Theory of Quantum Matter

Lecturer: Prof. Otfried Gühne (Mon 14:00, Fri 10:00, Room D120)
Exercises: Chau Nguyen (to be clarified) **Sheet 0**

Hand in: not required for this sheet
Discussion date: to be clarified

0. Elementary review of the Boltzmann distribution

Consider a quantum system in contact (weak interaction) with the environment at temperature T (also called heat bath). Statistical mechanics tells that the probability for the system to be in a state with energy E_n is given by the Boltzmann distribution

$$p(E_n) = \frac{1}{Z} e^{-\beta E_n}, \tag{1}$$

where $\beta = 1/T$, Z is the normalisation factor known as the *partition function*. Here we set the Boltzmann constant $k_B = 1$ for simplicity.

As a simple example, consider a single spin-$\frac{1}{2}$ with magnetic moment μ coupled to an external magnetic field h along the z-axis, so that the Hamiltonian operator is

$$H = -\mu h \sigma_z. \tag{2}$$

(a) What are the energy levels of the system?

(b) Find the partition function Z of the system.

(c) Show that the (average) magnetic moment of the system can be found as

$$M = \frac{1}{\beta} \frac{\partial \ln Z}{\partial h}. \tag{3}$$

Compute the magnetic moment M of the system.

(d) Show that the (average) energy of the system can be found as

$$E = -\frac{\partial \ln Z}{\partial \beta}. \tag{4}$$

Compute the energy of the system.

(e) Compute the heat capacity at constant magnetic field of the system.

(f) The entropy of the system is defined as $S = -\sum_n p(E_n) \ln p(E_n)$. Show that

$$F = E - TS, \tag{5}$$

where $F = -T \ln Z$ is the free energy of the system. Compute the entropy of the system.