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# Quantum Clocks and Quantum Causality

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Seminar, Naturwissenschaftlich-Technische Fakultaet, Universitaet Siegen, April 16th, 2015

Gravity

Quantum physics

All tests thus far are consistent with either **quantum mechanics with Newtonian gravity** or **general relativity with classical mechanics.**

Gravity

Quantum physics

How about the atomic clocks?

Time of the clock is a **classical parameter**.

Gravity

Quantum physics

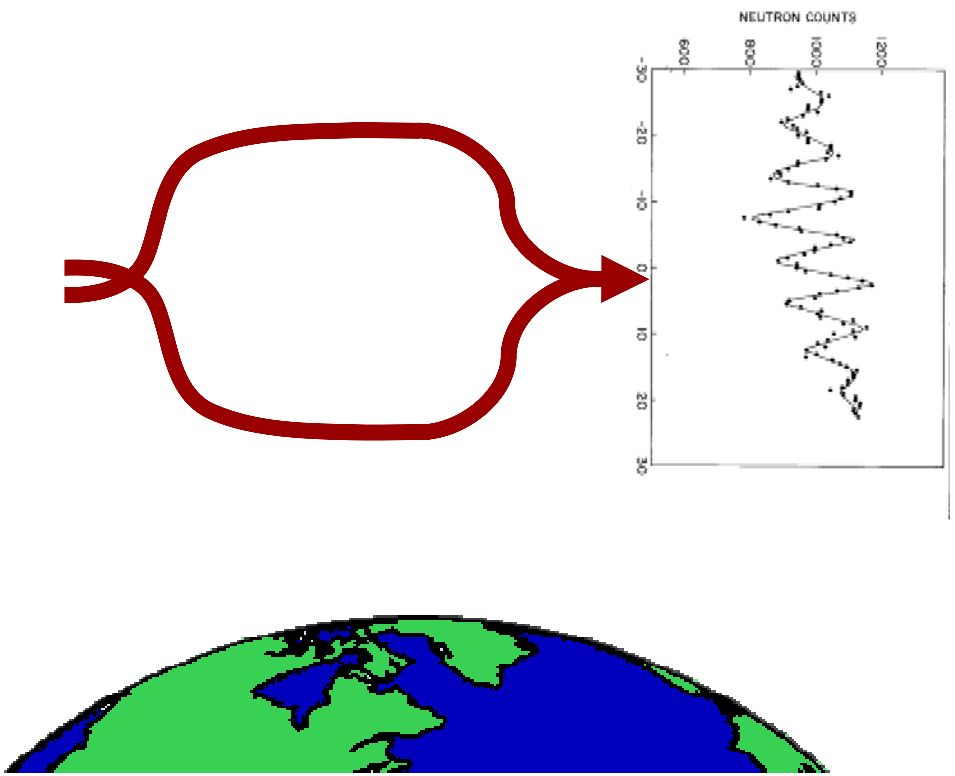
Importance of testing the foundational principles of the two theories jointly, e.g. **quantum superposition + time dilation**

# Outline

Probing the overlap between quantum mechanics and general relativity:

- Superpositions of „clocks“
- Decoherence due to time dilation
- Superposition of space-times ?

# Matter-wave intereference in a gravitational field




## Neutron interferometry

R. Colella, A. W. Overhauser, S. A. Werner,

PRL **34**, 1472-1474 (1975)

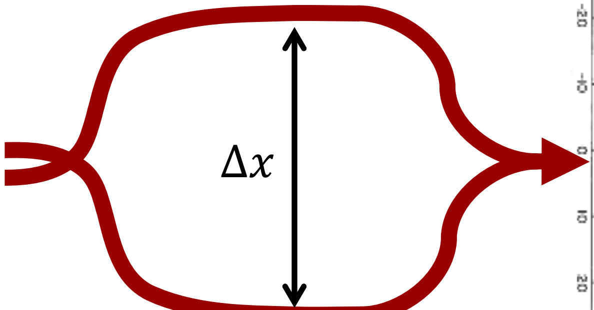
## Atomic fountains

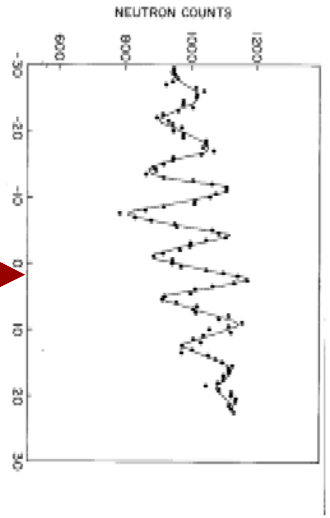
H. Müller, A. Peters, S. A. Chu,

Nature **463**, 926-929 (2010)

# Matter-wave intereference in a gravitational field

Time of flight $t$




Gravitational potential induces a relative phase

$$\Delta\phi = \frac{m\, g\Delta x t}{\hbar}$$

Explainable by:

- a potential in absolute time
- analogue to a charged particle in EM field
- independent of whether a particle is a „clock“ or a „rock“

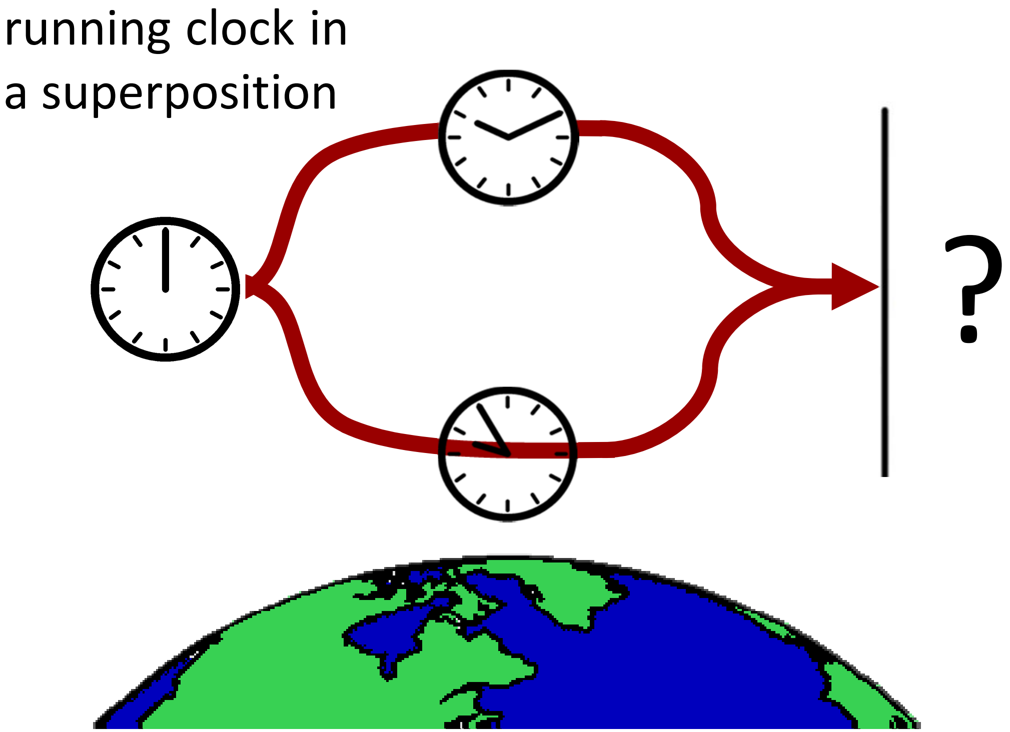
# Beyond Newtonian gravity: Time-dilation

Initially synchronized clocks will eventually show **different times** when placed at different gravitational potentials.

Clock closer to a massive body ticks slower than the clock further away from the mass.

# Superposition of a clock

“**Clock**”- a system with an evolving in time degree of freedom

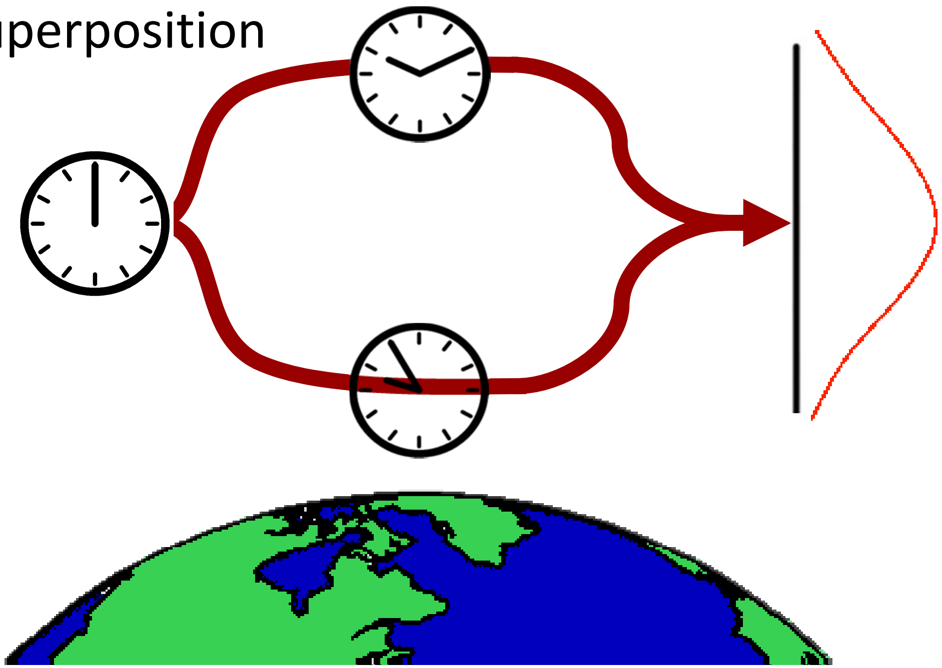



**GR**: time shown by the clock depends on the path taken

**QM**: either path or interference

# Superposition of a clock

running clock in
a superposition

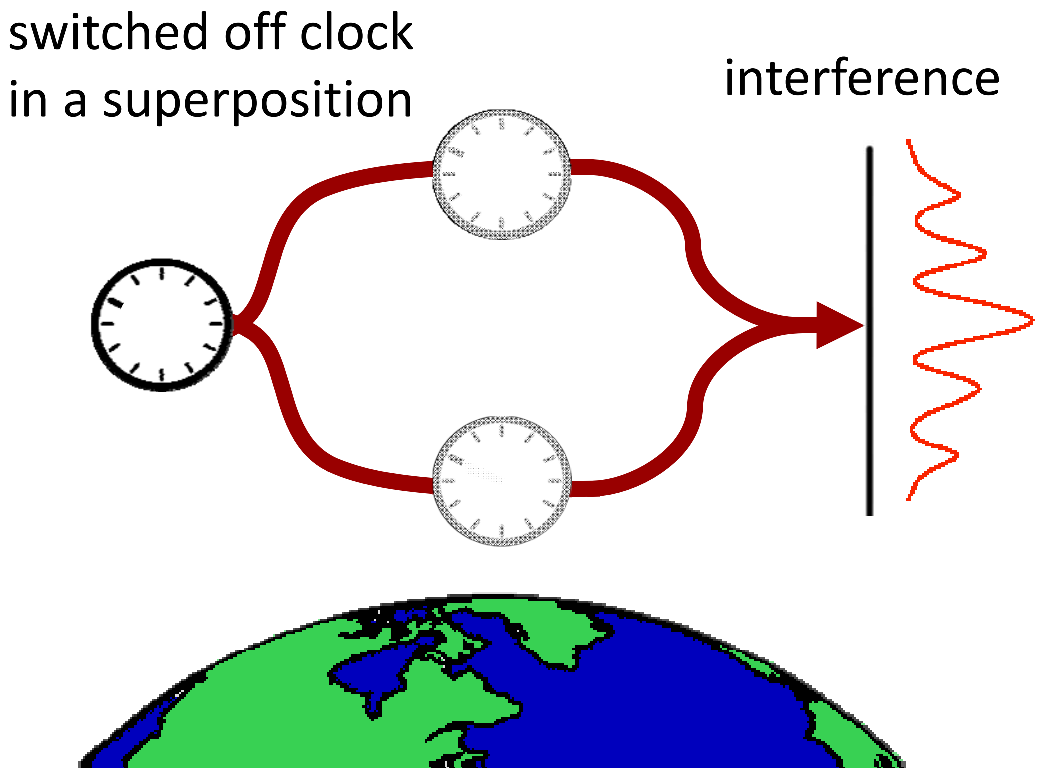


**GR**: time shown by the clock depends on the path taken

**QM**: either path or interference

**QM+GR**: interference cannot be observed since the which-path information is stored in the clock time

# Clock switched off




**GR**: time shown by the clock depends on the path taken

**QM**: either path or interference

**QM+GR**: interference cannot be observed since the which-path information is stored in the clock time

# Relativistic time dilation

The rest energy $E_{rest}$ is an invariant quantity:

$$p_\mu p^\mu = \boxed{g^{\mu\nu} p_\mu p_\nu} = \boxed{-(E_{rest}/c)^2}$$

In laboratory frame

The total mass-energy in the co-moving frame

Hamiltonian in lab frame: $H^2 = c^2 p_0^2 = -g_{00}(E_{rest}^2 + c^2 p_i p_j g^{ij})$

Mass-energy equivalence: $E_{rest} = mc^2 + H_{clock}$

Rest mass

Rest energy of the clock (internal degree of freedom)

Stationary metric, weak-field approximation

$$g_{00} = -(1 + 2\Phi(x)/c^2 + 2\Phi^2(x)/c^4) \text{ and } g_{ij} = \delta_{ij}(1 - 2\Phi(x)/c^2)$$

Potential

# Relativistic time dilation

The low energy expansion of the Klein-Gordon field:

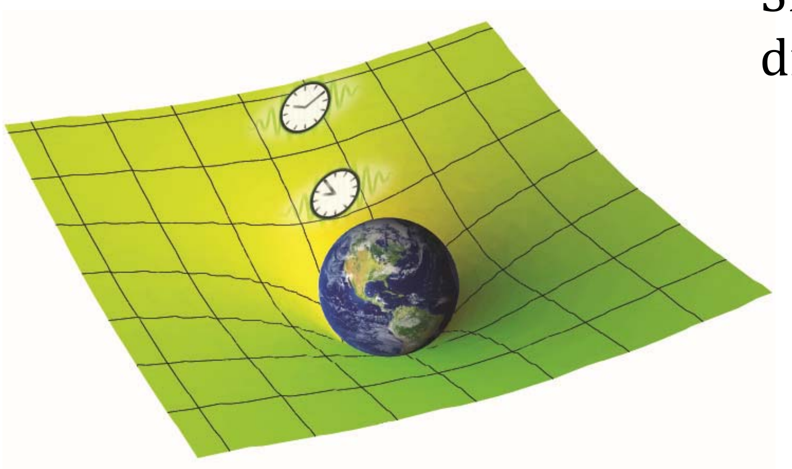
Lämmerzahl, C. A Hamilton operator for quantum optics in gravitational fields, Phys.Lett. A **203**, 12-17 (1996)

$$\hat{H} = mc^2 + \hat{H}_{clock} + \frac{\hat{p}^2}{2m}\left(1 - \boxed{\frac{\hat{H}_{clock}}{mc^2}}\right) + \left(\boxed{m} + \boxed{\frac{\hat{H}_{clock}}{c^2}}\right)\phi(\hat{x})$$

SR time dilation

Newtonian coupling

Grav. time dilation



QM on fixed background space-time with time dilation (but no curvature)

[Can be obtained simply from

$$\hat{H} = m_{rest}c^2 + \frac{\hat{p}}{2m_{rest}} + m_{rest}\phi(\hat{x})$$

and the mass-energy equivalence

$$\hat{m}_{rest} \to m + \frac{\hat{H}_{clock}}{c^2}$$ ]

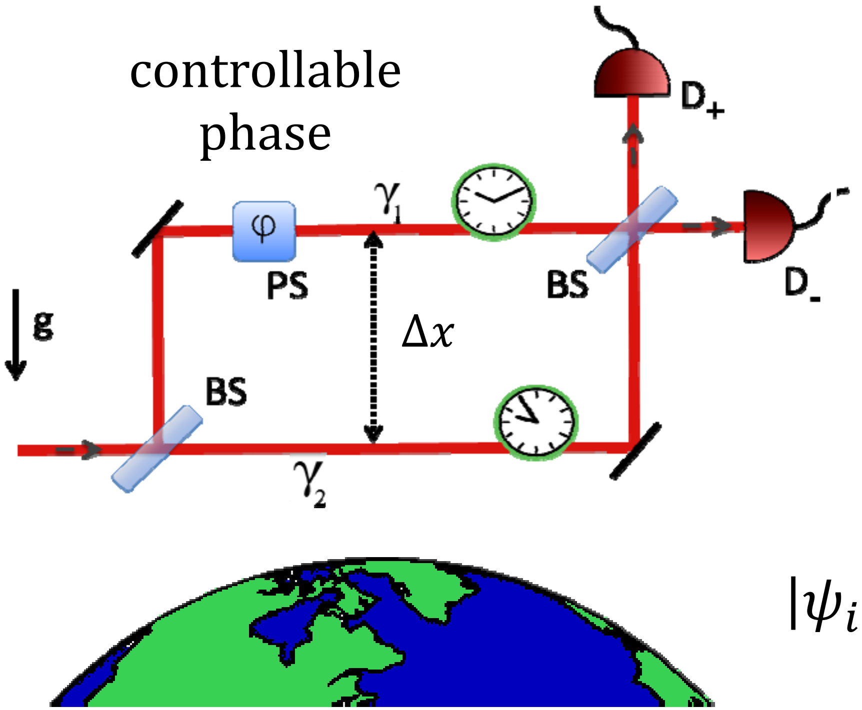
# Superposition of a clock

$$\hat{H} = mc^2 + \hat{H}_{clock} + \frac{\hat{p}^2}{2m}\left(1 - \boxed{\frac{\hat{H}_{clock}}{mc^2}}\right) + \left(\boxed{m} + \boxed{\frac{\hat{H}_{clock}}{c^2}}\right)\phi(\hat{x})$$

SR time dilation

Newtonian coupling

Grav. time dilation




$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$$ Spatial wave function

$$|\tau_{in}\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$ Clock

$$|\psi_i\rangle = e^{-\frac{i}{\hbar}\int_{\gamma_i} dt \frac{\phi(x)}{c^2}(mc^2 + \hat{H}_{clock} + \cdots)}|x_{in}\rangle|\tau_{in}\rangle$$

# Superposition of a clock

GR corrections

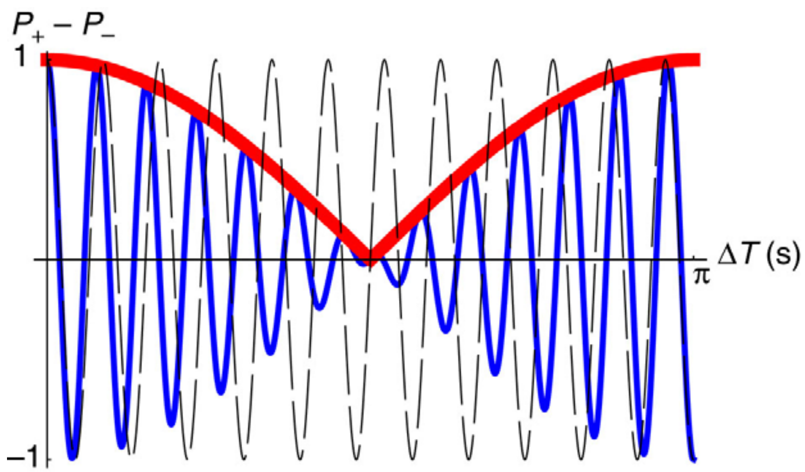
$$p = \frac{1}{2} \pm \frac{1}{2} \boxed{\cos\left(\frac{\Delta E g \Delta x t}{2\hbar c^2}\right)} \cos\left(\left(\boxed{mc^2} + \boxed{\langle H_{clock} \rangle} + \cdots\right) \frac{g \Delta x t}{\hbar c^2}\right)$$

Visibility

Phase from the Newtonian potential

Phase proportional to average internal energy

$$\Delta E = E_1 - E_0$$



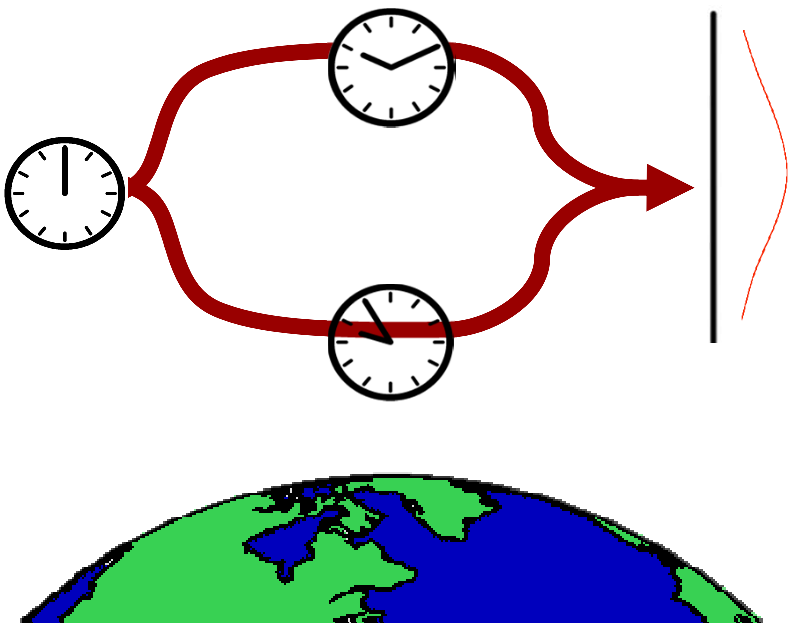

- dashed, black line: the “clock” switched off: $\cos\left(mgh\,\Delta T/\hbar\right)$
- blue line: phase with the “clock” switched on
- thick, red line: modulation in the visibility

# Visibility loss

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\gamma_1\rangle|\tau_1\rangle + e^{-i(\varphi+\Delta\phi)}\,|\gamma_2\rangle|\tau_2\rangle\,\right)$$

Entanglement between the path and the clock

$$P_{\pm} = \frac{1}{2} \pm \frac{1}{2}|\langle\tau_1|\tau_2\rangle| \cos\left(\Delta\phi + \alpha + \varphi\right)$$



Visibility:

$$\mathcal{V} = |\langle\tau_1|\tau_2\rangle|$$

Distunguishability:

$$\mathcal{D} = \sqrt{1 - |\langle\tau_1|\tau_2\rangle|^2}$$

$$\mathcal{V}^2 + \mathcal{D}^2 = 1$$

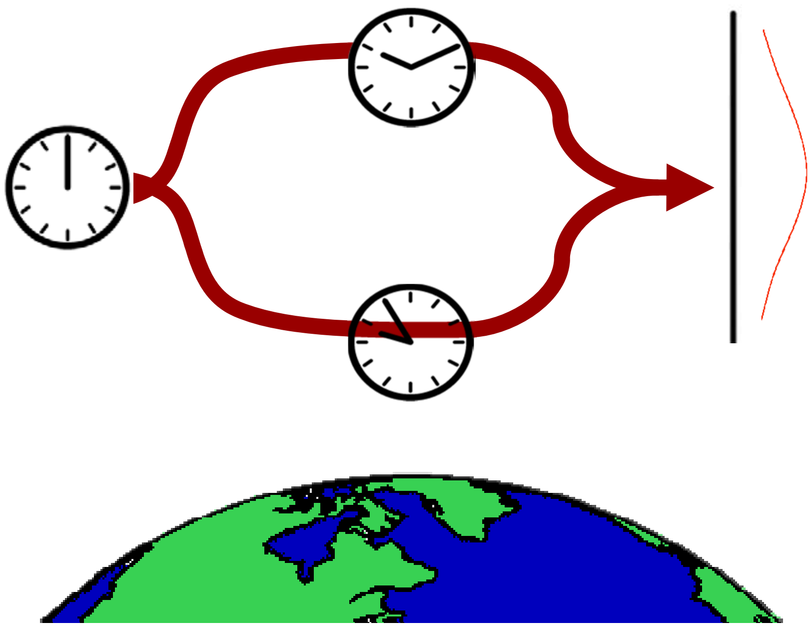
Interferometric visibility drops to the extent to which path information becomes available from the "clock".

# Visibility loss

Difference in proper time

$$\mathcal{V} = \left|\cos\left(\frac{\Delta E g \Delta x t}{2\hbar c^2}\right)\right| = \left|\cos\left(\frac{\Delta\tau}{t_\perp}\frac{\pi}{2}\right)\right|$$

Orthogonalization time $t_\perp = \dfrac{\hbar\pi}{\Delta E}$



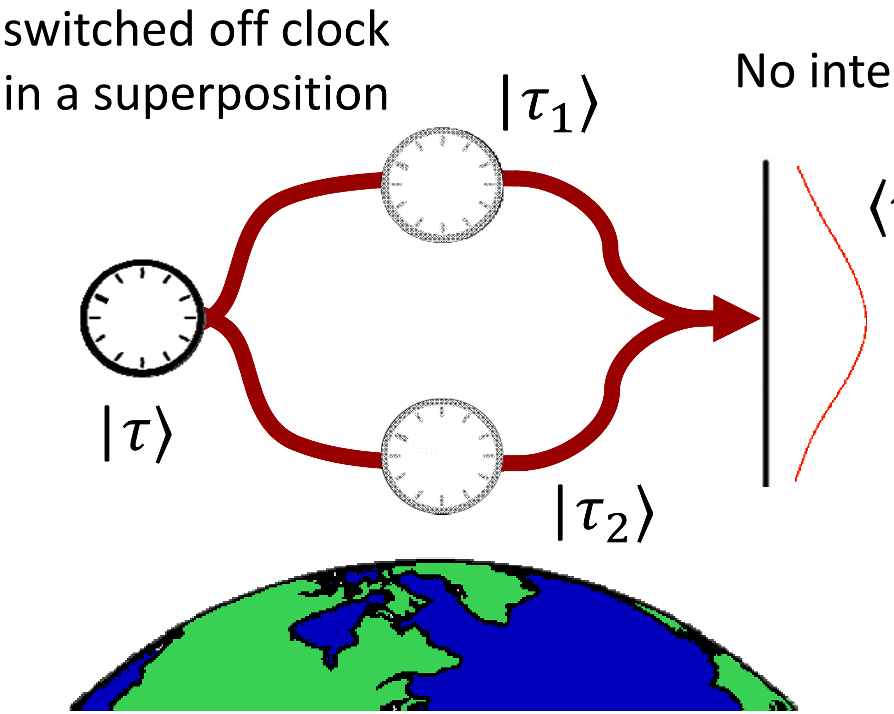
For $\omega = 10^{15}$Hz one needs
$\Delta x t = 10\, m \cdot s$ (still challenging)

Not explanaible without:

- gravity as metric theory,
- proper time τ flows at different rates – time dilation
- iff a particle is an operationally well defined „clock“

# Testing alternative theories
## (proper time as a degree of freedom)

$[\hat{\tau}, \hat{m}] = i\hbar$ Mass and proper time as conjugate observables




- Proper time as a hidden additional quantum degree of freedom?
- Internal „clock“? „Compton clock“?

D. M. Greenberger, J. Math. Physics **11**, 2329 (1970), ibid **11**, 2341 (1970).

# Testing the couplings

| Coupling | Effect | What is needed? | Done? |
|---|---|---|---|
| $m\phi(x)/c^2$ | Newtonian gravity | Localized rock | Yes |
| $\hat{H}_{clock}\phi(x)/c^2$ | Classical time dilation | Localized clock | Yes |
| $m\phi(\hat{x})/c^2$ | Gravitationally induced phase | „Rock“ in a spatial superposition | Yes |
| $\hat{H}_{clock}\phi(\hat{x})/c^2$ | Visibility loss | Clock in a spatial superposition | **No** |

M. Zych, F. Costa, I. Pikovski, Č.B.,
Nature Communication 2:505, (2011)

M. Zych, F. Costa, I. Pikovski, T. C. Ralph and Č.B.,
Class. Quantum Grav. 29 224010 (2012) (Highlight of CQG 2012/2013)

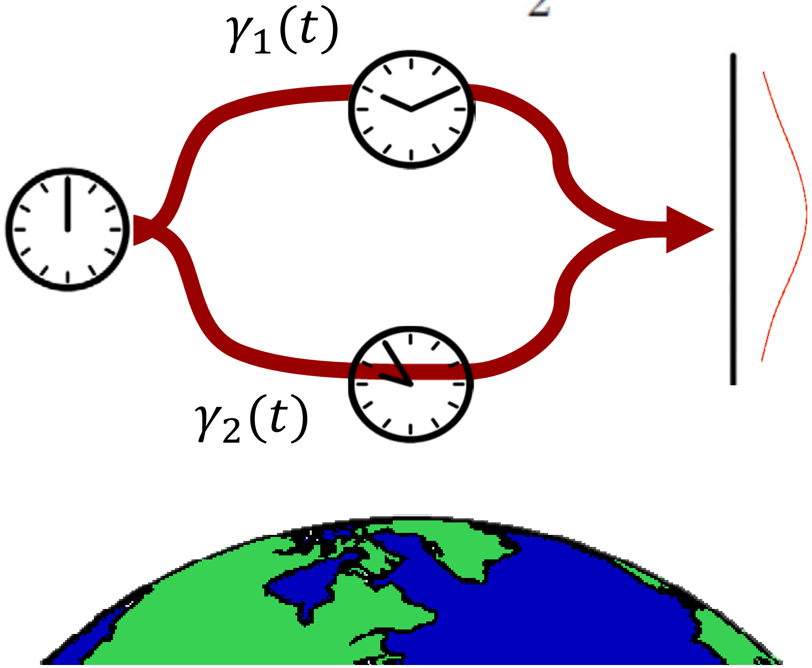
# Visibility modulation: general state

Center-of-motion degree of freedom

Initial state: $\rho(0) = \boxed{\rho_{cm}(0)} \otimes \boxed{\rho_{int}(0)}$

Internal degree-of-freedom

State at time t:
$$\rho(t) = \frac{1}{2}|\gamma_1(t)\rangle\langle\gamma_1(t)| \otimes \rho_{int,\gamma_1}(t) + \frac{1}{2}|\gamma_2(t)\rangle\langle\gamma_2(t)| \otimes \rho_{int,\gamma_2}(t)$$
$$+\frac{1}{2}\{|\gamma_2(t)\rangle\langle\gamma_1(t)| \otimes e^{-\frac{i}{\hbar}\int_{\gamma_2} H_{int} d\tau} \rho_{int}(0) e^{\frac{i}{\hbar}\int_{\gamma_1} H_{int} d\tau} + h.c.\},$$




$$\rho_{int,\gamma_i}(t) := e^{-\frac{i}{\hbar}\int_{\gamma_i} H_{int} d\tau} \rho_{int}(0) e^{\frac{i}{\hbar}\int_{\gamma_i} H_{int} d\tau}$$

$$\mathcal{V} = |\langle\gamma_2(t)|\mathrm{Tr}_{\mathrm{int}}[\rho(t)]|\gamma_1(t)\rangle|$$

$$\Downarrow$$

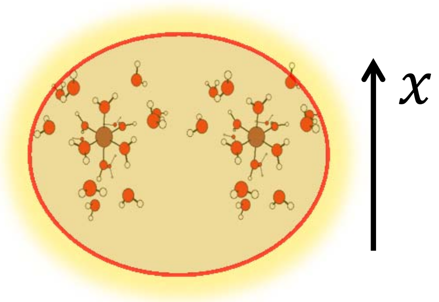
$$\boxed{\mathcal{V} = |\langle e^{-\frac{i}{\hbar}H_{int}\Delta\tau}\rangle_{int}|}$$

Proper time difference: $\Delta\tau = \int_{\gamma_2} d\tau - \int_{\gamma_1} d\tau$

# Composite system under time dilation

Model: Particle has N internal harmonic oscillators:

$$\hat{H}_{int} = \sum_{i=1}^{N} \hat{n}_i \hbar\omega_i$$




Each constituent in equilibrium at temperature T:

$$\rho_i = \frac{1}{\pi \bar{n}_i} \int d^2\alpha_i \, e^{-|\alpha_i|^2/\bar{n}_i} \, |\alpha_i\rangle\langle\alpha_i|$$

$$\bar{n}_i = (e^{\hbar\omega_i/k_B T} - 1)^{-1}$$

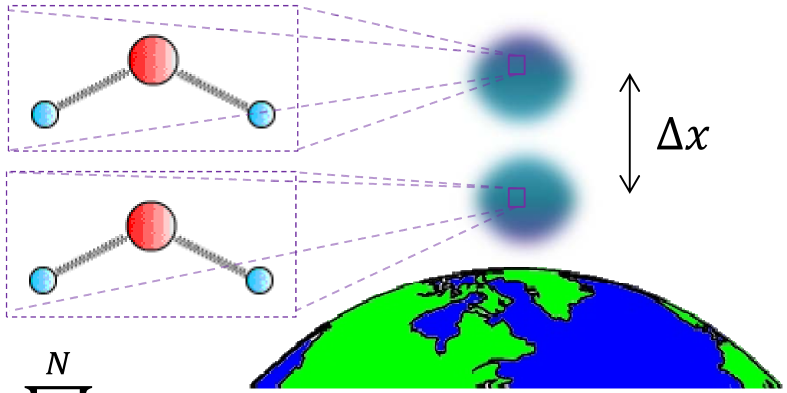
GR time-dilation induces interaction with center-of-mass position $\hat{x}$:

$$\hat{H} = mg\hat{x} + \frac{\hbar g \hat{x}}{c^2} \sum_{i=1}^{N} n_i \omega_i$$

# Decoherence due to time dilation

Initial superposition state:

$$|\psi_{cm}\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$$




Total state

$$\rho(0) = \frac{1}{2}(|x_1\rangle\langle x_1| + |x_2\rangle\langle x_2| + |x_2\rangle\langle x_1| + |x_1\rangle\langle x_2|) \otimes \prod_{i=1}^{N} \rho_i$$

Quantum coherence reduces:

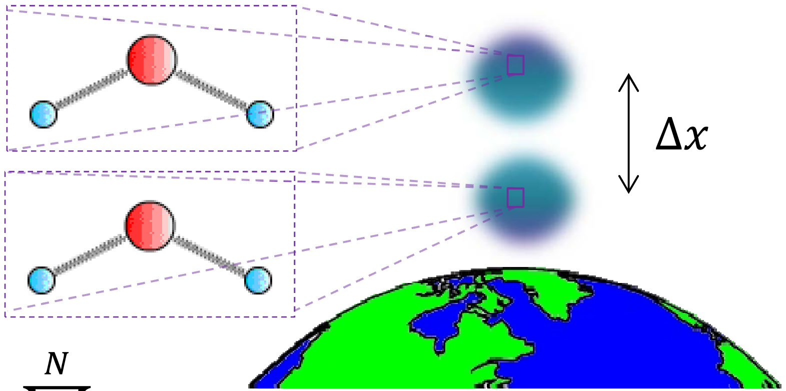
$$V(t) = \left|\langle x_1 | Tr_{int}^{\otimes N}[\rho(t)] | x_2\rangle\right| \approx e^{-\left(\frac{t}{\tau_{dec}}\right)^2}$$

$$\boxed{\tau_{dec} = \sqrt{\frac{2}{N}}\frac{\hbar c^2}{k_B T g \Delta x}}$$

| $N$ | $\tau_{dec}$ |
|---|---|
| 1 | $116\ days$ |
| 2424 | $3.5\ 10^4 s$ |
| $10^{18}$ | $1.7\ 10^{-5} s$ |
| $10^{23}$ | $10^{-6} s$ |

$T = 300K, \Delta x = 1mm, g = 10m/s^2$

# Decoherence due to time dilation




Initial superposition state:

$$|\psi_{cm}\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$$

Total state

$$\rho(0) = \frac{1}{2}(|x_1\rangle\langle x_1| + |x_2\rangle\langle x_2| + |x_2\rangle\langle x_1| + |x_1\rangle\langle x_2|) \otimes \prod_{i=1}^{N} \rho_i$$

Relativistic, quantum & thermodynamical effect

Quantum coherence reduces:

$$V(t) = \left|\langle x_1 | Tr_{int}^{\otimes N}[\rho(t)] | x_2\rangle\right| \approx e^{-\left(\frac{t}{\tau_{dec}}\right)^2}$$

$$\boxed{\tau_{dec} = \sqrt{\frac{2}{N}}\frac{\hbar c^2}{k_B T g \Delta x}}$$

- No collapse, regular QM+GR
- Proper time difference
- Different from Anastopoulos and B. L. Hu, Blancowe
- No „external“ environment

Igor Pikovski

# Master-equation

$$\dot{\rho}_{cm}(t) = \underbrace{-\frac{i}{\hbar}\left[H_{cm} + \left(m + \frac{Nk_BT}{c^2}\right)gx, \rho_{cm}(t)\right]}_{\text{Unitary part. „A piece of iron weighs more when red-hot than when cool"}} \underbrace{- Nt\left(\frac{k_BTg}{\hbar c^2}\right)^2 [x,[x,\rho_{cm}(t)]]}_{\text{Decoherence into position basis}}$$

Off-diagonal elements supressed:

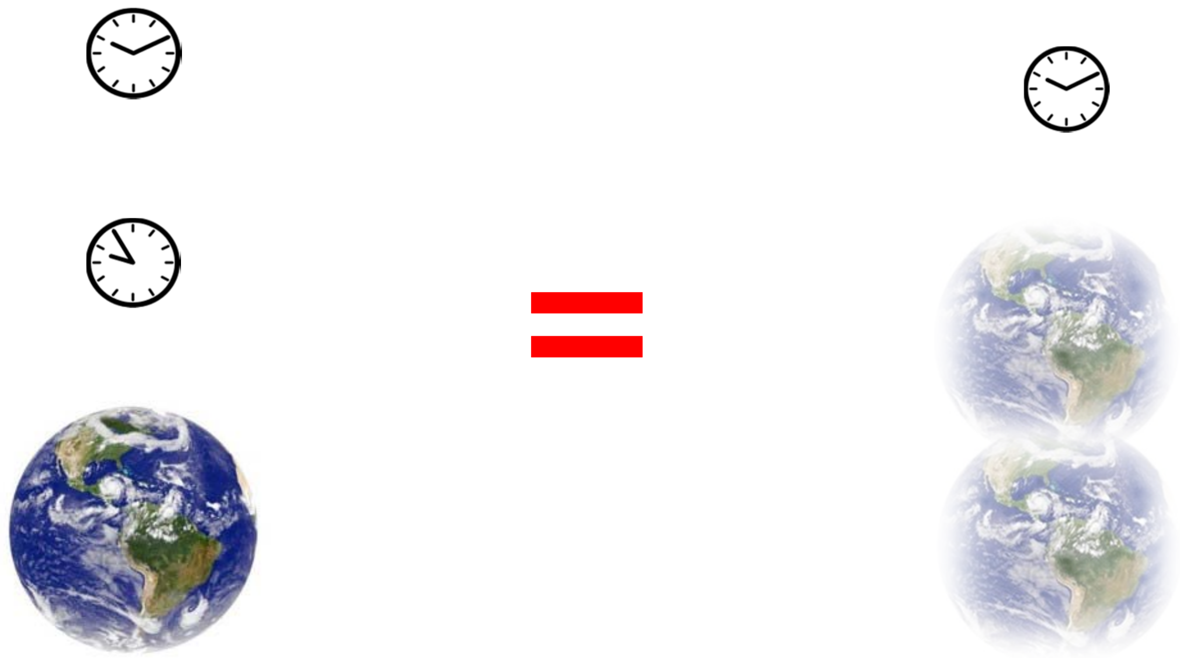
$$\langle x_1|\rho_{cm}(t)|x_2\rangle \sim \rho_{cm}(0)e^{-\left(\frac{t}{\tau_{dec}}\right)^2}$$

- No dissipation
- Gaussian decay
- Position pointer-basis

$$\boxed{\tau_{dec} = \sqrt{\frac{2}{N}}\frac{\hbar c^2}{k_BTg\Delta x}}$$

I. Pikovski, F. Costa, M. Zych, Č.B., Decoherence due to time dilation, ArXiv:1311.1095

# Quantum Causality

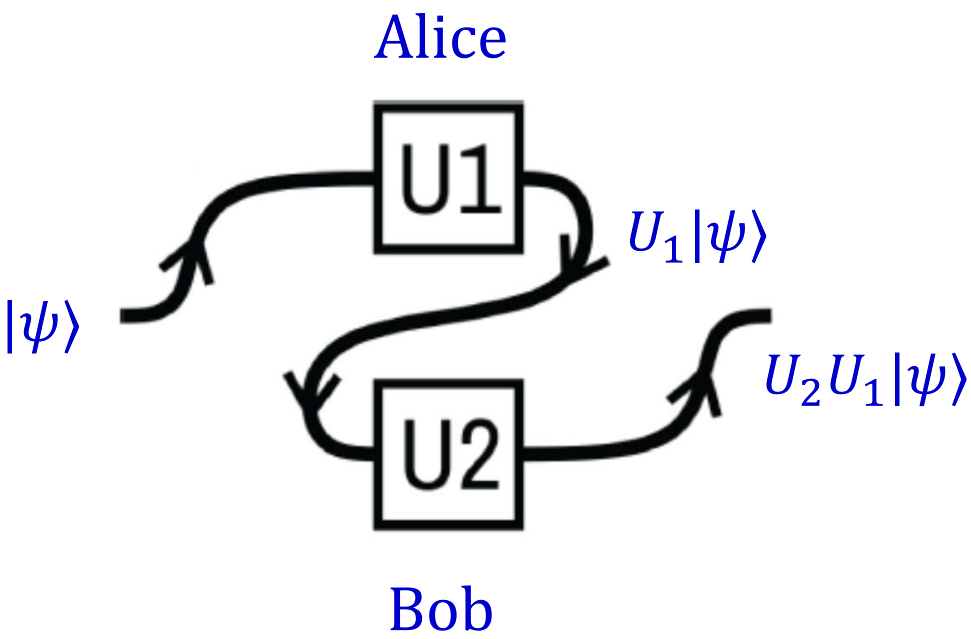


Superposition of a clock
Fixed backgroud

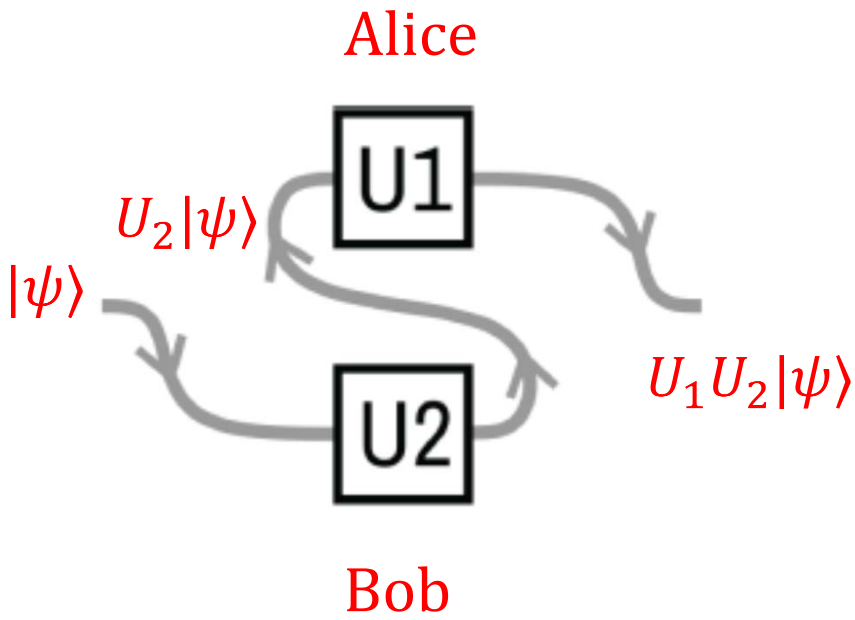
Localized clock
Superposition of backgrouds

Entanglement between clock's time and the relative positition to the mass

# Causal Processes




Channel from Alice to Bob

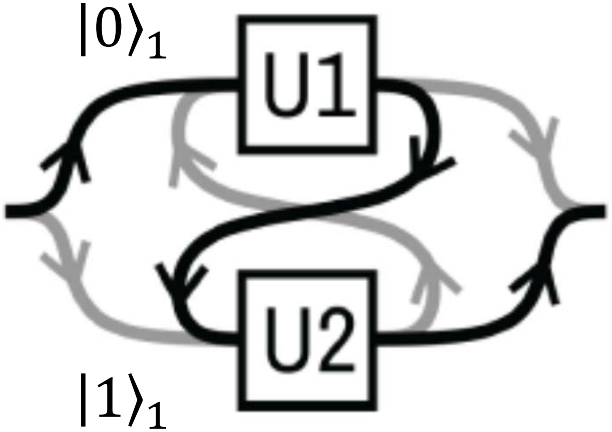



Channel from Bob to Alice

Time-like separated events;
Quantum circuits

# „Superposition of Quantum Circuits“

## Quantum control of gate order




Path degree of freedom (control)

$$\frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)|\psi\rangle_2 \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle_1 U_2 U_1 |\psi\rangle_2 + |1\rangle_1\, U_1 U_2 |\psi\rangle_2)$$

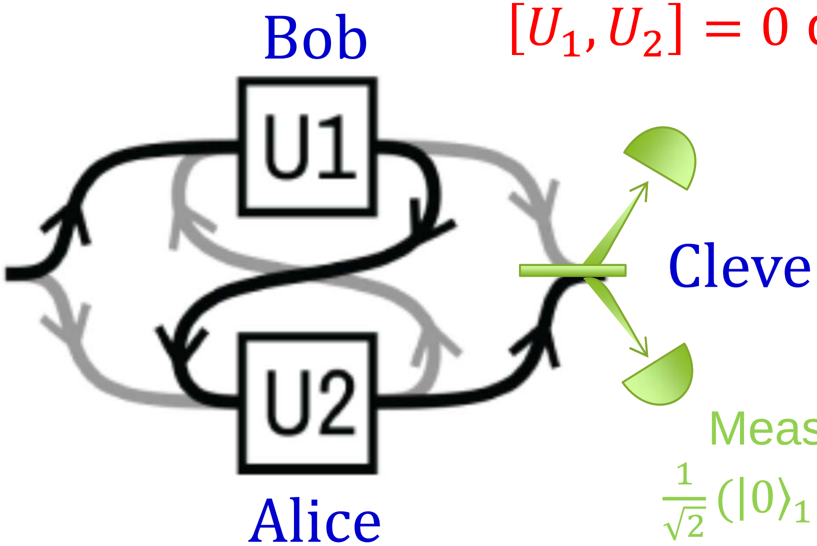
Internal degree of freedom

G. Chiribella, G.M. D'Ariano, P. Perinotti, & B. Valiron, Phys. Rev. A **88**, 022318 (2013).

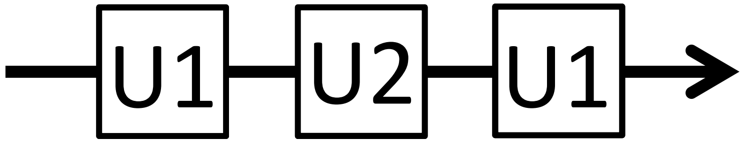
# Computational advantage

Promise:

$[U_1, U_2] = 0$ or $\{U_1, U_2\} = 0$




Measurement in $\frac{1}{\sqrt{2}}(|0\rangle_1 \pm |1\rangle_1)$ basis

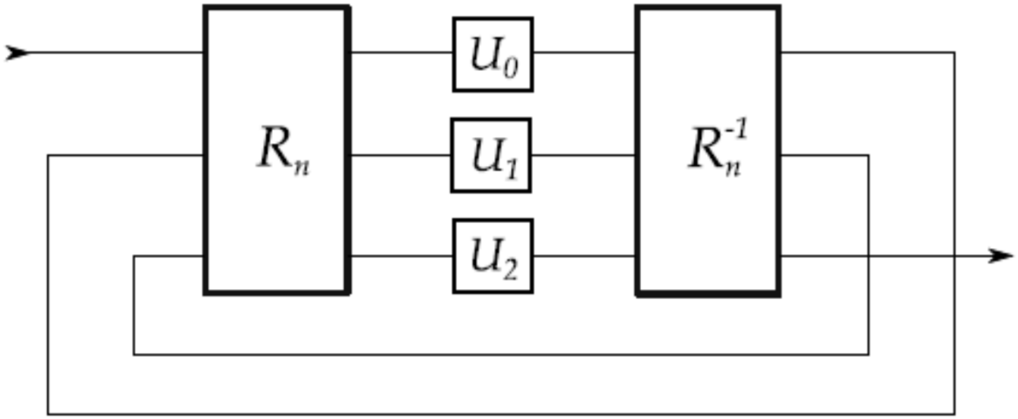



$$\frac{1}{\sqrt{2}}(|0\rangle_1 U_1 U_2 |\psi\rangle_2 + |1\rangle_1 \, U_2 U_1 |\psi\rangle_2) \longrightarrow |0\rangle_1 [\hat{U}_1, \hat{U}_2] |\psi\rangle_2 + |1\rangle_1 \{\hat{U}_1, \hat{U}_2\} |\psi\rangle_2$$

**Single** query of each gate enough.
Causal quantum algorithms require at least **two** queries of one of the gates.

G. Chiribella, G.M. D'Ariano, P. Perinotti, & B. Valiron, Phys. Rev. A **88**, 022318 (2013).

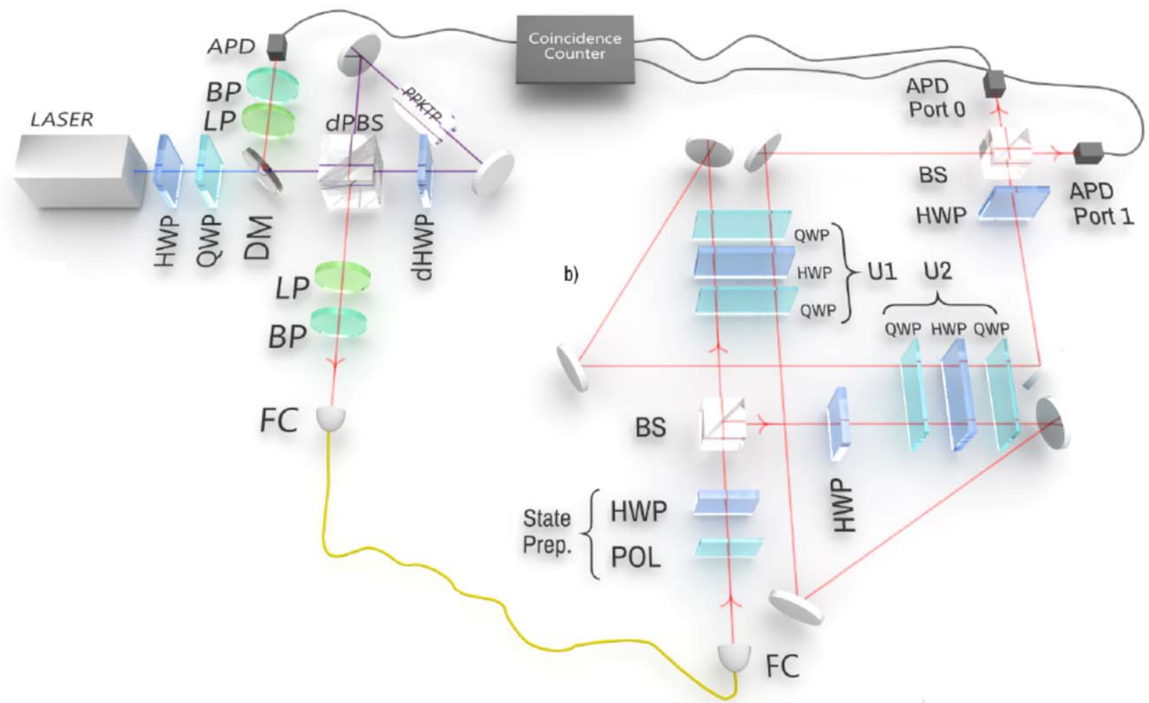
# Reduction of query complexity




Reduction of the **query complexity** from $O(n^2)$ to $O(n)$ for $n$ gates using quantum controlled ordering of gates.

M. Araujo, F. Costa, C.B., Phys. Rev. Lett. **113**, 250402 (2015) (Editor's Suggestion).

# Experimental Demonstration

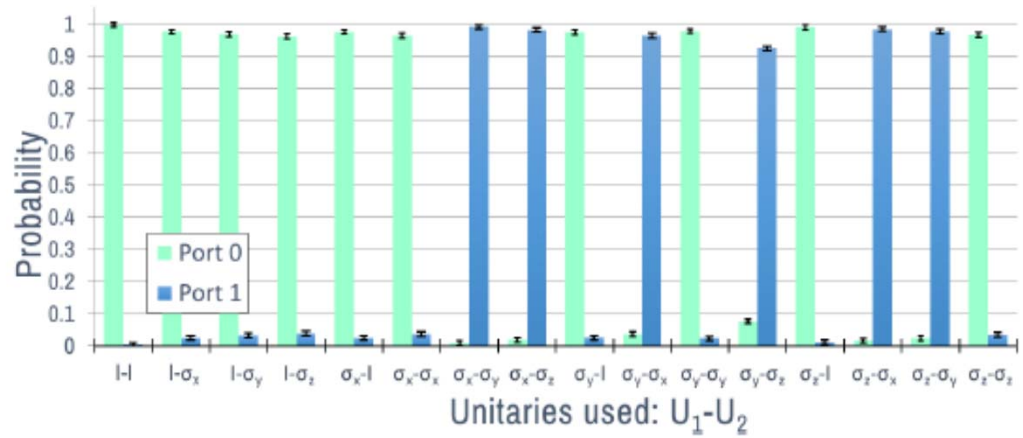



Probability of success

Average measured: $0.970 \pm 0.024$

Max. causal: 0.92

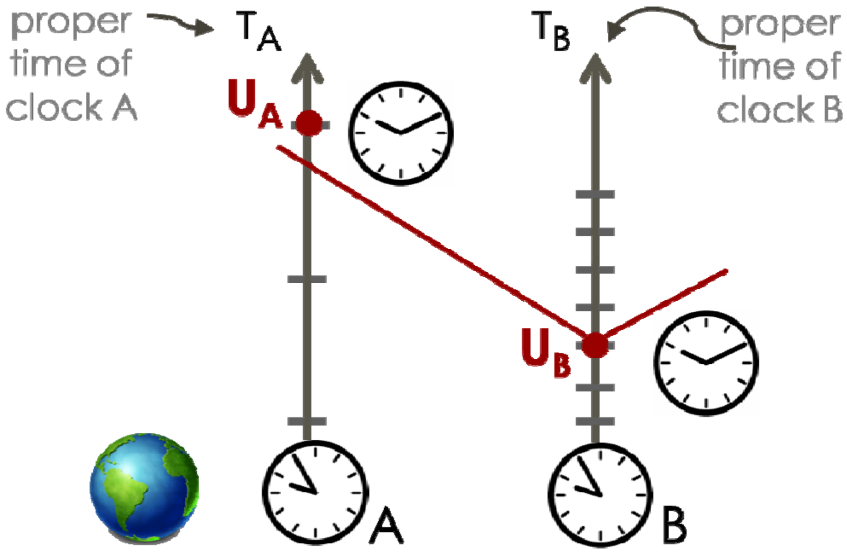
Group of P. Walther, University of Vienna, arXiv:1412.4006



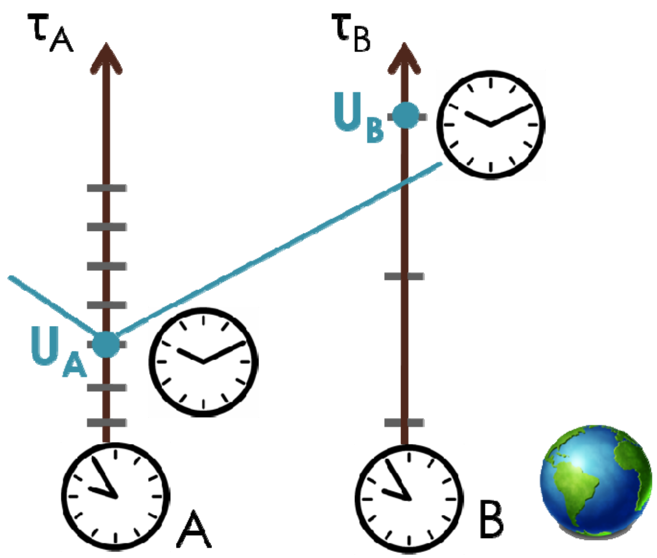
# Quantum control of gate order

## via superposition of masses

Consider two events $U_A$ and $U_B$ defined with respect to the clock A and B, respectively.




$U_A$ is in a causal future of $U_B$ when mass is placed closer to A

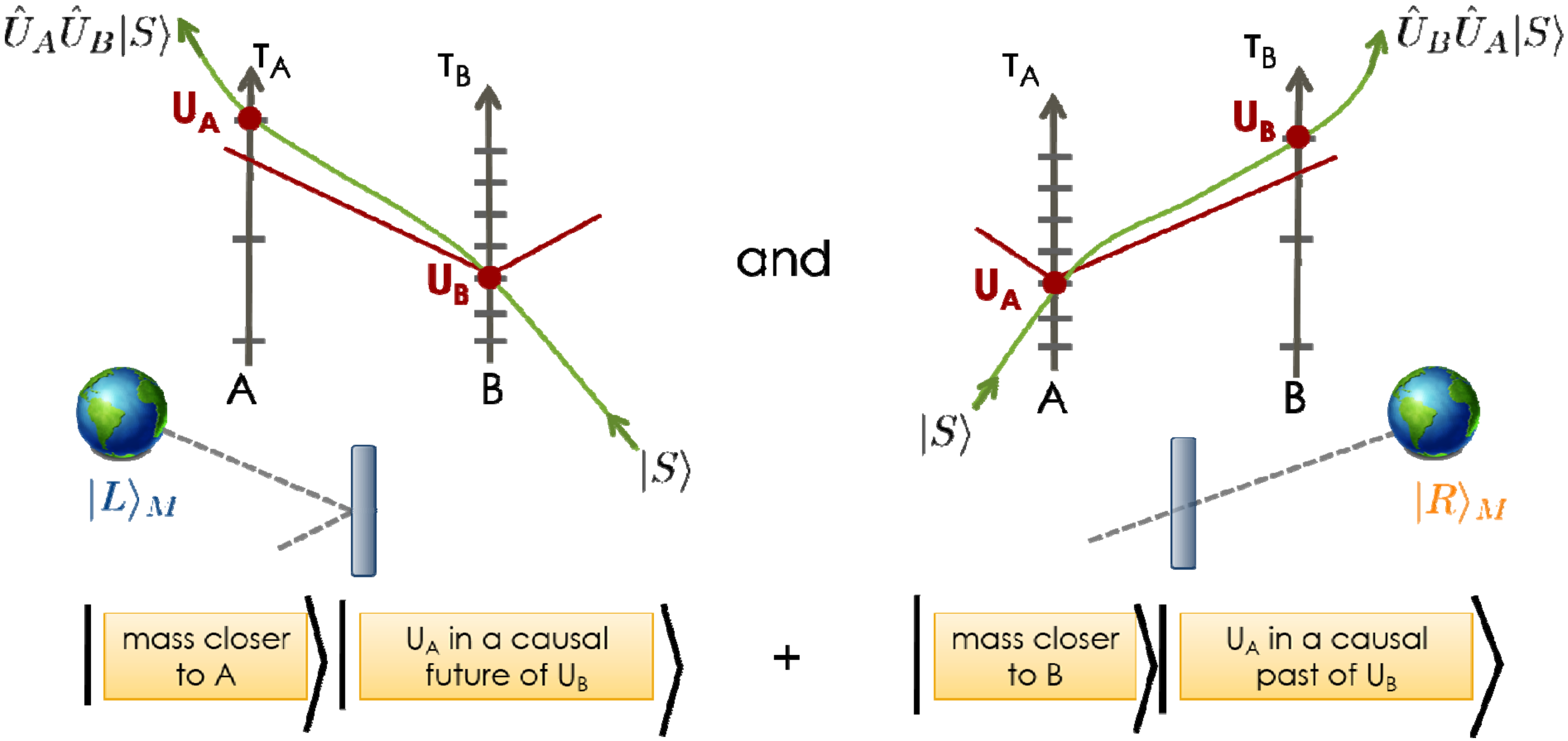



$U_A$ is in a causal past of $U_B$ when mass is placed closer to B

Causal order between events $U_A$, and $U_B$ can be changed due to gravitational time dilation.

# Quantum control of gate order

## via superposition of masses

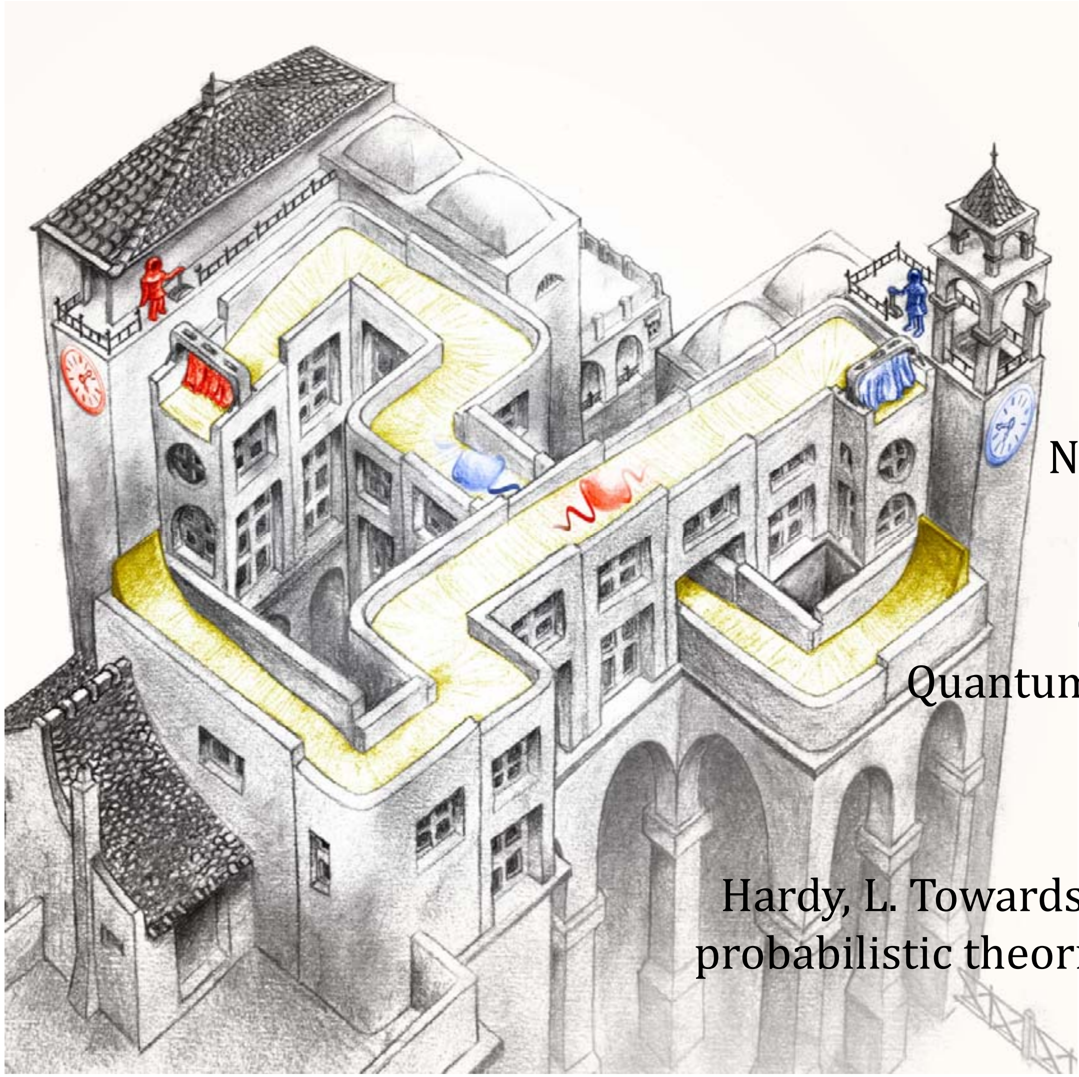



Mass in superposition: $(|L\rangle_M + |R\rangle_M)|S\rangle \rightarrow |L\rangle_M \hat{U}_A\hat{U}_B|S\rangle + |R\rangle_M \hat{U}_B\hat{U}_A|S\rangle$

The mass measured in the superposition basis :

$(|L\rangle_M \pm |R\rangle_M)(U_A U_B \pm\ U_B U_A)|S\rangle$ **=** „Superposition of quantum circuits“

# Quantum Causality




Brukner, C. Quantum Causality,
Nature Physics **10**, 259- 263 (2014).

Oreshkov, O., Costa, F. & Brukner, C.
Quantum correlations with no causal order,
Nature Commun. **3**, 1092 (2012).

Hardy, L. Towards quantum gravity: a framework for
probabilistic theories with non-fixed causal structure,
J. Phys. A **40**, 3081 (2007).


by Jonas Schmöle

# Summary

- Testing the overlap between QM and GR with clocks
    - interference visibility as a witness of GR time dilation
    - tests of alternative theories
    - decoherence due to time dilation

- Superpositions of Space-Time?
    - new resource for quantum computation
    - indefinite order of operations

- A generalization of concept of space-time? Relevance for quantum gravity?
- A new resource for quantum information processing?

# Thank you!


CoQuS Complex Quantum Systems

FWF Der Wissenschaftsfonds.

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FQXi

quantumfoundations.org