(0.59\text{ nm})} = 999\text{ rulings}. \qquad \text{(Answer)}$$

Solid state physics

UNIVERSITÄT SIEGEN
FESTKÖRPERPHYSIK

Lecture 2: X-ray diffraction

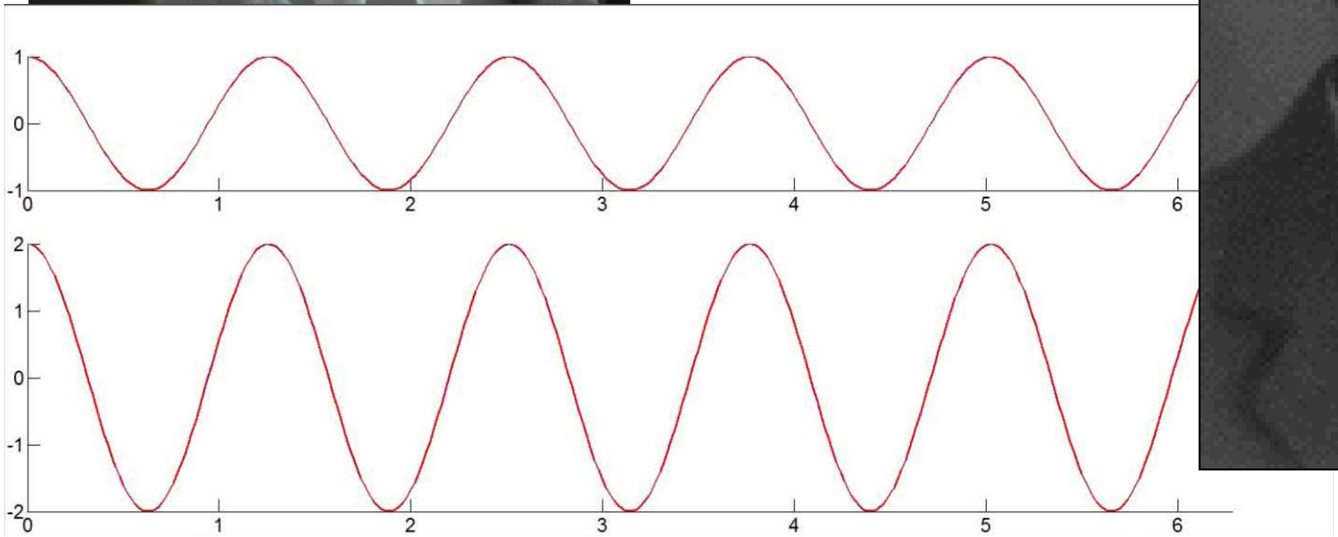
Prof. Dr. U. Pietsch

The mean aim of Max von Laue (1912)

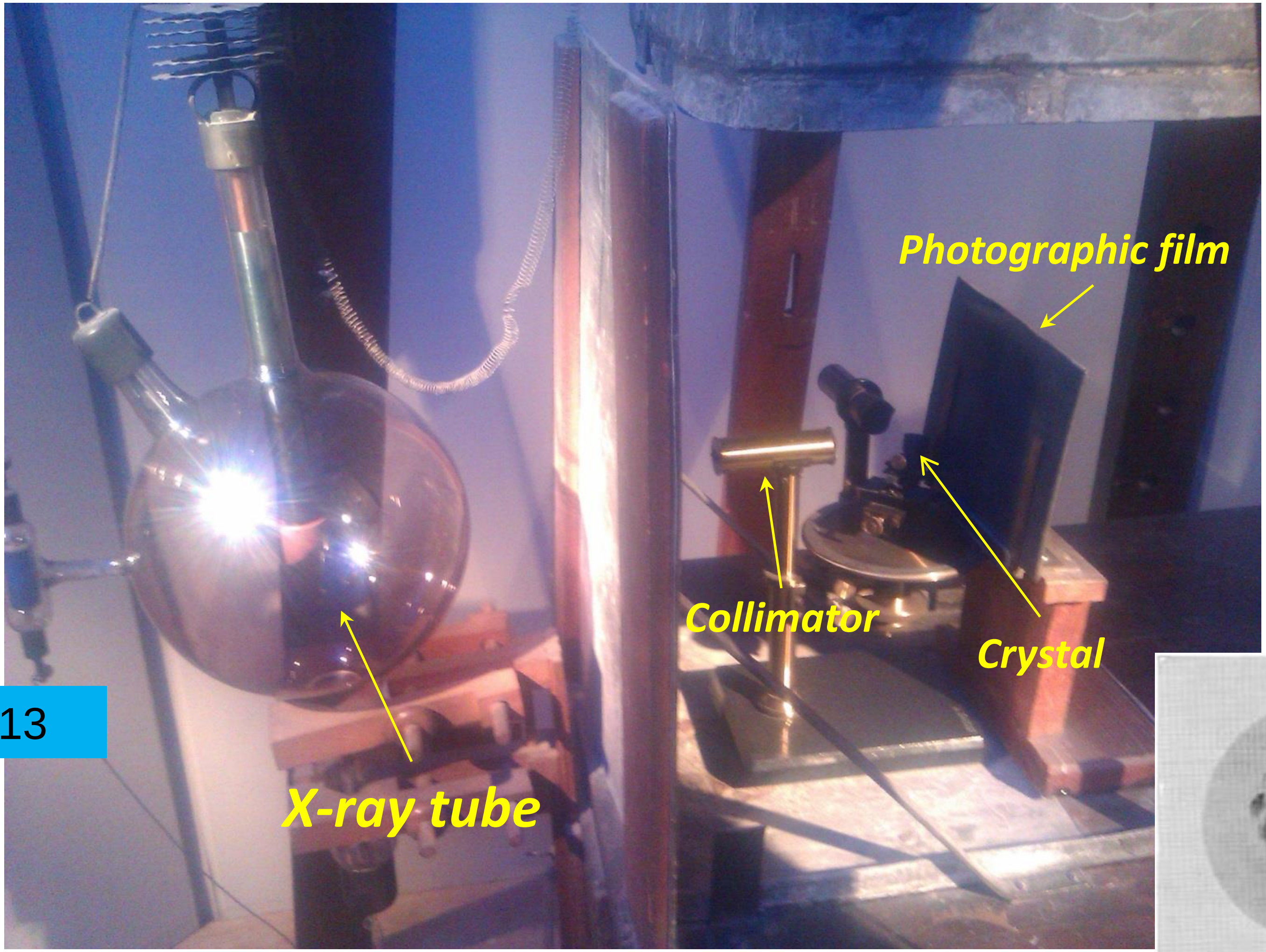
X-rays are electromagnetic wave with wave length much smaller than wave length of visible light. X-rays are diffracted a crystal lattice

1914

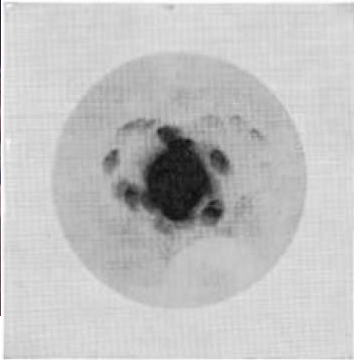
Nobelpreis für Physik



Original experiment von Laue, Friedrich und Knipping

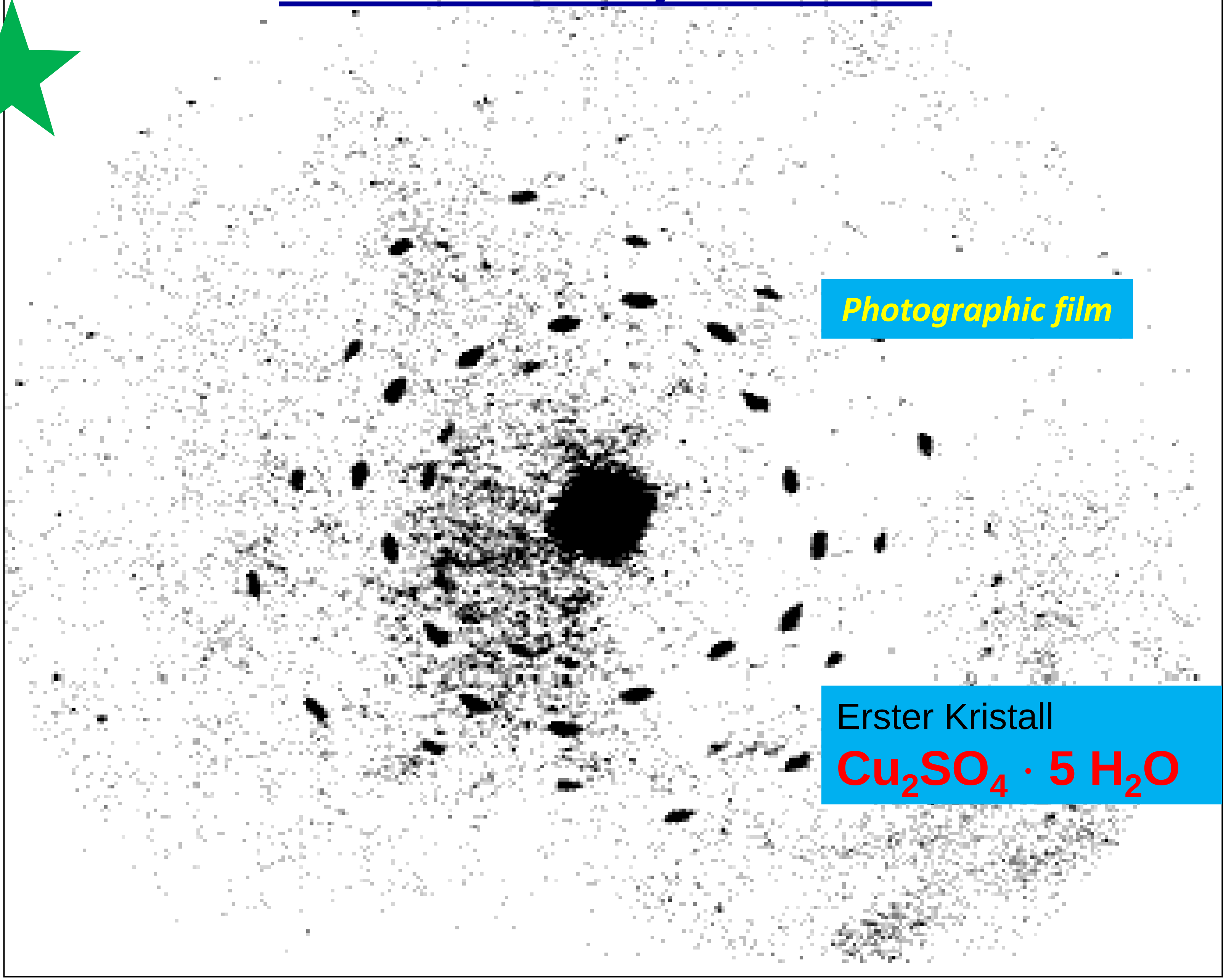


20.April 1913



Displayed at *Deutsche Museum* in Munich

First Laue Experiment



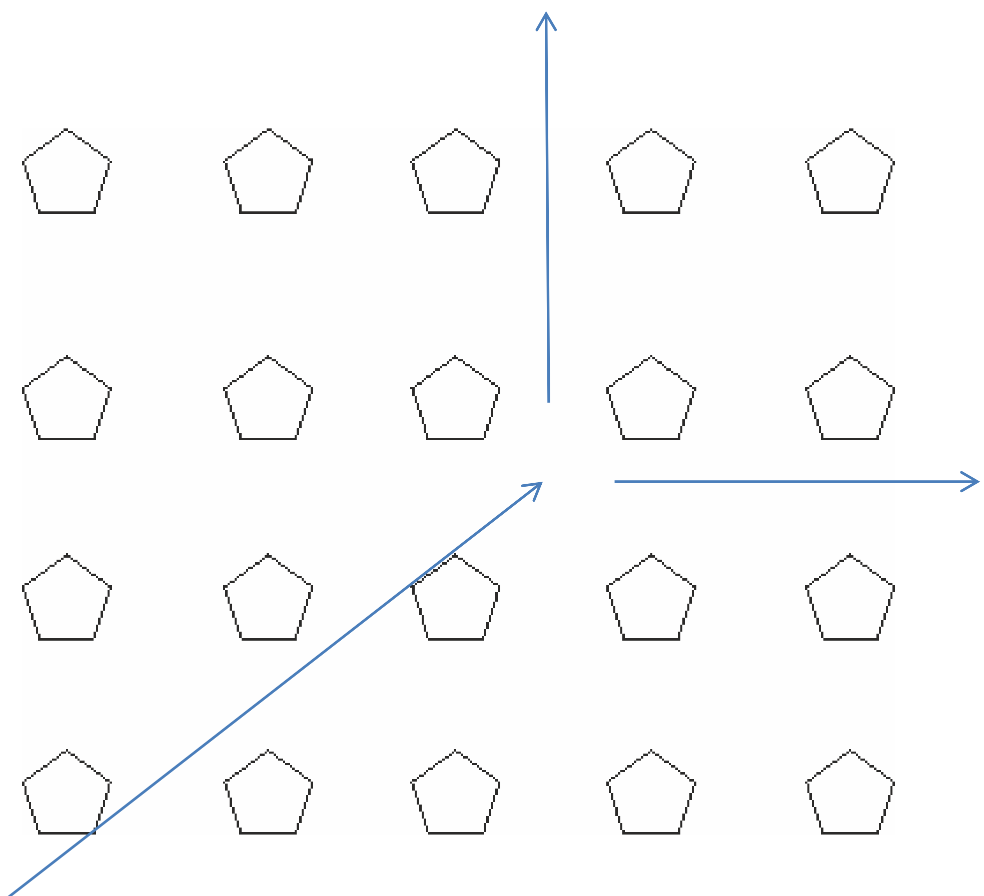
1912: Begin of modern Crystallography

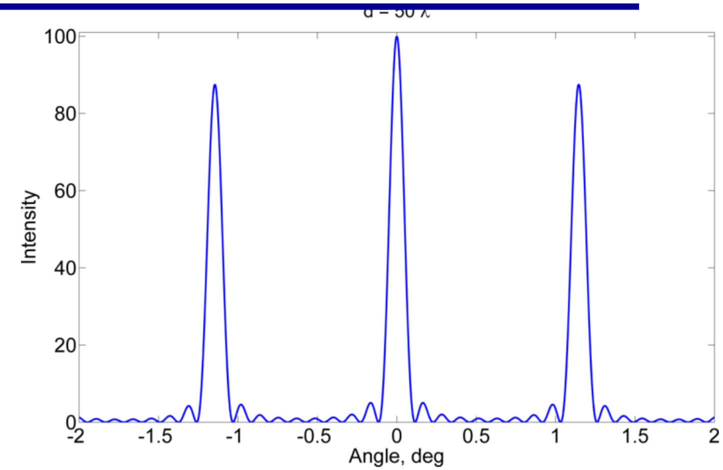
X-rays are electromagnetic waves of very short wavelength (~ 1 Å = 10^{-10} m).

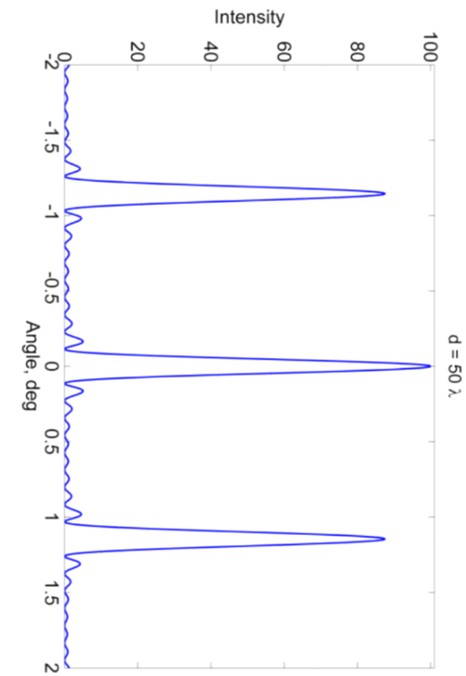
Crystals are periodic structures in 3D : interatomic distances are of similar order of magnitude as x-ray wave length

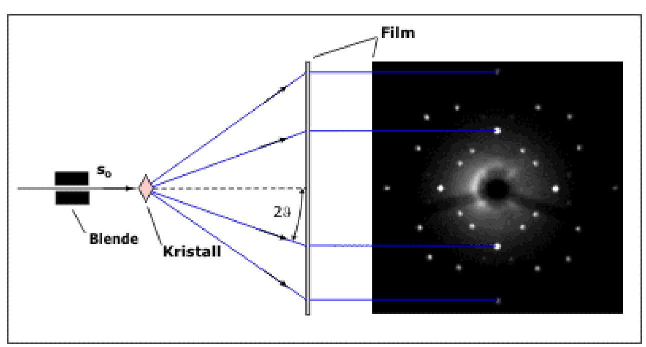
X-ray diffraction is a method to determine the geometric structure of solids !

Explaination by interference at 3D lattice





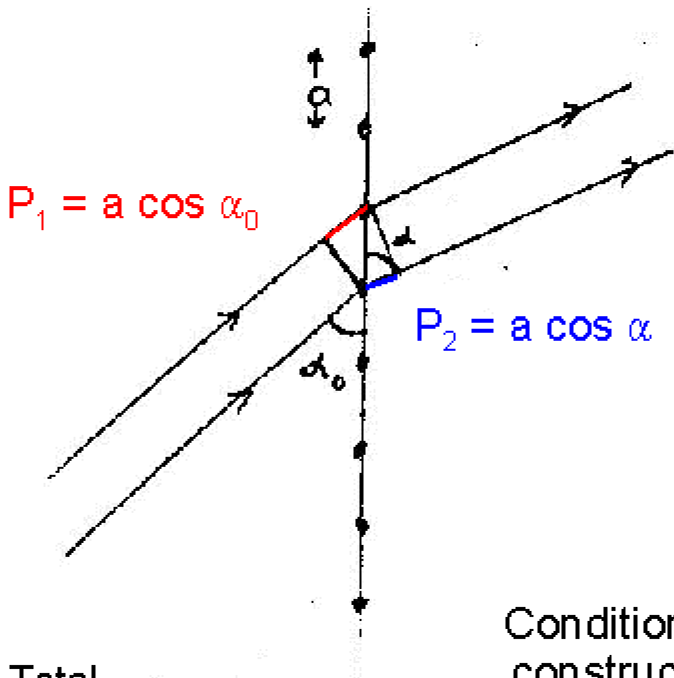




Explanation of Laue pattern

von Laue Equation

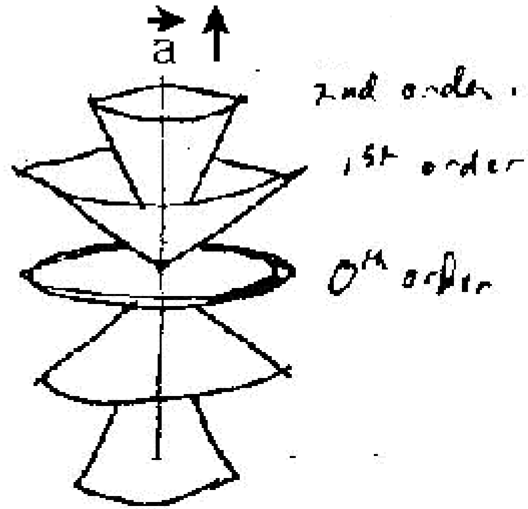
Scattering from a line of atoms along "a"



Total
Path difference

Condition for constructive interference

$$P_2 - P_1 = a\,(\cos \alpha - \cos \alpha_0) = h\,\lambda$$

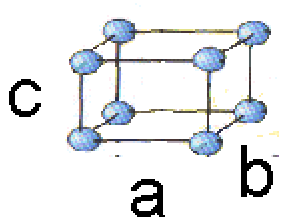


Constructive interference along conical surfaces about a-axis

von Laue Equation

For a crystal with cell parameters

$\vec{a}\ ,\ \vec{b}\ ,\ \vec{c}$



We have three von Laue equations

$$a\ (\cos\alpha - \cos\alpha_0) = h\ \lambda$$
$$b\ (\cos\beta - \cos\beta_0) = k\ \lambda \qquad ①$$
$$c\ (\cos\gamma - \cos\gamma_0) = l\ \lambda$$

Where $\cos\alpha_0$, $\cos\beta_0$, $\cos\gamma_0$ are the direction cosines of the incident ray and $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of the reflected ray in the crystal axis.

So, we must also satisfy

$$\cos^2\alpha_0 + \cos^2\beta_0 + \cos^2\gamma_0 = 1$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad ②$$

Using that the angle between the incident and reflected ray is $2\ \theta$

$$③ \qquad \cos 2\theta = \cos\alpha\ \cos\alpha_0 + \cos\beta\ \cos\beta_0 + \cos\gamma\ \cos\gamma_0$$

von Laue Equation

We square the von Laue Equations

① ^{2}

$$\frac{h^2\lambda^2}{a^2} = \cos^2\alpha - 2\cos\alpha\cos\alpha_0 + \cos^2\alpha_0$$

$$\frac{k^2\lambda^2}{b^2} = \cos^2\beta - 2\cos\beta\cos\beta_0 + \cos^2\beta_0$$

$$\frac{l^2\lambda^2}{c^2} = \cos^2\gamma - 2\cos\gamma\cos\gamma_0 + \cos^2\gamma_0$$

$$\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)\lambda^2 = 1 - 2\left(\cos\alpha\cos\alpha_0 + \cos\beta\cos\beta_0 + \cos\gamma\cos\gamma_0\right) + 1$$

Using eq. ③

$$\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)\lambda^2 = 2(1 - \cos 2\theta)$$

Trig. Identity

$$\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)\lambda^2 = 4\sin^2\theta$$

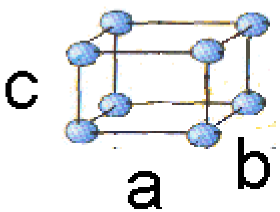
General Bragg law

$$\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right)^{1/2}\lambda = 2\sin\theta$$

Reciprocal Lattice

For a crystal with cell parameters

$\vec{a}, \vec{b}, \vec{c}$



A vector connecting 2 lattice points in reciprocal space can be written

Reciprocal lattice is constructed using vectors

$\vec{a}^*, \vec{b}^*, \vec{c}^*$

$\vec{a}^*$ is normal to the bc plane and $\left|\vec{a}^*\right| = \frac{2\pi}{a}$

$\vec{b}^*$ is normal to the ac plane and $\left|\vec{b}^*\right| = \frac{2\pi}{b}$

$\vec{c}^*$ is normal to the ab plane and $\left|\vec{c}^*\right| = \frac{2\pi}{c}$

recip. lattice vector $\vec{G} = h\vec{a}^* + k\vec{b}^* + l\vec{c}^*$

$$\left|\vec{G}\right| = \left[\left(h\vec{a}^*\right)^2 + \left(k\vec{b}^*\right)^2 + \left(l\vec{c}^*\right)^2\right]^{1/2} = 2\pi\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right)^{1/2}$$

General Bragg law

$$2\pi\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right)^{1/2}=\frac{4\pi}{\lambda}\sin\theta$$

$\vec{G}$

are the reciprocal vectors of the reciprocal lattice

$$|\vec{G}|=\frac{4\pi}{\lambda}\sin\theta$$

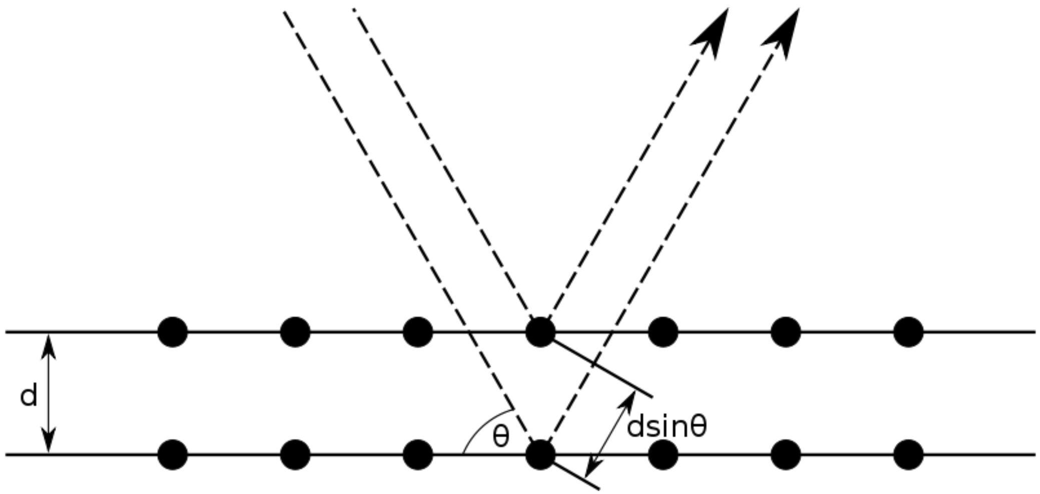
$a = b = c$

$$2\pi\frac{\sqrt{h^2+k^2+l^2}}{a}=\frac{4\pi}{\lambda}\sin\theta$$

$$\lambda = 2\frac{a}{\sqrt{h^2+k^2+l^2}}\sin\theta$$

$$\lambda = 2d\sin\theta$$

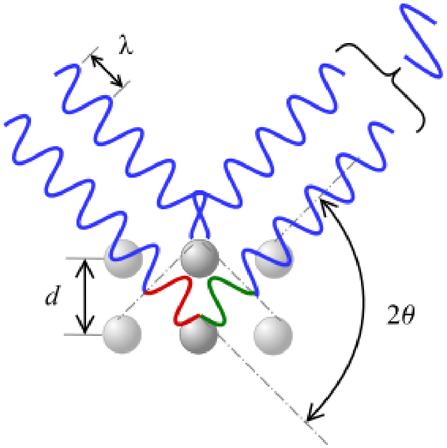
Alternative description of Laue-pattern by W.H.Bragg und W.L.Bragg

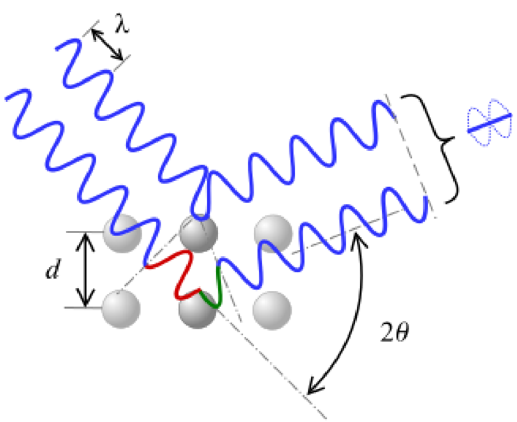


Interference at dense backed „lattice planes“

Bragg equation

$$N\lambda = 2d\sin\Theta$$



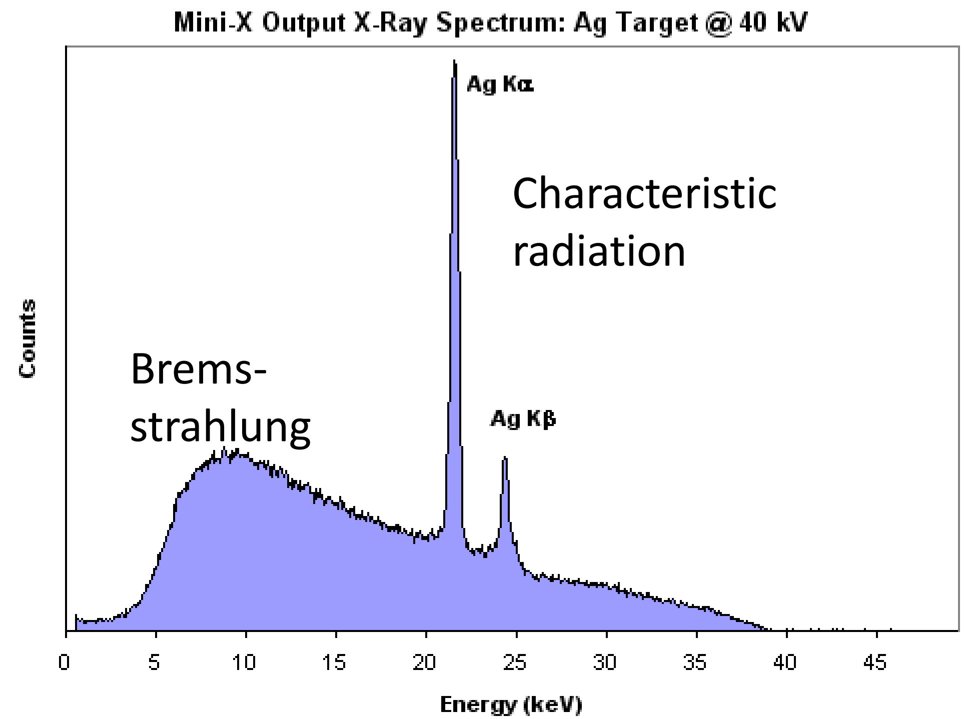


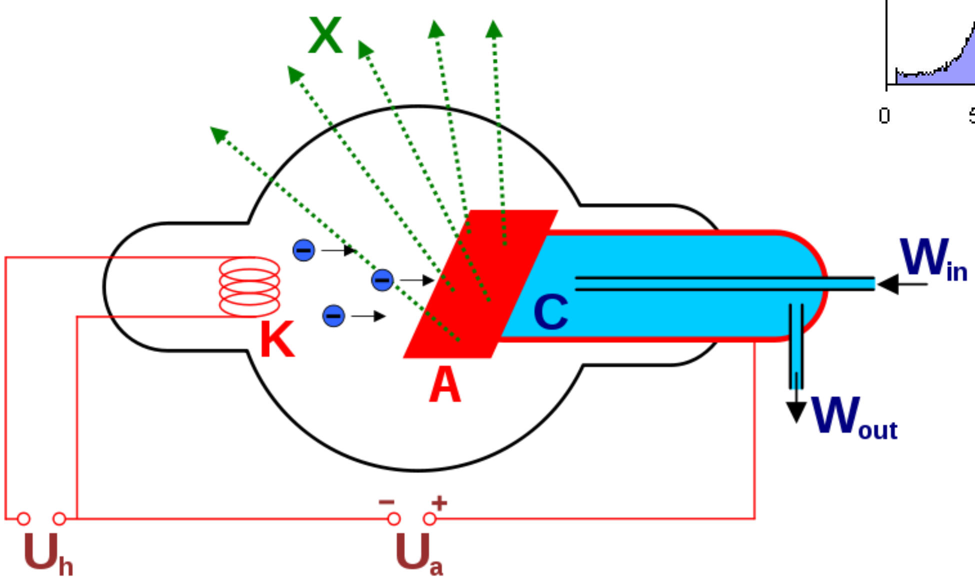
X-ray tube and tube spectrum

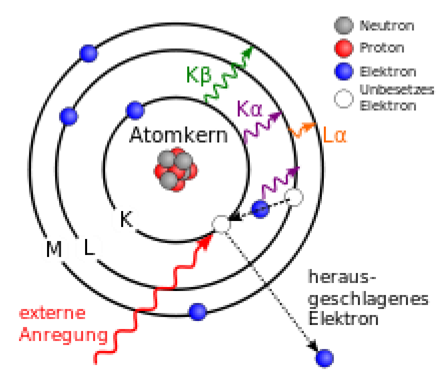
$E = h\nu = eU$

$\lambda_{min} = hc / eU_a$

$\lambda_{min} = 12.4/U_a$



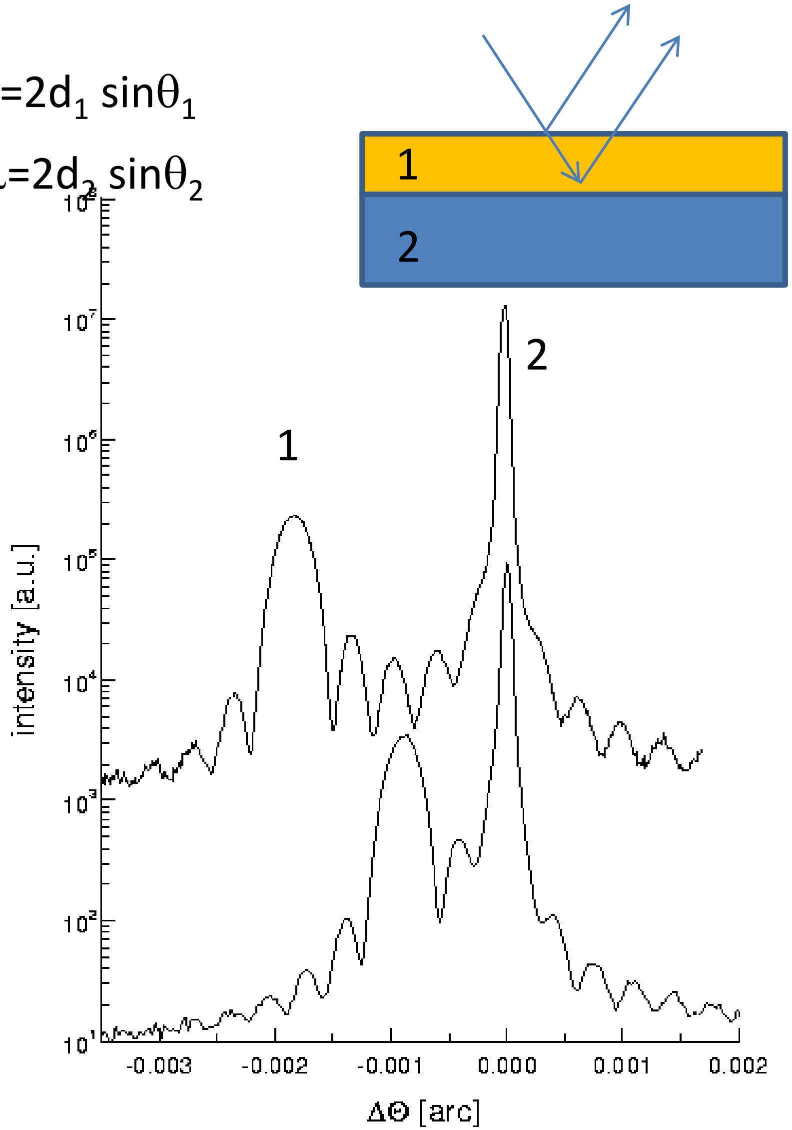


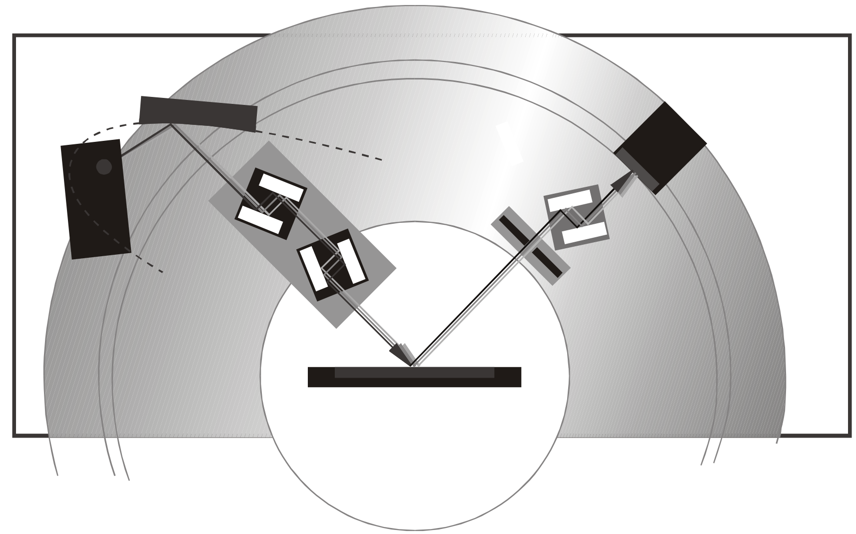


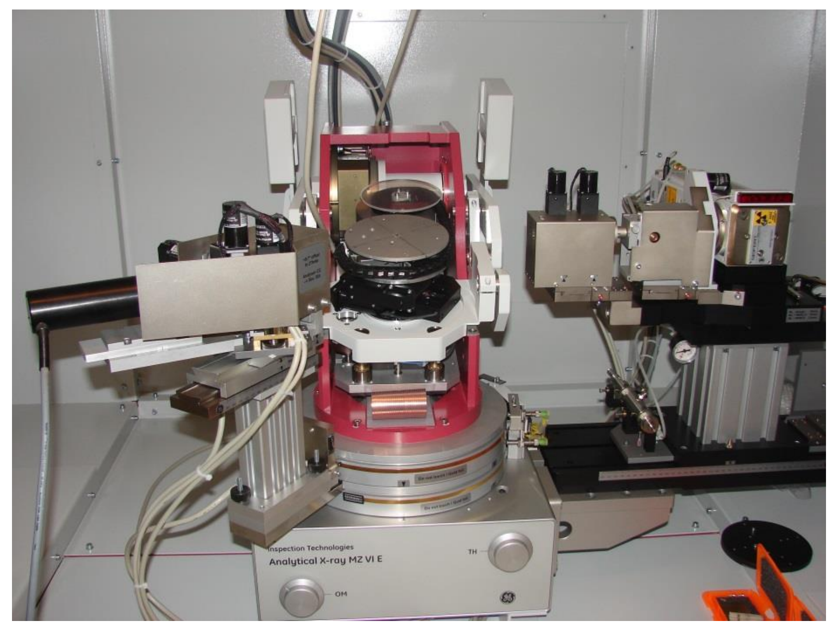
Measuring lattice parameters

$\lambda = 2d_1 \sin\theta_1$

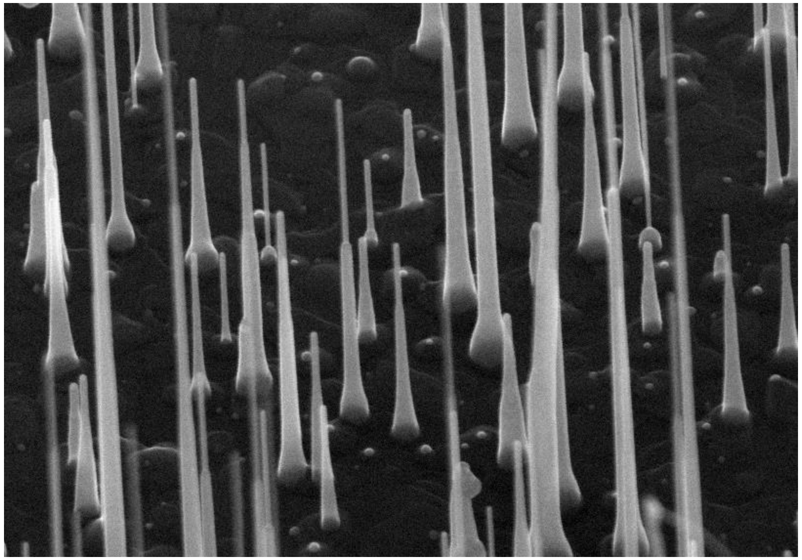
$\lambda = 2d_2 \sin\theta_2$

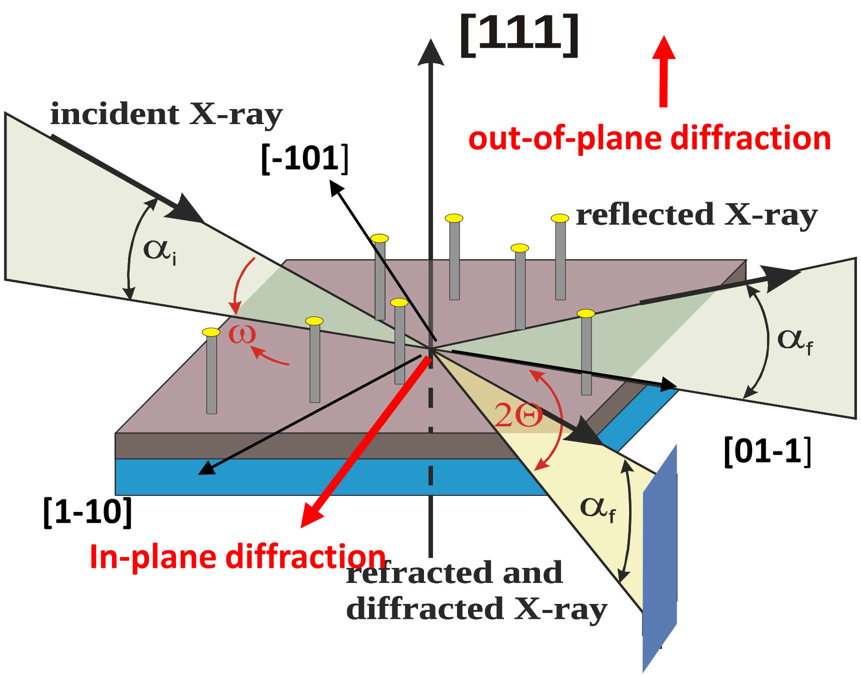


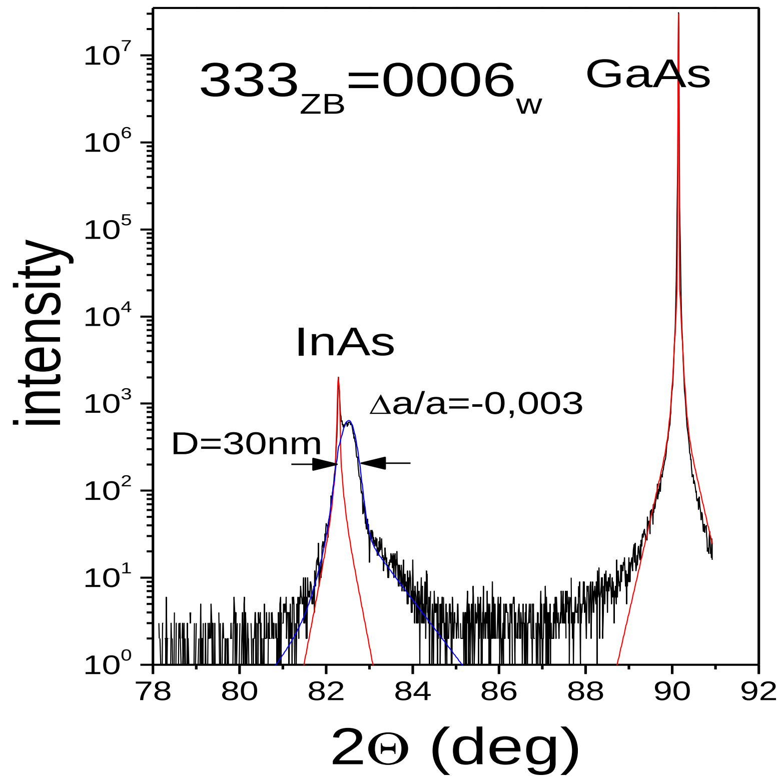




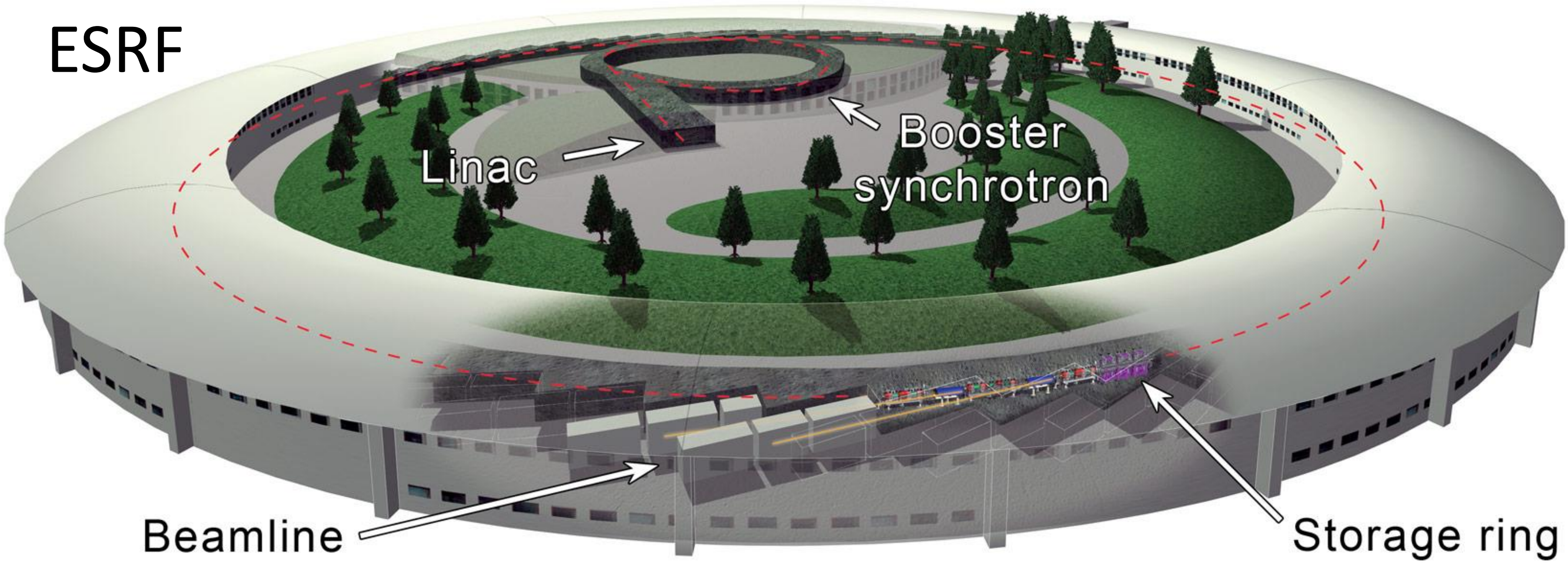
X-ray diffraction of InAs Nanowires on GaAs[111]

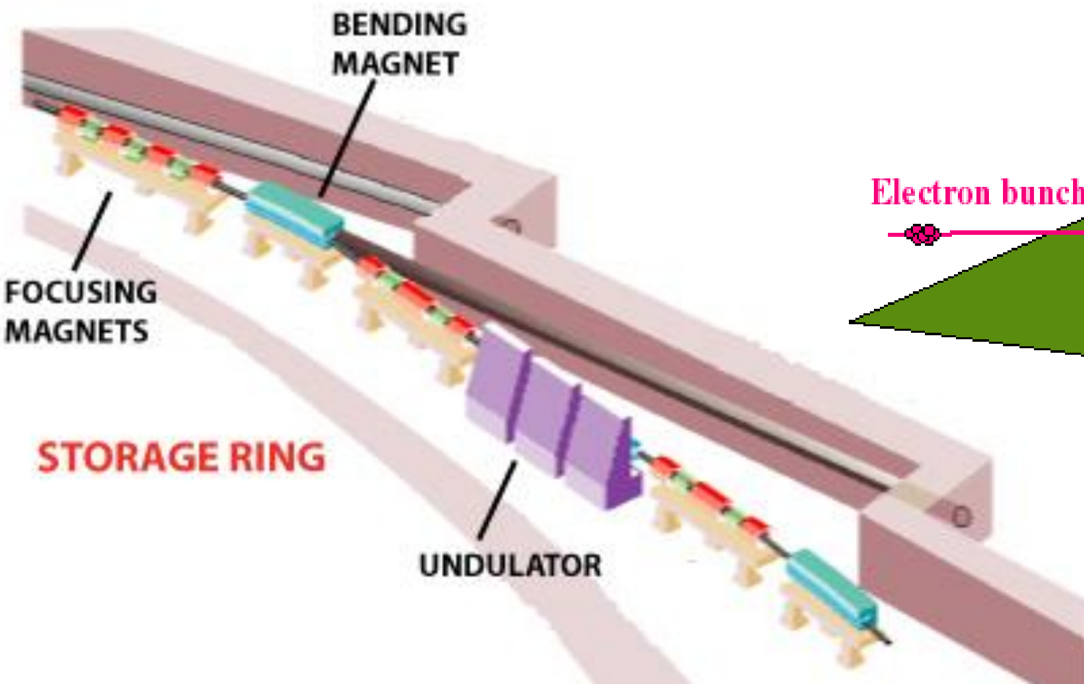


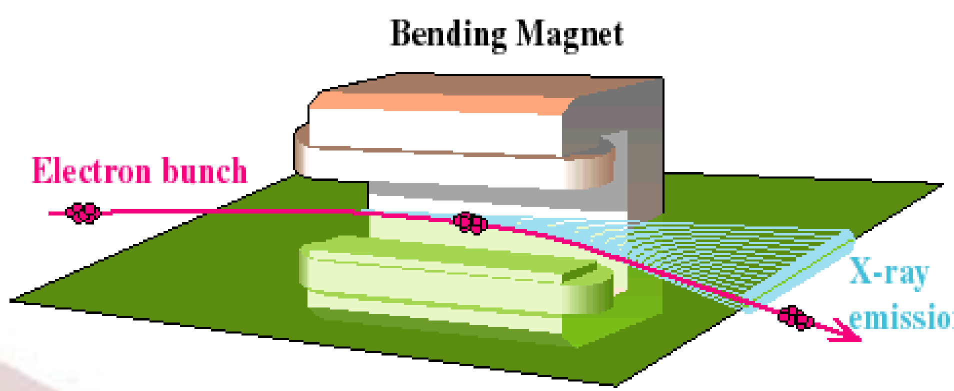




ESRF


Linac
Booster
synchrotron
Beamline
Storage ring


BENDING
MAGNET
FOCUSING
MAGNETS
STORAGE RING
UNDULATOR


Bending Magnet
Electron bunch
X-ray
emission