Incompatibility, steering, and channel-state duality

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Incompatibility and steering – short background

- Both based on Heisenberg's ideas but precisely formulated later:
  - Incompatibility of POVMs goes back to Ludwig[1]
  - Quantum steering due to Wiseman et al.[2]
- They are connected:
  - Starting point by Wolf et al.[3]: Joint measurability connected to hidden variable models / Bell inequalities in the CHSH case
  - Crucial observation[4]: Joint measurement is up to a normalization a hidden *state* model -> "**incompatibility and steering are equivalent"**

[1] G.Ludwig, Zeitschrift fur Physik 181 233-260 (1964)

[2] H. M. Wiseman, S. J. Jones, and A. C. Doherty, PRL 98, 140402 (2007)

[3] M.M. Wolf, D. Perez-Garcia, C. Fernandez, PRL 103 230402 (2009)

[4] R.Uola, T. Moroder, O.Guhne, PRL 113 160403 (2014); M.T. Quintino, T. Vertesi, N. Brunner, PRL 113 160402 (2014); R. Uola, C. Budroni, O. Guhne, and J.-P. Pellonpaa, PRL 115, 230402 (2015).

This talk

- Solving steering problems using *incompatibility breaking channels* (IBC)[1]
  - Steerability of **any** bipartite state by a fixed set of measurements becomes an IBC-problem for the same set of measurements (and vice versa)[2]
  - Works also in the infinite-dimensional (separable) case
- Examples / applications
  - Extreme cases: separable states and pure states with full Schmidt rank
  - Isotropic states (briefly)
  - Gaussian states
  - (NOON-states with vacuum noise)

[1] T. Heinosaari, J. Kiukas, D. Reitzner, and J. Schultz, J. Phys. A: Math. Theor. 48, 435301 (2015); T. Heinosaari, J. Kiukas, and J. Schultz, J. Math. Phys. 56, 082202 (2015).
[2] J. Kiukas, C. Budroni, R. Uola, and J.-P. Pellonpaa, arXiv:1704.05734

Incompatibility of measurements - definition

- Given a set of measurements (POVMs) $\mathcal{M}$ on a single system
- **Def**. $\mathcal{M}$ is *jointly measurable* if there is a POVM $G(d\lambda)$ and postprocessing functions $D_M(da|\lambda),\ M \in \mathcal{M}$ so that

$$M(da) = \int D_M(da|\lambda)G(d\lambda) \text{ for all } M \in \mathcal{M}$$

otherwise $\mathcal{M}$ is called *incompatible*.

Steering - definition

- Given a bipartite state ρ and a set of Alice's measurements $\mathcal{M}$
- **Def.** ρ *is non-steerable by* $\mathcal{M}$ if there exist states σ_λ on Bob's side, and *response functions* $D_M(da|\lambda),\, M \in \mathcal{M}$ so that

$$\mathrm{tr}_A[\rho(M(da) \otimes \mathbb{I})] = \int D_M(da|\lambda)\, \sigma_\lambda\, \mu(d\lambda) \text{ for all } M \in \mathcal{M}$$

  otherwise ρ called *steerable*.
- σ_λ and $D_M(da|\lambda),\, M \in \mathcal{M}$ form a *local hidden state (LHS)* model for bipartite correlations of the form $\mathrm{tr}[\rho(M(da) \otimes M_B(db))]$

Incompatibility vs steering – full Schmidt rank pure states

- Fix any injective state σ on bob's side, & purify:

$$\sigma = \sum_n p_n |n\rangle\langle n| \longrightarrow \Omega_\sigma = \sum_n \sqrt{p_n}|n\rangle \otimes |n\rangle \in \mathcal{H}_B \otimes \mathcal{H}_B$$

fixed eigenbasis
$p_n > 0$

Theorem[1]: $\mathcal{M}$ incompatible <-> Ω_σ steerable by $\mathcal{M}$

- Proof:
  1) the usual trick: $\mathrm{tr}_A[|\Omega_\sigma\rangle\langle\Omega_\sigma|(M(da)\otimes\mathbb{I})] = \sigma^{\frac{1}{2}} M(da)^T \sigma^{\frac{1}{2}}$

     Transpose in the fixed eigenbasis
  2) Joint POVM G corresponds to LHS σ_λ :

$$\sigma^{\frac{1}{2}} G(d\lambda)^T \sigma^{\frac{1}{2}} = \sigma_\lambda \, \mu(d\lambda)$$
$$\mu(d\lambda) = \mathrm{tr}[\sigma \, G(d\lambda)]$$

  3) Postprocessings = response functions $D_M(da|\lambda),\ M \in \mathcal{M}$

Need to use:

- Radon-Nikodym property of the trace class!
- The range of $\sigma^{\frac{1}{2}}$ is dense

[1] R.Uola, T. Moroder, O.Guhne, Phys. Rev. Lett. 113 160403 (2014); M.T. Quintino, T. Vertesi, N. Brunner, Phys. Rev. Lett. 113 160402 (2014); J. Kiukas, C. Budroni, R. Uola, and J.-P. Pellonpaa, arXiv:1704.05734

Incompatibility vs steering - general case

- Fix an injective state σ on Bob's side and purify:

$$\sigma = \sum_n p_n |n\rangle\langle n| \longrightarrow \Omega_\sigma = \sum_n \sqrt{p_n}|n\rangle \otimes |n\rangle \in \mathcal{H}_B \otimes \mathcal{H}_B$$

- For any bipartite state ρ with $\mathrm{tr}_A[\rho] = \sigma$, define

$$\tilde{\mathcal{M}} := \{\tilde{M} \mid M \in \mathcal{M}\} \qquad \sigma^{\frac{1}{2}}\tilde{M}(da)\sigma^{\frac{1}{2}} := \mathrm{tr}_A[\rho(M(da) \otimes \mathbb{I})]^T$$

Effective observables on Bob's side

- **Theorem[1]** : $\tilde{\mathcal{M}}$ incompatible <-> ρ steerable by $\mathcal{M}$
- **Next observation[2]**: $\tilde{M}(da) = \Lambda(M(da))$ where Λ is the quantum channel such that $\rho = (\Lambda \otimes \mathrm{Id})(|\Omega_\sigma\rangle\langle\Omega_\sigma|)$
- Two connections emerge:
  - Incompatibility breaking channels
  - State-channel correspondence

[1] R. Uola, C. Budroni, O. Guhne, and J.-P. Pellonpaa, Phys. Rev. Lett. 115, 230402 (2015)
[2] J. Kiukas, C. Budroni, R. Uola, and J.-P. Pellonpaa, arXiv:1704.05734

Incompatibility breaking channels[1]

- Fix a set $\mathcal{M}$ of Alice's observables
- For a channel Λ (unital normal CP) from Alice to Bob, define

$$\Lambda(\mathcal{M}) := \{\Lambda(M) \mid M \in \mathcal{M}\}$$

Observables on Bob's side – Heisenberg picture!

$$\Lambda(M)(da) := \Lambda(M(da))$$

- **Def**: Λ *breaks the incompatibility of* $\mathcal{M}$ if $\Lambda(\mathcal{M})$ is jointly measurable.
- In particular, entanglement breaking channels[2] break the incompatibility of every set $\mathcal{M}$.

[1] T. Heinosaari, J. Kiukas, D. Reitzner, and J. Schultz, JPA 48, 435301 (2015).
[2] M. Horodecki, P.W. Shor, and M.B. Ruskai. Rev. Math. Phys., 15:629–641, 2003.

State-channel correspondence

- Choi-Jamiolkowski does not work
  - only applies to states with maximally mixed Bob-marginal
  - Generalisation to infinite-dimensional case requires unbounded forms[1]
- Instead, fix any injective state σ on Bob's side and purify

$$\sigma = \sum_n p_n |n\rangle\langle n| \longrightarrow \Omega_\sigma = \sum_n \sqrt{p_n} |n\rangle \otimes |n\rangle \in \mathcal{H}_B \otimes \mathcal{H}_B$$

- **Lemma**: For each ρ with $\mathrm{tr}_A[\rho] = \sigma$ there is a unique channel Λ from Bob to Alice such that

$$\rho = (\Lambda \otimes \mathrm{Id})(|\Omega_\sigma\rangle\langle\Omega_\sigma|)$$ or, equivalently, $$\sigma^{\frac{1}{2}} \Lambda(A) \sigma^{\frac{1}{2}} = \mathrm{tr}_A[\rho(A \otimes \mathbb{I})]^T \text{ for all } A$$

- less "canonical" than CJ but works also in infinite dimensions

[1] A. S. Holevo, JMP 52, 042202 (2011)

From steering to IBC

- Given a bipartite state ρ, Alice's measurements $\mathcal{M}$:
  1. purify the Bob-marginal: $\mathrm{tr}_A[\rho] =: \sigma = \sum_n p_n |n\rangle\langle n| \longrightarrow \Omega_\sigma = \sum_n \sqrt{p_n}|n\rangle \otimes |n\rangle$
  2. use the state-channel correspondence to find Λ such that $\rho = (\Lambda \otimes \mathrm{Id})(|\Omega_\sigma\rangle\langle\Omega_\sigma|)$
- **Theorem[1]**: The following are equivalent:
  i. ρ is steerable by $\mathcal{M}$
  ii. Λ breaks the incompatibility of $\mathcal{M}$
- States with different marginals may have the same channel – different steering problems may reduce to the same IBC-problem
- Appropriate robustness measures carry over (consistent SR[2] <-> IR[3])

[1] J. Kiukas, C. Budroni, R. Uola, and J.-P. Pellonpaa, arXiv:1704.05734
[2] D. Cavalcanti and P. Skrzypczyk, Phys. Rev. A 93, 052112 (2016)
[3] R. Uola, C. Budroni, O. Gühne, and J.-P. Pellonpaa, Phys. Rev. Lett. 115, 230402 (2015)

Examples / applications

- Extreme cases – separable and pure full Schmidt rank states
- Isotropic states (briefly)
- Noisy NOON-states with quadratures
- Gaussian states

Extreme cases

- Separable states <-> entanglement breaking channels (EBC)

$$\rho = \int \rho_\lambda \otimes \sigma_\lambda \, \mu(d\lambda) \longleftrightarrow \Lambda(A) = \int \mathrm{tr}[\rho_\lambda A] F(d\lambda) \qquad \sigma^{\frac{1}{2}} F(d\lambda) \sigma^{\frac{1}{2}} = \mu(d\lambda) \sigma_\lambda^T$$

  - Not steerable by any measurements - F is the joint POVM for all $\Lambda(M)$

- Pure states with full Schmidt rank <-> unitary channels

$$\Psi \longleftrightarrow \Lambda(A) = U^* A U \qquad \langle n | U \sigma^{\frac{1}{2}} | m \rangle = \langle nm | \Psi \rangle$$

  - Steerable by any incompatible set since unitaries preserve incompatibility

Isotropic states

$$\rho = t|\Psi_0\rangle\langle\Psi_0| + (1-t)\mathbb{I}/d^2, \quad \Psi_0 = \frac{1}{\sqrt{d}}\sum_n |nn\rangle \quad \longleftrightarrow \Lambda(A) = tA + (1-t)\mathrm{tr}[A]\mathbb{I}/d$$

- Breaks the incompatibility of the set of *all* Alice's POVMs for $t \leq t_0$
- Based on Werner's original model[1]:
  - Hidden variables are the unitary operators (with the Haar measure)
  - joint POVM is $G(dU) = d\ U|\phi\rangle\langle\phi|U^* dU$
- Postprocessing functions from Barrett's model[2]

[1] R.F. Werner, PRA, 40:4277–4281, 1989

[2] J. Barrett. PRA, 65:042302, 2002.

Noisy NOON-states

- Two-mode CV state $|N00N\rangle = \frac{1}{\sqrt{2}}(|N0\rangle - e^{i\alpha N}|0N\rangle)$ with noise
  a) classical vacuum noise: $\rho_\eta = \eta|N00N\rangle\langle N00N| + (1-\eta)|00\rangle\langle 00|$
  b) dynamical quantum noise: $\rho_t = (\mathcal{E}_t \otimes \mathrm{Id})(|1001\rangle\langle 1001|)$
  $$d\mathcal{E}_t(\rho_0)/dt = \gamma(t)\left[\sigma_-\mathcal{E}_t(\rho_0)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \mathcal{E}_t(\rho_0)\}\right]$$
- **Problem**: find the noise limits for steering by two quadratures[1]
- Different states – but both reduce to the same IBC problem
  - amplitude damping qubit channel
  $$\Lambda(A) = \sum_{i=0}^{1} K_{i,r}^* A K_{i,r}, \quad K_{0,r} = \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix}, \; K_{1,r} = \begin{pmatrix} 0 & \sqrt{1-r^2} \\ 0 & 0 \end{pmatrix}.$$
  a) $r = \eta/(2-\eta)$
  b) $r = r(t)$
  - A pair of qubit measurements with continuous outcome set
  - Breaks incompatibility for $r \leq 1/\sqrt{2}$ – SDP suggests steerable otherwise

[1] I. Kogias, P. Skrzypczyk, D. Cavalcanti, A. Acin, and G. Adesso, PRL 115, 210401 (2015)

The Gaussian case

- Both Alice and Bob have an N-mode continuous variable system
- Steerability criterion by Wiseman et al[1], Gaussian IBC criterion obtained in[2]
- Now:
  - independent proof of steerability criterion via the IBC problem
  - steering by quadratures
- Based on
  1) Characteristic function formalism for Gaussian states, POVMs and channels
  2) Joint measurability criterion for *pairs* of unsharp quadratures
  3) Gaussian version of the state-channel correspondence

[1] H. M. Wiseman, S. J. Jones, and A. C. Doherty, PRL 98, 140402 (2007)
[2] T. Heinosaari, J. Kiukas, and J. Schultz, JMP 56, 082202 (2015)

Gaussian case – basic elements

- Phase space coordinates $\mathbf{x} = (q_1, p_1, \ldots, q_N, p_N)^T \in \mathbb{R}^{2N}$
- symplectic form $\boldsymbol{\Omega} = \oplus_{j=1}^{N} \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$
- Standard quadratures $\mathbf{R} = (Q_1, P_1, \ldots, Q_N, P_N)^T$ $\quad [R_i, R_j] = i\Omega_{ij}\mathbb{I}$
- Weyl operators $W(\mathbf{x}) = e^{-i\mathbf{x}^T\mathbf{R}}$
- Symplectic matrices & unitaries: $\mathbf{S}^T\boldsymbol{\Omega}\mathbf{S} = \boldsymbol{\Omega}$ $\quad U_{\mathbf{S}}^* W(\mathbf{x}) U_{\mathbf{S}} = W(\mathbf{S}\mathbf{x})$

Gaussian states, POVMs and channels – the characteristic function approach[1]

- A state is Gaussian if $\hat{\rho}(\mathbf{x}) := \mathrm{tr}[\rho W(\mathbf{x})] = e^{-\frac{1}{4}\mathbf{x}^T\mathbf{V}\mathbf{x} - i\mathbf{r}^T\mathbf{x}}$
  - Determined by $(\mathbf{V}, \mathbf{r})$; $\mathbf{V}$ is the covariance matrix (CM)
  - Uncertainty relation $\mathbf{V} + i\boldsymbol{\Omega} \geq 0$
- A POVM is Gaussian if $\hat{M}(\mathbf{p}) := \int e^{i\mathbf{p}^T\mathbf{a}} M(d\mathbf{a}) = W(\mathbf{K}\mathbf{p}) e^{-\frac{1}{4}\mathbf{p}^T\mathbf{L}\mathbf{p} - i\mathbf{m}^T\mathbf{p}}$
  - Determined by $(\mathbf{K}, \mathbf{L}, \mathbf{m})$
  - Positivity condition $\mathbf{C}_{\mathbf{K},\mathbf{L}} := \mathbf{L} - i\mathbf{K}^T\boldsymbol{\Omega}\mathbf{K} \geq 0$
- A channel is Gaussian if $\hat{\Lambda}(\mathbf{x}) := \Lambda(W(\mathbf{x})) = W(\mathbf{M}\mathbf{x}) e^{-\frac{1}{4}\mathbf{x}^T\mathbf{N}\mathbf{x} - i\mathbf{c}^T\mathbf{x}}$
  - Determined by $(\mathbf{M}, \mathbf{N}, \mathbf{c})$
  - Complete positivity condition $\mathbf{C}_{\mathbf{M},\mathbf{N}} + i\boldsymbol{\Omega} \geq \mathbf{0}$

[1] A. S. Holevo and R. F. Werner, PRA 63, 032312 (2001) and others

Gaussian states, POVMs and channels – the characteristic function approach

- A postprocessing is Gaussian if $\hat{D}(\mathbf{p}|\mathbf{x}) := \int e^{i\mathbf{p}^T\mathbf{y}} D(d\mathbf{y}|\mathbf{x}) = e^{i(\mathbf{M}\mathbf{p})^T\mathbf{x}} e^{\frac{1}{4}\mathbf{p}^T\mathbf{N}\mathbf{p} - i\mathbf{c}^T\mathbf{p}}$
  - Determined by $(\mathbf{M}, \mathbf{N}, \mathbf{c})$
  - Positivity condition $\mathbf{N} \geq 0$

- A Gaussian channel $(\mathbf{M}, \mathbf{N}, \mathbf{c})$ transforms
  - Gaussian states (Schrodinger picture) $(\mathbf{V}, \mathbf{r}) \mapsto (\mathbf{M}^T\mathbf{V}\mathbf{M} + \mathbf{N}, \mathbf{M}^T\mathbf{r} + \mathbf{c})$
  - Gaussian POVMs (Heisenberg picture) $(\mathbf{K}, \mathbf{L}, \mathbf{m}) \mapsto (\mathbf{M}\mathbf{K}, \mathbf{L} + \mathbf{K}^T\mathbf{N}\mathbf{K}, \mathbf{m} + \mathbf{K}^T\mathbf{c})$
- A Gaussian postprocessing $(\mathbf{M}, \mathbf{N}, \mathbf{c})$ transforms Gaussian POVMs:

$$(\mathbf{K}, \mathbf{L}, \mathbf{m}) \mapsto (\mathbf{K}\mathbf{M}, \mathbf{N} + \mathbf{M}^T\mathbf{L}\mathbf{M}, \mathbf{c} + \mathbf{M}^T\mathbf{m})$$

Joint measurability of noisy quadratures

- A POVM is Gaussian if $\hat{M}(\mathbf{p}) := \int e^{i\mathbf{p}^T\mathbf{a}} M(d\mathbf{a}) = W(\mathbf{K}\mathbf{p}) e^{-\frac{1}{4}\mathbf{p}^T\mathbf{L}\mathbf{p} - i\mathbf{m}^T\mathbf{p}}$
  - Determined by $(\mathbf{K}, \mathbf{L}, \mathbf{m})$ satisfying $\mathbf{C}_{\mathbf{K},\mathbf{L}} := \mathbf{L} - i\mathbf{K}^T\mathbf{\Omega}\mathbf{K} \geq 0$
  - Outcomes $\mathbf{a} \in \mathbb{R}^d$
- For $d = 1$ this is a convolution of a quadrature:

$$M(da) = M_{\mathbf{x},\xi}(da) := \frac{1}{\xi\sqrt{2\pi}} \int e^{-\frac{1}{2}(a-a')^2/\xi^2} Q_{\mathbf{x}}(da')$$

$$Q_{\mathbf{x}} := \mathbf{x}^T\mathbf{R} := \int a\, Q_{\mathbf{x}}(da)$$

$\mathbf{K} = \mathbf{x} \in \mathbb{R}^{2N}$ Quadrature parameter

$\mathbf{L} = 2\xi^2 \in \mathbb{R}$ Noise parameter

- **Lemma 1:** $M_{\mathbf{x},\xi},\, M_{\mathbf{y},\xi'}$ jointly measurable iff $\xi\xi' \geq \mathbf{x}^T\mathbf{\Omega}\mathbf{y}/2$. In this case they have a joint *Gaussian* observable.
  - A generalisation of known position-momentum joint measurability criterion[1]

[1] R. F. Werner, Qu. Inf. Comp. 4, 546 (2004); C. Carmeli, T. Heinonen, and A. Toigo, JPA 38, 5253 (2005); P. Busch, T. Heinonen, and P. Lahti, Physics Reports 452, 155 (2007).

Gaussian state-channel correspondence

- Fix a Gaussian state σ on Bob's side with full symplectic rank & diagonalise symplectically[1]:

$$\mathbf{V}_\sigma = \mathbf{S}^T \mathbf{D} \mathbf{S}, \quad \mathbf{D} = \oplus_{k=1}^N \nu_k \mathbb{I}_2 \qquad \nu_i > 1 \qquad \sigma = \sum_{\mathbf{n}} p_{\mathbf{n}} \, U_{\mathbf{S}} |\mathbf{n}\rangle\langle\mathbf{n}| U_{\mathbf{S}}^*$$

number basis of the N-mode system

- The purification Ω_σ is Gaussian with CM

$$\mathbf{V}_\Omega = \begin{pmatrix} \mathbf{V}_\sigma & \mathbf{S}^T\mathbf{Z}\mathbf{S} \\ \mathbf{S}^T\mathbf{Z}\mathbf{S} & \mathbf{V}_\sigma \end{pmatrix} \qquad \mathbf{Z} = \oplus_{i=1}^N \sqrt{\nu_i^2 - 1}\, \sigma_z$$

- **Lemma**: the correspondence $\rho \leftrightarrow \Lambda$, $\rho = (\Lambda \otimes \mathrm{Id})(|\Omega_\sigma\rangle\langle\Omega_\sigma|)$ is bijective when restricted to Gaussians:

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_A & \mathbf{\Gamma}^T \\ \mathbf{\Gamma} & \mathbf{V}_\sigma \end{pmatrix} \leftrightarrow \begin{cases} \mathbf{M} & = (\mathbf{S}^T\mathbf{Z}\mathbf{S})^{-1}\mathbf{\Gamma} \\ \mathbf{N} & = \mathbf{V}_A - \mathbf{M}^T \mathbf{V}_\sigma \mathbf{M} \end{cases}, \qquad \mathbf{C}_{\mathbf{M},\mathbf{N}} := \mathbf{N} - i\mathbf{M}^T \mathbf{\Omega} \mathbf{M}$$

$$\mathbf{V} + i\mathbf{\Omega} \geq 0 \Leftrightarrow \mathbf{C}_{\mathbf{M},\mathbf{N}} + i\mathbf{\Omega} \geq 0,$$

[1] J. Williamson, Am. J. Math. 58, 141 (1936)

IBC vs steering for Gaussians

- The following are *all* equivalent:

IBC side – Gaussian channel $\Lambda \simeq (\mathbf{M}, \mathbf{N})$

i. breaks the incompatibility of the set of all Gaussian measurements
ii. breaks the incompatibility of every pair of canonical quadratures
iii. its parameters satisfy $\mathbf{C}_{\mathbf{M},\mathbf{N}} \geq 0$

Steering side – Gaussian state $\rho \simeq (\mathbf{V}, \mathbf{r})$

i. non-steerable by the total set of Gaussian measurements
ii. not steerable by any pair of canonical quadratures
iii. its CM satisfies $\mathbf{V} + i(\mathbf{0} \oplus \mathbf{\Omega}) \geq 0$

- If these hold, then
  - the matrices $(\mathbf{M}, \mathbf{N})$ define a *Gaussian* joint POVM for $\Lambda(\mathcal{M}_G)$
  - Note: the conditions $\mathbf{C}_{\mathbf{M},\mathbf{N}} + i\mathbf{\Omega} \geq 0 \Leftrightarrow \mathbf{V} + i\mathbf{\Omega} \geq 0$ always hold

IBC for Gaussians - proof

Gaussian channel $\Lambda \simeq (\mathbf{M}, \mathbf{N})$

i. breaks the incompatibility of the set of all Gaussian measurements
ii. breaks the incompatibility of every pair of canonical quadratures
iii. its parameters satisfy $\mathbf{C}_{\mathbf{M},\mathbf{N}} \geq 0$

$$\mathbf{C}_{\mathbf{M},\mathbf{N}} := \mathbf{N} - i\mathbf{M}^T\boldsymbol{\Omega}\mathbf{M}$$

- Proof:
  - (iii) -> $(\mathbf{M}, \mathbf{N})$ defines a Gaussian joint POVM for $\Lambda(\mathcal{M}_G)$ -> (i)
  - (i) -> (ii) trivial
  - (ii) -> (iii): If (iii) is not true we can find $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{2N}$, $\mathbf{x}^T\boldsymbol{\Omega}\mathbf{y} = 1$ for which

$$\xi\xi' < (\mathbf{M}\mathbf{x})^T\boldsymbol{\Omega}\mathbf{M}\mathbf{y}/2 \quad \text{where} \quad \xi^2 = \mathbf{x}^T\mathbf{N}\mathbf{x},\ \xi'^2 = \mathbf{y}^T\mathbf{N}\mathbf{y}$$

So $\begin{array}{l} Q_{\mathbf{x}} = \mathbf{x}^T\mathbf{R} \\ Q_{\mathbf{y}} = \mathbf{y}^T\mathbf{R} \end{array}$ is canonical and $\begin{array}{l} \Lambda(Q_{\mathbf{x}}) = M_{\mathbf{M}\mathbf{x},\xi} \\ \Lambda(Q_{\mathbf{y}}) = M_{\mathbf{M}\mathbf{y},\xi'} \end{array}$ incompatible by Lemma 1.

-> Λ does not break the incompatibility of $(Q_{\mathbf{x}}, Q_{\mathbf{y}})$ -> (ii) does not hold

Summary

- Idea: solving steering problems using *incompatibility breaking channels*
  - Steerability of **any** bipartite state by a fixed set of measurements becomes an IBC-problem for the same set of measurements (and vice versa)
  - Works also in the infinite-dimensional (separable) case
- Examples / applications
  - Extreme cases: separable states and pure states with full Schmidt rank
  - Isotropic states
  - Noisy NOON-states
  - Gaussian states

Thank you!